Aleksa Cvetković

TELEKOMUNIKACIJE 1

TELEKOMUNIKACIJE 1

- SPEKTRALNA ANALIZA PERIODIČNIH SIGNALA (FURIJEOVI REDOVI)

$X(t)$ – KONTINUALNI SIGNAL

$(\forall t) \quad X(t+T) = X(t)$

$e^{jt} = \cos t + j\sin t \quad ; \quad (e^{jt})' = je^{jt}$

T – OSNOVNI PERIOD (NAJMANJI), $2T, 3T, \ldots$

$\omega_0 = \frac{2\pi}{T}$ KRUŽNA UČESTANOST (OSNOVNA)

$\omega_n = n \cdot \omega_0$ KRUŽNA UČESTANOST VIŠIH HARMONIKA

$$\boxed{X(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}}$$

FURIJEOVI KOEFICIJENTI

$X(t) \longrightarrow X_n = ?$

$\cos t = \frac{e^{jt} + e^{-jt}}{2}$

$\sin t = \frac{e^{jt} - e^{-jt}}{2j}$

$n = 0$

$n = \pm 1$

$n = \pm 2, 3, \ldots$

SPEKTAR PERIODIČNIH FUNKCIJA JE DISKRETAN

$n = 0$
JEDNOSMERNA KOMPONENTA

$n = \pm 1$
OSNOVNI HARMONIK

$n = \pm 2, \ldots$
VIŠI HARMONICI

$\omega_0, 2\omega_0, 3\omega_0, \ldots$

- BESKONAČAN
- DISKRETAN
- EKVIDISTANTAN

ETF.BEASTWEB.ORG

$X_n = ?$

$$x(t)e^{-jm\omega_0 t} = \sum_{n=-\infty}^{\infty} X_n e^{j(n-m)\omega_0 t} \quad \Big/ \int_0^T dt$$

$$\int_0^T x(t)e^{-jm\omega_0 t}\,dt = \sum_{n=-\infty}^{\infty} \int_0^T X_n e^{j(n-m)\omega_0 t}\,dt = X_m \cdot T$$

$$\int_0^T e^{j(n-m)\omega_0 t}\,dt = \begin{cases} T, & n=m \\ 0, & n\neq m \end{cases}$$

$$X_m = \frac{1}{T}\int_0^T x(t)e^{-jm\omega_0 t}\,dt$$

KAD SE m ZAMENI SA n

$$\boxed{X_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}\,dt}$$

(INTEGRAL NE MORA IĆI OD 0 DO T, MOŽE I $-T/2$ DO $T/2$ ...)

$n=0$ $\quad X_0 = \frac{1}{T}\int_0^T x(t)\cancel{e^{-j0\omega_0 t}}^{\,1}\,dt \Rightarrow X_0 = X_{SR}$

$x(t) \rightarrow$ REALAN SIGNAL $\Rightarrow$ $x^*(t) = x(t)$

$$\sum_{n=-\infty}^{\infty} X_n^* e^{-jn\omega_0 t} = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$$\boxed{X_{-n} = X_n^*}$$

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$$X_n = |X_n| e^{j \arg X_n}$$

FAZNI SPEKTAR

AMPLITUDSKI SPEKTAR

$$\left.\begin{aligned} |X_n| &= |X_{-n}| \\ \arg X_n &= -\arg(X_{-n}) \end{aligned}\right\}$$

MODUO - PARNA F-JA

ARGUMENT - NEPARNA

- PARSEVALOVA TEOREMA

$X(t) = i(t)$

$P_R(t) = R\, i^2(t)$ $(R = 1\Omega)$

$P(t) = x^2(t)$ TRENUTNA SNAGA

$$E = \int_0^T P(t)\,dt = \int_0^T x^2(t)\,dt$$ ENERGIJA JEDNOG PERIODA

$$\int_0^T x^2(t)\,dt = \sum_{n=-\infty}^{\infty} (X_n)^2$$

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(INTEGRAL NE MORA IĆI OD 0 DO T, MOŽE I $-T/2$ DO $T/2$ ...)

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$$\boxed{X_{-n} = X_n^*}$$

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$$X_n = |X_n| e^{j \arg X_n}$$

FAZNI SPEKTAR

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1. PERIODIČNA POVORKA UNIPOLARNIH PRAVOUGAONIH IMPULSA

$X(t) = \Pi_p(t)$

$\Pi_n = ?$

$\omega_0 = \frac{2\pi}{T}$

$$\Pi_n = \frac{1}{T}\int_{-T/2}^{T/2} \Pi_p(t) e^{-jn\omega_0 t}\,dt = \frac{1}{T} E \int_{-\tau/2}^{\tau/2} e^{-jn\omega_0 t}\,dt$$

$$\Pi_n = \frac{E}{T} \left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_{-\tau/2}^{\tau/2} = \frac{E}{T} \frac{\underbrace{e^{jn\omega_0 \tau/2} - e^{-jn\omega_0 \tau/2}}_{= 2j\sin\left(\frac{n\omega_0\tau}{2}\right)}}{jn\omega_0}$$

$$\Pi_n = \frac{2E}{T} \frac{\sin\frac{n\omega_0\tau}{2}}{n\omega_0} = E\frac{\tau}{T} \frac{\sin\frac{n\omega_0\tau}{2}}{\frac{n\omega_0\tau}{2}}$$

$$\frac{\sin t}{t} \overset{def}{=} \operatorname{sinc} t$$

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$$|\Pi_n| = E\frac{\tau}{T}\left|\frac{\sin\frac{n\omega_0\tau}{2}}{\frac{n\omega_0\tau}{2}}\right|$$

$$\frac{n\omega_0\tau}{2} = \pi \qquad n\omega_0 = \frac{2\pi}{\tau} \qquad \omega_0 = \frac{2\pi}{T}$$

$$\frac{\omega}{\omega_0} = \frac{T}{\tau}$$

T - VIŠE NA GUSTINU

τ - NA ŠIRINU

$\tau \nearrow \rightarrow \Delta\omega \swarrow$

$\tau \swarrow \rightarrow \Delta\omega \nearrow$

HAJZENBERGOVE RELACIJE NEODREĐENOSTI

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② PERIODIČNA POVORKA DIRAKOVIH IMPULSA

DIRAKOVA F-JA

$$\delta(t) = \begin{cases} +\infty, & t=0 \\ 0, & t\neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\delta(t) = \frac{du(t)}{dt}$$

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\,dt$$

$$\int_{-\infty}^{\infty} \delta(t-t_0)\,f(t)\,dt = f(t_0)$$

$$\frac{2}{\tau}\cdot\frac{\tau}{2} = 1$$

$$\tau\cdot\frac{1}{\tau} = 1$$

$$\left(\frac{\tau}{n}, \frac{n}{\tau}\right) \xrightarrow[\to \delta(t)]{n\to\infty}$$

$\delta_p(t)$

$$\Delta_n = \frac{1}{T}\int_{-T/2}^{T/2} \delta_p(t)\, e^{-jn\omega_0 t}\,dt = \frac{1}{T}\cdot 1 = \frac{1}{T} = \text{const}$$

ISTO KAO I $\delta(t)$

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AMPLITUDSKI SPEKTAR JE KONSTANTAN I IZNOSI $\frac{1}{T}$

FAZNI SPEKTAR JE NULA

- POMERANJE PERIODIČNOG SIGNALA U VREMENU

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$$y(t) = x(t-t_0) = \sum_{n=-\infty}^{\infty} Y_n e^{jn\omega_0 t}$$

$$x(t-t_0) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0(t-t_0)} = \sum_{n=-\infty}^{\infty} \underbrace{\left(X_n e^{-jn\omega_0 t_0}\right)}_{Y_n} e^{jn\omega_0 t}$$

$$Y_n = X_n e^{-jn\omega_0 t_0} \rightarrow \boxed{\begin{aligned} &|X_n| = |Y_n| \\ &\arg Y_n = \arg X_n - n\omega_0 t_0 \end{aligned}}$$

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3.

$X_n = ?$

I NAČIN

$$X_n = \frac{1}{T}\int_0^T X(t)\, e^{-jn\omega_0 t}\, dt \qquad \omega_0 = \frac{2\pi}{T}$$

$$X_n = \frac{E}{T}\int_{t_1}^{t_2} e^{-jn\omega_0 t}\, dt = \frac{E}{T}\, \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Bigg|_{t_1}^{t_2}$$

$$X_n = \frac{E}{T}\, \frac{e^{-jn\omega_0 t_1} - e^{-jn\omega_0 t_2}}{jn\omega_0}$$

$$X_n = \frac{E}{T}\, e^{-jn\omega_0 \frac{t_1+t_2}{2}} \left[\frac{\overbrace{e^{-jn\omega_0 \frac{(t_1-t_2)}{2}} - e^{-jn\omega_0 \frac{(t_2-t_1)}{2}}}^{2\cancel{j}\sin \frac{n\omega_0(t_2-t_1)}{2}}}{\cancel{j}n\omega_0} \right]$$

$$X_n = \frac{2E}{T}\, \frac{\sin \frac{n\omega_0(t_2-t_1)}{2}}{n\omega_0}\, e^{-jn\omega_0 \frac{(t_1+t_2)}{2}}$$

$$X_n = E\, \frac{t_2-t_1}{T}\, \frac{\sin \frac{n\omega_0(t_2-t_1)}{2}}{\frac{n\omega_0(t_2-t_1)}{2}}\, e^{-jn\omega_0 \frac{(t_2+t_1)}{2}}$$

$$|X_n| = E\, \frac{t_2-t_1}{T} \left| \frac{\sin \frac{n\omega_0(t_2-t_1)}{2}}{\frac{n\omega_0(t_2-t_1)}{2}} \right| \qquad (t_2-t_1=\tau)$$

$$\arg X_n = -\frac{n\omega_0(t_2+t_1)}{2} \quad \text{I}(0,\pi)$$

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II NAČIN:

$\tau = t_2 - t_1$

$$X(t) \longrightarrow X(t-t_0)$$

$$X_n \longrightarrow X_n e^{-jn\omega_0 t}$$

$X_n = \cdots$ 12 FORMULE

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4. BIPOLARNI PRAVOUGAONI IMPULS

PREDSTAVIĆEMO SIGNAL KAO ZBIR DVA SIGNALA

$$X(t)=X_1(t)+X_2(t) \rightarrow X_n=X_{n1}+X_{n2}$$

$$X_n=\frac{E\tau/2}{T}\frac{\sin\frac{n\omega_0\tau}{4}}{\frac{n\omega_0\tau}{4}}e^{j\frac{\omega_0 n\tau}{4}}-\frac{E\tau/2}{T}\frac{\sin\frac{n\omega_0\tau}{4}}{\frac{n\omega_0\tau}{4}}e^{-j\frac{\omega_0 n\tau}{4}}$$

$$X_n=\frac{E\tau}{\cancel{2}T}\frac{\sin\frac{n\omega_0\tau}{2}}{\frac{n\omega_0\tau}{4}}\underbrace{\left(e^{j\frac{n\omega_0\tau}{4}}-e^{-j\frac{n\omega_0\tau}{4}}\right)}_{\cancel{2}j\sin\frac{\omega_0 n\tau}{4}}$$

$$X_n=\frac{E\tau}{T}\frac{j\sin^2\frac{n\omega_0\tau}{4}}{\frac{n\omega_0\tau}{4}}$$

POŠTO JE U OVOM SLUČAJU $\frac{\sin^2 x}{x}$ NEĆE BITI JEDNOSMERNE KOMPONENTE

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$$|X_n| = \frac{E\tau}{T}\,\frac{\sin^2\frac{n\omega_0\tau}{4}}{\left|\frac{n\omega_0\tau}{4}\right|} \qquad \arg X_n = \pi/2 \pm (0,\pi)$$

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– SPEKTRALNA ANALIZA APERIODIČNIH SIGNALA (FURIJEOVA TRANSFORMACIJA)

$$X(t+T) = X(t) \; (\forall t) \qquad W_0 = \frac{2\pi}{T}$$

$$\left.\begin{aligned} X(t) &= \sum_{n=-\infty}^{\infty} X_n e^{jnw_0 t} \\ X_n &= \frac{1}{T}\int_{-T/2}^{T/2} X(t) e^{-jnw_0 t} dt \end{aligned}\right\} \quad \begin{aligned} X_{-n} &= X_n^* \\ X(t) &\to X_n \end{aligned}$$

APERIODIČNA FUNKCIJA

NE POSTOJI T

$\exists T \to \infty$

$W_0 = \frac{2\pi}{T} \to 0$

$\Delta W = W_0 \to 0 \Rightarrow$ DISKRETNI SPEKTAR PRELAZI U KONTINUALNI

$$\sum_{-\infty}^{\infty} \to \int_{-\infty}^{\infty} dw$$

$X_n \to 0$

$$T \cdot X_n = \int_{-T/2}^{T/2} X(t) e^{-jnw_0 t} dt \quad , \; T \to +\infty$$

$$T X_n = \int_{-\infty}^{\infty} X(t) e^{-jnw_0 t} dt$$

NOVA AMPLITUDA

$$X_n \to T X_n \qquad X(-jw) = X^*(jw)$$

$T X_n = X(jw)$ – SPEKTRALNA GUSTINA

$$X(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} (T X_n) e^{jnw_0 t} = \frac{1}{2\pi}\sum_{-\infty}^{\infty} (T X_n) e^{jnw_0 t} \Delta w$$

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$$\frac{\omega_0}{2\pi} = \frac{\Delta\omega}{2\pi}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \quad \left[FT^{-1}(X(j\omega))\right]$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \quad FT(x(t))$$

FURIJEOVA TRANSFORMACIJA

- OSOBINE FT:

1) LINEARNOST

$$C_1 x_1(t) + C_2 x_2(t) \rightarrow C_1 X_1(j\omega) + C_2 X_2(j\omega)$$

2) POMERANJE U VREMENU

$$x(t-t_0) \rightarrow X(j\omega)e^{-j\omega t_0}$$

3) POMERANJE U FREKVENTNOM DOMENU

$$e^{j\omega_0 t} \cdot x(t) \longrightarrow X(j(\omega-\omega_0))$$

4) IZVOD U VREMENSKOM DOMENU

$$\frac{dx(t)}{dt} \longrightarrow j\omega X(j\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \longrightarrow \frac{dx}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} (j\omega X(j\omega))e^{j\omega t}d\omega$$

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5) INTEGRAL U VREMENSKOM DOMENU

$$\int_0^t x(t)\,dt \longrightarrow \frac{X(j\omega)}{j\omega}$$

1. USAMLJENI PRAVOUGAONI IMPULS

$\Pi(j\omega) = ?$

$$\Pi(j\omega) = \int_{-\infty}^{+\infty} \Pi(t) e^{-j\omega t}\,dt = E\int_{-\tau/2}^{\tau/2} e^{-j\omega t}\,dt$$

$$\Pi(j\omega) = E\cdot \frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-\tau/2}^{\tau/2} = E\,\frac{e^{-j\omega\frac{\tau}{2}} - e^{j\omega\frac{\tau}{2}}}{-j\omega} = E\,\frac{e^{j\omega\frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}}}{j\omega}$$

$$E\,\frac{\cancel{2j}\sin\frac{\omega\tau}{2}}{\cancel{j}\omega}$$

$$\boxed{\Pi(j\omega) = E\tau\,\frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}}$$

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② USAMLJENI DELTA IMPULS

$\Delta(j\omega) = ?$

$$\Delta(j\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1 \qquad \Delta(j\omega) = 1$$

UNIFORMNO KONSTANTNA

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Delta(j\omega) e^{j\omega t} d\omega \qquad \arg(\Delta(j\omega)) = 0$$

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} d\omega$$

FURIJEOVA REPREZENTACIJA DELTA FUNKCIJE

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$$\int_{-\infty}^{\infty} e^{j\omega t}\, d\omega = ?$$

$$\left.\frac{e^{j\omega t}}{jt}\right|_{-\infty}^{\infty} = \frac{e^{j\infty} - e^{-j\infty}}{jt} = \frac{2\cancel{j}\sin\infty}{\cancel{j}t}$$

NE MOŽE TAKO

$$\boxed{\int_{-\infty}^{\infty} e^{j\omega t}\, d\omega = 2\pi\delta(t)}$$

$$\int e^{j\omega t} = \int(\cos\omega t + \cancel{j\sin\omega t})$$

$$\int_{-\infty}^{\infty} \sin\omega t\, d\omega = 0 \qquad \int_{-\infty}^{\infty} \cos\omega t\, d\omega = ?$$

$$\int_{-\infty}^{\infty} \cos\omega t\, d\omega = 2\pi\delta(t)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

DIRIHLEOVI USLOVI:

$$1^\circ \int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

$$2^\circ \int_{-\infty}^{\infty} x^2(t)\, dt < \infty$$

KONAČNA SNAGA I KONAČNA ENERGIJA

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– FURIJEOVA TRANSFORMACIJA PERIODIČNIH SIGNALA

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$$\overset{?}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$X(j\omega) = 2\pi \sum_{n=-\infty}^{\infty} X_n \delta(\omega - n\omega_0)$$

③ SPEKTRALNA GUSTINA $x(t) = \cos\omega_0 t$, $X(j\omega) = ?$

$$X(j\omega) = 2\pi\left(\frac{1}{2}\delta(\omega-\omega_0) + \frac{1}{2}\delta(\omega+\omega_0)\right) \; ; \; x(t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$$X_1 = \frac{1}{2}, \; X_{-1} = +\frac{1}{2}$$

$$\boxed{X(j\omega) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)} \qquad X_{k\neq\pm1} = 0$$

SPEKTRALNA GUSTINA KOSINUSA

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4.

$x(t) = E = \text{const}$ KONSTANTNI SIGNAL

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = E \underbrace{\int_{-\infty}^{\infty} e^{-j\omega t} dt}_{= 2\pi\delta(\omega)} = E \left. \frac{e^{-j\omega t}}{-j\omega} \right|_{-\infty}^{\infty} = \dots$$

OVAJ NAČIN DAJE NEODREĐENO REŠENJE!

Ex NIJE KONAČNA, NIJE ZADOVOLJEN DIRIHLEOV USLOV

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega t} d\omega$$

$$\int_{-\infty}^{\infty} e^{-j\omega t} d\omega = 2\pi\delta(t)$$

$$\Downarrow$$

$$\int_{-\infty}^{\infty} e^{-j\omega t} dt = 2\pi\delta(\omega)$$

$$\boxed{X(j\omega) = 2\pi E \delta(\omega)}$$

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DUALNOST:

$$x(t) \longrightarrow X(j\omega)$$

$$x(t) \longrightarrow 2\pi X(j\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

SIMETRIJA FORMULE

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

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5. $x(t) = \mathrm{sinc}(t) = \frac{\sin t}{t}$

$X(j\omega) = ?$

$$E(x) = E_x = \int_{-\infty}^{\infty} \frac{\sin^2 t}{t^2}\,dt = 2\int_{0}^{\infty} \frac{\sin^2 t}{t^2}\,dt < \infty$$

ZADOVOLJEN D. USLOV

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} \frac{\sin t}{t}\, e^{-j\omega t}\,dt$$

PREKO DUALNOSTI

NIJE VREMENSKO τ

$$X(j\omega) = 2\pi\left(u(\omega+1) - u(\omega-1)\right)$$

VREDNOST INTEGRALA

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⑥ EKSPONENCIJALNI SIGNAL

$$x(t) = \begin{cases} e^{-\lambda t}, & t>0 \\ 0, & t<0 \end{cases} \qquad \lambda>0$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} \cdot x(t) = \int_{0}^{\infty} e^{-(\lambda+j\omega)t}\, dt$$

$$X(j\omega) = \frac{e^{-(\lambda+j\omega)t}}{-(\lambda+j\omega)}\Bigg|_{0}^{\infty} = \frac{1-\cancel{e^{-(\lambda+j\omega)\cdot\infty}}^{\,0}}{\lambda+j\omega} = \frac{1}{\lambda+j\omega}$$

$$X(j\omega) = \frac{1}{\lambda+j\omega} \qquad |X(j\omega)| = \frac{1}{\sqrt{\lambda^2+\omega^2}}$$

$$\arg(X(j\omega)) = -\operatorname{arctg}\frac{\omega}{\lambda}$$

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7. HEVISAJDOVA FUNKCIJA

$$x(t)=u(t)=\begin{cases}1, & t>0\\ 0, & t<0\end{cases}$$

$$E_x=\int_0^\infty 1\,dt=\infty$$

NIJE ZADOVOLJEN D. USLOV

PA NE MOŽE PREKO INTEGRALA

IZ PRETHODNOG PRIMERA VIDIMO DA ZA PARAMETAR $\lambda\to 0$ DOBIJAMO DA $e^{-\lambda t}\to 1$

$\lambda\to 0$

$$X(j\omega)=\frac{1}{\lambda+j\omega}=\frac{1}{j\omega}$$

$$X(j\omega)=\frac{1}{j\omega}\quad,\ \omega\neq 0$$

ZA $\omega=0$ PRIKAZAĆEMO HEVISAJDOV SIGNAL KAO ZBIR DVA SIGNALA

$$U(j\omega)=X_1(j\omega)+X_2(j\omega)$$

$$\omega=0\qquad U(j0)=\underbrace{X_1(j0)}_{2\pi\cdot\frac{1}{2}\delta(0)}+\underbrace{X_2(j0)}_{0}$$

$$X_2(j\omega)=\int_{-\infty}^{\infty}\cancelto{0}{x_2(t)}\,dt\,e^{-j\omega t}$$

NEPARNA F-JA

$$U(j0)=\pi\delta(0)$$

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$$X(j\omega) = \begin{cases} \frac{1}{j\omega}, & \omega \neq 0 \\ \pi\delta(0), & \omega = 0 \end{cases}$$

$$X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

ZAPIS U PREDMETU SIGNALI I SISTEMI

ETF.BEASTWEB.ORG

8. $x(t) = \operatorname{sgn} t$

$X(j\omega) = ?$

$X(j0) = 0$

ZA $\omega \neq 0$ PREDSTAVITI KAO ZBIR DVA SIGNALA (ILI RAZLIKU)

$$X(j\omega) = \begin{cases} \frac{2}{j\omega}, & \omega \neq 0 \\ 0, & \omega = 0 \end{cases}$$

NEMA CENTRALNU TAČKU

II NAČIN

$u(t),\ \delta(t) \qquad \delta(t) = \frac{du(t)}{dt}$

$\rightarrow \Delta(j\omega) = j\omega U(j\omega)$

$\downarrow$

$U(j\omega) = \frac{1}{j\omega},\ \omega \neq 0$

ETF.BEASTWEB.ORG

9. BIPOLARNI PRAVOUGAONI IMPULS

$$x(t) = x_1(t) + x_2(t)$$

$$X(j\omega) = \underbrace{\frac{E\tau}{2}\,\frac{\sin\frac{\omega\tau}{4}}{\frac{\omega\tau}{4}}\,e^{j\frac{\omega\tau}{4}}}_{X_1(j\omega)} - \underbrace{E\frac{\tau}{2}\,\frac{\sin\frac{\omega\tau}{4}}{\frac{\omega\tau}{4}}\,e^{-j\omega\frac{\tau}{4}}}_{X_2(j\omega)}$$

$$X(j\omega) = \frac{E\tau}{2}\,\frac{\sin\frac{\omega\tau}{4}}{\frac{\omega\tau}{4}}\left[\underbrace{e^{j\frac{\omega\tau}{4}} - e^{-j\omega\frac{\tau}{4}}}_{2j\sin\frac{\omega\tau}{4}}\right]$$

$$X(j\omega) = \underbrace{j}_{e^{j\pi/2}}\,\frac{E\tau\sin^2\frac{\omega\tau}{4}}{\frac{\omega\tau}{4}}$$

0

1 POZITIV. 1 NEG.

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10. USAMLJENI TROUGAONI IMPULS

$$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt=\int_{-\tau/2}^{0}\left(\frac{2E}{\tau}\cdot t+E\right)e^{-j\omega t}dt$$

$$+\int_{0}^{\tau/2}\left(E-\frac{2E}{\tau}t\right)e^{-j\omega t}dt=\dots$$

II NAČIN, PREKO IZVODA

$$\frac{dx}{dt}=\begin{cases}\frac{2E}{\tau}, & t\in[-\tau/2,0]\\ -\frac{2E}{\tau}, & t\in[0,\tau/2]\end{cases}$$

$$X(j\omega)=\frac{\cancel{j}2E\sin^2\frac{\omega\tau}{4}}{\cancel{j}\omega\frac{\omega\tau}{4}}=\frac{2E}{\tau}\frac{\sin^2\frac{\omega\tau}{4}}{4\frac{\omega^2\tau^2}{16}}$$

$$X(j\omega)=\frac{E\tau}{2}\frac{\sin^2\frac{\omega\tau}{4}}{\frac{\omega^2\tau^2}{16}}$$

FAZNI NULA!

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(11) $x(t) = \begin{cases} \cos w_0 t & , t \in [-\tau/2, \tau/2] \\ 0 & , \text{INACE} \end{cases}$

$$x(t) = \Pi(t) \cos w_0 t$$

$$\downarrow$$

$$X(jw) = \int_{-\infty}^{\infty} (\Pi(t) \cos w_0 t) e^{-jwt} dt$$

$$X(jw) = \frac{1}{2} \left(\int_{-\infty}^{\infty} \Pi(t) e^{-j(w-w_0)t} dt + \int_{-\infty}^{\infty} \Pi(t) e^{-j(w+w_0)t} dt \right)$$

DOMERANJE U FREKVENTNOM DOMENU

$$X(jw) = \frac{1}{2} \left(\Pi(j(w-w_0)) + \Pi(j(w+w_0)) \right)$$

$\tau \to \infty$ NISU ČISTI TONOVI

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LTI - SISTEM

$$x(t) \longrightarrow \boxed{} \longrightarrow y(t)$$

1) LINEARNOST

$$C_1 x_1(t) + C_2 x_2(t) \longrightarrow C_1 y_1(t) + C_2 y_2(t)$$

2) VREMENSKA INVARIJANTNOST

$$x(t-\tau) \longrightarrow y(t-\tau)$$

$$\delta(t) \longrightarrow \boxed{LTI} \longrightarrow h(t) \Rightarrow x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t) = h(t) * x(t)$$

$$x(t) = \int_{-\infty}^{\infty} \delta(t-\tau) x(\tau) d\tau = \delta(t) * x(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = h(t) * x(t)$$

$$x_1(t) \longrightarrow X_1(j\omega)$$

$$x_2(t) \longrightarrow X_2(j\omega)$$

$$\Rightarrow x_1(t) * x_2(t) \longrightarrow X_1(j\omega) \cdot X_2(j\omega)$$

ALGORITAM ZA REŠAVANJE PROBLEMA

I) ODZIV SISTEMA NA $\delta(t)$

$$\delta(t) \longrightarrow \boxed{LTI} \longrightarrow h(t)$$

II) OD $h(t)$ NAĆI $H(j\omega)$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = \dots$$

$$x(t) \longrightarrow X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \dots$$

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III)

$$X(j\omega) \longrightarrow \boxed{H(j\omega)} \longrightarrow Y(j\omega) = X(j\omega)H(j\omega)$$

IV) PREBACITI U VREMENSKI DOMEN

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\omega)e^{j\omega t}d\omega = \dots$$

$h(t) \rightarrow H(j\omega)$ FREKVENTNI ODZIV (PRENOSNA F-JA)

$$H(j\omega) = |H(j\omega)|e^{j\arg H(j\omega)}$$

$$X(j\omega) = |X(j\omega)|e^{j\arg X(j\omega)}$$

$$Y(j\omega) = |H(j\omega)||X(j\omega)|e^{j(\arg H(j\omega) + \arg X(j\omega))}$$

$$|Y(j\omega)| = |H(j\omega)||X(j\omega)|$$

$$\arg Y(j\omega) = \arg H(j\omega) + \arg X(j\omega)$$

$|H(j\omega)|$ – POJAČANJE NA FREK. ω

$\arg H(j\omega)$ – POMERANJE U FAZI

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PRIMER 1 :

$$x(t)=\sin\omega_0 t \longrightarrow \boxed{H(j\omega)} \longrightarrow y(t)=?$$

$$H(j\omega)=5\omega e^{j\pi/4}$$

I NAČIN

$$y(t)=h(t)*x(t)$$

$$H(j\omega)\longrightarrow h(t)=\dots$$

II NAČIN

$$x(t)\longrightarrow X(j\omega)=j\pi\left(\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right)$$

$$Y(j\omega)=X(j\omega)\cdot H(j\omega)=\dots$$

$$y(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}Y(j\omega)e^{-j\omega t}dt=\dots$$

III NAČIN

$$|H(j\omega)|=5|\omega| \quad \text{– POJAČANJE}$$

$$\arg(H(j\omega))=\pi/4 \quad \text{– FAZNO POMERANJE}$$

$$y(t)=5\omega_0\sin(\omega_0 t+\pi/4)$$

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- KARAKTERISTIČNI LTI SISTEMI

1) $H(j\omega) = K = const$ MNOŽAČ

$$x(t) \longrightarrow \boxed{K} \xrightarrow{y(t)=?}$$

$y(t) = K \cdot x(t)$

2.) $H(j\omega) = e^{-j\omega t_0}$

$$x(t) \longrightarrow \boxed{e^{-j\omega t_0}} \xrightarrow{y(t)=?}$$

$y(t) = x(t - t_0)$

POMERANJE U VREMENU

$$Y(j\omega) = H(j\omega) X(j\omega)$$

$$Y(j\omega) = e^{-j\omega t_0} X(j\omega)$$

$$x(t - t_0) \longrightarrow e^{-j\omega t_0} X(j\omega)$$

3.) $H(j\omega) = j\omega$

$$x(t) \longrightarrow \boxed{j\omega} \xrightarrow{y(t) = \frac{dx}{dt}}$$

DIFERENCIJATOR

4.) $H(j\omega) = \frac{1}{j\omega}$ $\quad y(t) = \int_{-\infty}^{t} x(t)\,dt$ INTEGRATOR

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- SLOŽENI LTI SISTEMI (BLOK DIJAGRAMI)

1)

$$H(j\omega) = H_1(j\omega) \cdot H_2(j\omega)$$

REDNA VEZA (MNOŽENJE)

2.)

$$H(j\omega) = H_1(j\omega) \pm H_2(j\omega)$$

PARALELNA VEZA

3)

POVRATNA SPREGA

$$Y(j\omega) = H_1(j\omega)\left(X(j\omega) \pm H_2(j\omega)\, Y(j\omega)\right)$$

$$Y(j\omega) = \frac{H_1(j\omega)}{1 \mp H_1(j\omega) H_2(j\omega)} \cdot X(j\omega)$$

$$H(j\omega) = \frac{H_1(j\omega)}{1 \mp H_1(j\omega) H_2(j\omega)}$$

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- NEIZOBLIČEN PRENOS

- NEIZOBLIČEN ODZIV OD X(t)

VERNA REPRODUKCIJA ORIGINALA

DEF:

$$y(t) = A \cdot x(t)$$

$$y(t) = A \cdot x(t-\tau)$$

SIGNAL NEĆE BITI IZOBLIČEN UKOLIKO JE POMNOŽEN NEKOM KONSTANTOM I\ILI POMEREN U VREMENU (ZAKAŠNJEN ZA NEKO τ)

$$Y(j\omega) = A \cdot e^{-j\omega\tau} X(j\omega)$$

$$H(j\omega) = A \cdot e^{-j\omega\tau}$$ - KARAKTERISTIČNA F-JA PRENOSNE F-JE

$|H(j\omega)| = A$ - NA SVIM ω PODJEDNAKO

NEMOGUĆE U PRAKSI

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- FILTRI

- IDEALNI FILTRI

1) PROPUSNIK NISKIH FREKVENCIJA

LPF (LOW-PASS FILTER)

2) PROPUSNIK VISOKIH FREKVENCIJA

HPF (HIGH-PASS FILTER)

3) PROPUSNIK OPSEGA

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ZADATAK 1.26. ZBORNIK

$H_1(j\omega) = e^{-j\omega\tau}$

$H_2(j\omega) = A_2$

$H_3(j\omega) = A_3$

a) ODREDITI $H(j\omega)$

b) NACRTATI $|H(j\omega)|$, $\arg H(j\omega)$, $A_2 = 1/2$ $A_3 = 1$

c) NAĆI $y(t)$ I NACRTATI GA AKO JE

$x(t) =$

a) $H(j\omega) = H_2(j\omega) + H_1(j\omega)\cdot\left(H_1(j\omega)\cdot H_2(j\omega) + H_3(j\omega)\right)$

$$H(j\omega) = A_2 + e^{-j\omega\tau}\left(e^{-j\omega\tau}\cdot A_2 + A_3\right)$$

b)

$$H(j\omega) = 1/2 + e^{-j\omega\tau}\left(e^{-j\omega\tau}\cdot\frac{1}{2} + 1\right)$$

$$= \frac{1}{2}\left(1 + e^{-2j\omega\tau}\right) + e^{-j\omega\tau}$$

$$= \frac{1}{2}e^{-j\omega\tau}\left(e^{j\omega\tau} + e^{-j\omega\tau}\right) + e^{-j\omega\tau}$$

$$= e^{-j\omega\tau}\left(\cos\omega\tau + 1\right)$$

$$\arg H(j\omega) = -\omega t$$

$$|H(j\omega)| = 1 + \cos\omega\tau$$

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b)

c) <u>INAČIN</u>

$$Y(j\omega) = H(j\omega)X(j\omega) \qquad X(j\omega) = \tau \frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}} = \tau \operatorname{sinc}\left(\frac{\omega\tau}{2}\right) e^{-j\omega\frac{\tau}{2}}$$

$$Y(j\omega) = (1+\cos\omega\tau)e^{-j\omega\tau} \cdot \tau \frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}} e^{-j\omega\tau}$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\omega)e^{j\omega t}\,d\omega = \dots$$

LAKŠI PUT → $Y(j\omega) = \left(e^{-j\omega\tau} + \frac{1}{2}e^{-j\omega\frac{\tau}{2}} + \frac{1}{2}e^{-j\omega 2\tau}\right)X(j\omega)$

$$Y(j\omega) = e^{-j\omega\tau}X(j\omega) + \frac{1}{2}X(j\omega) + \frac{1}{2}e^{-j\omega 2\tau}X(j\omega)$$

$x(t-\tau)$ $\quad$ $\frac{1}{2}x(t)$ $\quad$ $\frac{1}{2}x(t-2\tau)$

$$y(t) = x(t-\tau) + \frac{1}{2}x(t) + \frac{1}{2}x(t-2\tau)$$

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II NAČIN – PREKO DIJAGRAMA, PRATITI TAČKE I KAKO SE MENJA SIGNAL X(t)

ZADATAK 1.28 ZBORNIK

$y(t) = ?$

$X_B(t) = X(t-\tau)$

$X_C(t) = X(t) - X_B(t)$

$= X(t) - X(t-\tau)$

$y(t) = \frac{1}{\tau}\int_{-\infty}^{t} X_C(t)\,dt$

$y(t) = \left(\frac{1}{\tau}t + \frac{1}{2}\right)\left(u(t+\tau/2) - u(t-\tau/2)\right) + \left(-\frac{1}{\tau}t + \frac{3}{2}\right)\cdot\left(u(t-\tau/2) - u(t-3\tau/2)\right)$

POMEREN I SIMETRIČAN TROUGAONI IMPULS

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ZADATAK 1.32 ZBORNIK

a) $x(t)=\delta(t)$

b) $x(t)=u(t)$

POD USLOVOM $\frac{\omega_1+\omega_2}{2}=\omega_0 \gg \omega_2-\omega_1$

$B \ll \omega_0$!!!

VEOMA UZAK FREKVENTNI OPSEG

a)

$$Y(j\omega)=H(j\omega)\underbrace{\Delta(j\omega)}_{1}=H(j\omega)$$

$$y(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}Y(j\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\left[\int_{-\omega_2}^{-\omega_1}e^{j\omega t}d\omega+\int_{\omega_1}^{\omega_2}e^{j\omega t}d\omega\right]$$

$$y(t)=\frac{1}{2\pi}\left[\frac{e^{j\omega t}}{jt}\Big|_{-\omega_2}^{-\omega_1}+\frac{e^{j\omega t}}{jt}\Big|_{\omega_1}^{\omega_2}\right]$$

$$y(t)=\frac{1}{2\pi}\left[\frac{e^{-j\omega_1 t}-e^{-j\omega_2 t}}{jt}+\frac{e^{j\omega_2 t}-e^{j\omega_1 t}}{jt}\right]$$

$$y(t)=\frac{1}{2\pi}\left[e^{-j\frac{\omega_1+\omega_2}{2}t}\cdot\underbrace{\frac{e^{-j\frac{(\omega_1-\omega_2)}{2}t}-e^{j\frac{\omega_1-\omega_2}{2}t}}{jt}}_{2j\sin\frac{\omega_2-\omega_1}{2}t}+e^{j\frac{\omega_1+\omega_2}{2}t}\cdot\underbrace{\frac{e^{j\frac{\omega_2-\omega_1}{2}t}-e^{-j\frac{\omega_2-\omega_1}{2}t}}{jt}}_{2j\sin\frac{\omega_2-\omega_1}{2}t}\right]$$

$$y(t)=\frac{2}{\pi t}\sin\left(\frac{\omega_2-\omega_1}{2}t\right)\cdot\cos\left(\frac{\omega_2+\omega_1}{2}t\right)$$

ETF.BEASTWEB.ORG

$$y(t) = \frac{2}{\pi t} \sin\left(\frac{B}{2} t\right) \cos\omega_0 t$$

$$y(t) = \frac{B}{\pi} \frac{\sin \frac{Bt}{2}}{\frac{Bt}{2}} \cos\omega_0 t$$

b) HEVISIDE

$$u(t) = \int_{-\infty}^{t} \delta(t)\,dt$$

$$y(t) = \int_{-\infty}^{\infty} h(t)\,dt$$

ODZIV NA DIRAKOV IMPULS

$$y(t) = \frac{B}{\pi} \frac{\sin \frac{Bt}{2}}{\frac{Bt}{2}} \int_{-\infty}^{\infty} \cos\omega_0 t\,dt = \dots$$

ETF.BEASTWEB.ORG

- HILBERTOV TRANSFORMATOR

$$x(t) \longrightarrow \boxed{HT} \longrightarrow y(t)$$

$$H_{HT}(j\omega) = -j\,\mathrm{sgn}\,\omega \qquad \mathrm{sgn}\,\omega = \begin{cases} 1, & \omega > 0 \\ -1, & \omega < 0 \end{cases}$$

1. $|H_{HT}(j\omega)| = 1$ AMPLITUDSKO POJAČANJE SIGNALA JE 1

2. $$H_{HT}(j\omega) = -j\,\mathrm{sgn}\,\omega = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases} = \begin{cases} e^{-j\pi/2}, & \omega > 0 \\ e^{j\pi/2}, & \omega < 0 \end{cases}$$

$$\arg H_{HT}(j\omega) = \begin{cases} -\pi/2, & \omega > 0 \\ \pi/2, & \omega < 0 \end{cases}$$

(1.) $x(t) = \cos\omega_0 t$, $y(t) = HT(x(t)) = ?$

$$x(t) = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

$$y(t) = \frac{1}{2}(-j)e^{j\omega_0 t} + j\cdot\frac{1}{2}e^{-j\omega_0 t} = \underbrace{\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2}}_{\sin\omega_0 t}$$

$$HT(\cos\omega_0 t) = \sin\omega_0 t$$

(2.) $x(t) = \sin\omega_0 t$ $y(t) = HT(\sin\omega_0 t) = ?$

$$x(t) = \frac{1}{2j}e^{j\omega_0 t} - \frac{1}{2j}e^{-j\omega_0 t}$$

$$y(t) = \underbrace{-\frac{1}{2}e^{j\omega_0 t} - \frac{1}{2}e^{-j\omega_0 t}}_{\cancel{\sin\omega_0 t}\ -\cos\omega_0 t} = -\cos\omega_0 t$$

$$HT(\sin\omega_0 t) = -\cos\omega_0 t$$

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3.)

$$\xrightarrow{\delta(t)} \boxed{HT} \xrightarrow{h(t)=?}$$

$$Y(j\omega) = H_{HT}(j\omega)\cdot \underbrace{\Delta(j\omega)}_{=1} = H_{HT}(j\omega)$$

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_{HT}(j\omega)\, e^{j\omega t}\, d\omega$$

$$h(t) = \frac{1}{2\pi}\left(j\int_{-\infty}^{0} e^{j\omega t}\, d\omega - j\int_{0}^{\infty} e^{j\omega t}\, d\omega\right)$$

$$h(t) = \frac{1}{2\pi}\left(j\,\frac{e^{j\omega t}}{jt}\Big|_{-\infty}^{0} - j\,\frac{e^{j\omega t}}{jt}\Big|_{0}^{\infty}\right)$$

$$\int_{0}^{\infty} e^{j\omega t}\, d\omega = \lim_{\alpha\to 0}\int_{0}^{\infty} e^{(-\alpha+j\omega)t}\, d\omega = \frac{-1}{-\alpha+jt} = -\frac{1}{jt} = \frac{j}{t}$$

$$h(t) = \frac{1}{2\pi}\left(\frac{1}{t}+\frac{1}{t}\right) = \frac{t}{\pi}$$

IMPULSNI ODZIV HILBERTOVOG TRANSFORMATORA

ETF.BEASTWEB.ORG

-ODABIRANJE (SEMPLOVANJE) CT SIGNALA

$$\delta(t)=\delta_p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$y(t)=x(t)\cdot\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$y(t)=\sum_{n=-\infty}^{\infty}x(nT)\cdot\delta(t-nT)$$

$$X(j\omega)\longrightarrow Y(j\omega)\ ?$$

$$\delta(t)=\delta_p(t)=\sum_{n=-\infty}^{\infty}\underbrace{\Delta_n}_{\frac{1}{T}}e^{jn\omega_0 t}\qquad \omega_0=\frac{2\pi}{T}$$

$$\delta(t)=\frac{1}{T}\sum_{n=-\infty}^{+\infty}e^{jn\omega_0 t}\qquad y=\frac{1}{T}\sum_{n=-\infty}^{\infty}x(t)e^{jn\omega_0 t}$$

$$Y(j\omega)=\int_{-\infty}^{\infty}y(t)e^{-j\omega t}dt$$

ETF.BEASTWEB.ORG

$$Y(j\omega) = \frac{1}{T}\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty} x(t)e^{-j\omega t}e^{jn\omega_0 t}\,dt$$

$$Y(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{+\infty} x(t)e^{-j(\omega-n\omega_0)t}\,dt$$

$$\boxed{Y(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(j(\omega-n\omega_0))}$$

MOŽE DOĆI DO PREKLAPANJA

$$x_{iz} = A\cdot x(t-\tau)$$

$$\omega_0 - \omega_n > \omega_n$$

TEOREMA ODABIRANJA

$$\omega_0 > 2\omega_n$$

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ZADATAK 1.23 ZBORNIK

SPEKTAR SIGNALA $f(t)$ JE OGRANICEN $\omega_1 \le |\omega| \le \omega_2$

$x(t) = f(t) \cdot s(t)$ $\qquad s(t) = \Pi_p(t)$, τ, $T = \frac{1}{f_0}$

$$H(j\omega) = \begin{cases} 1, & |\omega| \in [\omega_1, \omega_2] \\ 0, & |\omega| \notin [\omega_1, \omega_2] \end{cases}$$

$f_0 = ?$

$$S(j\omega) = \Pi_p(j\omega) = \frac{\tau}{T} \frac{\sin \frac{n\omega_0 \tau}{2}}{\frac{n\omega_0 \tau}{2}}$$

$\omega_0 > 2\omega_2$

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$$x(t) = \delta(t)\, f(t)$$

$S(t)$

$$x(t) = \sum_{n=-\infty}^{\infty} \Pi_n e^{jn\omega_0 t} f(t)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \Rightarrow X(j\omega) = \sum_{n=-\infty}^{\infty} \Pi_n \int_{-\infty}^{\infty} x(t) e^{-j(\omega - n\omega_0)t} dt$$

$$X(j\omega) = \sum_{n=-\infty}^{\infty} \Pi_n \cdot X(j(\omega - n\omega_0))$$

$n\omega_0 - \omega_1 < \omega_1 \qquad \omega_2 < (n+1)\omega_0 - \omega_2$

$n\omega_0 < 2\omega_1 \qquad (n+1)\omega_0 > 2\omega_2$

$$\frac{2\omega_2}{n+1} < \omega_0 < \frac{2\omega_1}{n}$$

$$\Rightarrow \frac{2\omega_2}{n+1} < \omega_0$$

$$\boxed{\omega_0 > 2\omega_2}$$

$$\omega_{0,min} = \frac{2\omega_2}{1 + \frac{\omega_1}{\omega_2 - \omega_1}}$$

$$\frac{\omega_2}{n+1} < \frac{\omega_1}{n}$$

$$\frac{\omega_2}{\omega_1} < \frac{n+1}{n}$$

$$\frac{\omega_2}{\omega_1} < 1 + \frac{1}{n}$$

$$\frac{1}{n} > \frac{\omega_2 - \omega_1}{\omega_1}$$

$$n < \frac{\omega_1}{\omega_2 - \omega_1}$$

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AMPLITUDSKE MODULACIJE

$\omega_0 = \frac{2\pi}{24h} = \dots$

$r_0 > R_Z$

$\sim 10\,GHz$

$kHz \rightarrow GHz$

MODULACIJA

$GHz \rightarrow kHz$

DEMODULACIJA

- MODULACIJA
  - AM (AMPLITUDSKE)
    - AM-DSB
    - AM-CAM
    - AM-SSB
      - AM-USB
      - AM-LSB
  - UM (UGAONE)
    - PM (FAZNE)
    - FM (FREKVENCIJSKE)

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AM-DSB (DOUBLE-SIDE BAND)

$u(t) = k_m u_m(t) \cdot u_0(t)$ → MODULISANI SIGNAL

$u_0(t) = U_0 \cos \omega_0 t$

SIGNAL NA NISKIM UČESTANOSTIMA

PRODUKTNA MODULACIJA

NOSILAC

MODULIŠUĆI SIGNAL

$\omega_0 \gg \omega_m$

VISOKA FREKVENCIJA

$U_0 = const$, $\omega_0 = const$ } NOSILAC

$\rightarrow U_0 = u_0(t)$

AMPLITUDNO MODULISANI SIGNAL

$u_m(t)$

FM $\left\{ \boxed{u_m(t) \rightarrow \omega_0 - FM \quad \omega_0 = \omega_0(t)} \right.$

$U_0(j\omega) = \pi(\delta(\omega-\omega_0) + \delta(\omega+\omega_0))$

$u(t) = k_m u_m(t) u_0(t) = k_m U_0 u_m(t) \cos \omega_0 t$

$u(t) = \frac{1}{2} k_m U_0 \left(u_m(t) e^{j\omega_0 t} + u_m(t) e^{-j\omega_0 t} \right)$

$U_m(j(\omega-\omega_0))$ $\quad$ $U_m(j(\omega+\omega_0))$

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$$U(j\omega) = \frac{1}{2} k_m U_0 \left(U_m(j(\omega-\omega_0)) + U_m(j(\omega+\omega_0)) \right)$$

- KOHERENTNA DEMODULACIJA

NA ISTOJ FREKVENCIJI

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$u_m(t) = U_m \cos \omega_m t \qquad \omega_0 \gg \omega_m$

$u_0(t) = U_0 \cos \omega_0 t$

$u(t) = k_m U_m U_0 \cos(\omega_m t) \cos(\omega_0 t)$

$u(t) = \underbrace{(k_m U_m U_0 \cos(\omega_m t))}_{\text{NOVA AMPLITUDA } u_b(t)} \cos \omega_0 t$

$u_m(t) \longrightarrow u_b(t)$

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AM-CAM MODULACIJA

$$u(t) = U_0 \cos\omega_0 t + k_m u_m(t) \cos\omega_0 t$$

$$u_m(t) = U_m\, m(t)$$

U_m – AMPLITUDA

$|m(t)| < 1$ – NORMIRANI SIGNAL

$$u(t) = U_0 \left(1 + \frac{k_m U_m}{U_0} m(t)\right) \cos\omega_0 t$$

m_0 – STEPEN (INDEKS) MODULACIJE

$$u_{AM\text{-}CAM}(t) = \underbrace{U_0 (1 + m_0 m(t))}_{\text{SPORA PROMENA}} \cos\omega_0 t$$

$$u_{AM\text{-}CAM}(t) = U_0(t) \cos\omega_0 t$$

$$U_0(t) = U_0 (1 + m_0 m(t))$$

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$m(t) = \cos\omega_m t$

$u_m(t) = U_m \cos\omega_m t$

$u(t) = U_0(1 + m_0 m(t))\cos\omega_0 t$

$u(t) = U_0(1 + m_0 \cos\omega_m t)\cos\omega_0 t$

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AM-SSB MODULACIJA

USB (LSB $\mp$)

$$u(t) = u_1(t) - u_2(t) \qquad HT(u_m(t)) = \hat{u}_m(t)$$

$$u_1(t) = u_m(t)\cos\omega_0 t \longrightarrow \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

$$u_2(t) = \hat{u}_m(t)\underbrace{\cos(\omega_0 t - \pi/2)}_{\sin\omega_0 t} = \hat{u}_m(t)\sin(\omega_0 t)$$

$$u_m(t) = \frac{1}{2\pi}\int_{-\omega_m}^{\omega_m} U_m(j\omega)e^{j\omega t}\,d\omega$$

$$u_1(t) = \frac{1}{2\pi}\int_{-\omega_m}^{\omega_m} \frac{U_m(j\omega)}{2}e^{j(\omega+\omega_0)t}\,d\omega + \frac{1}{2\pi}\int_{-\omega_m}^{\omega_m} \frac{U_m(j\omega)}{2}e^{+j(\omega-\omega_0)t}\,d\omega$$

$\lambda = \omega + \omega_0$ $\qquad$ $\mu = \omega - \omega_0$

$d\lambda = d\omega$ $\qquad$ $d\mu = d\omega$

$\omega = \lambda - \omega_0$ $\qquad$ $\omega = \mu + \omega_0$

$$\Downarrow$$

$$u_1(t) = \frac{1}{2\pi}\int_{\omega_0-\omega_m}^{\omega_0+\omega_m} \frac{U(j(\lambda-\omega_0))}{2}e^{j\lambda t}\,d\lambda + \frac{1}{2\pi}\int_{-\omega_0-\omega_m}^{-\omega_0+\omega_m} \frac{U(j(\mu+\omega_0))}{2}e^{j\mu t}\,d\mu$$

$$u_1(t) = \frac{1}{2\pi}\left[\int_{\omega_0-\omega_m}^{\omega_0+\omega_m} \frac{U_m(j(\omega-\omega_0))}{2}e^{j\omega t}\,d\omega + \int_{-\omega_0-\omega_m}^{-\omega_0+\omega_m} \frac{U_m(j(\omega+\omega_0))}{2}e^{j\omega t}\,d\omega\right]$$

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$$\sin\omega_0 t = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t}$$

$$\hat{u}_m(t) = \frac{1}{2\pi}\int_0^{\omega_m} (-j U_m(j\omega)) e^{j\omega t} d\omega + \frac{1}{2\pi}\int_{-\omega_m}^{0} j U_m(j\omega) e^{j\omega t} d\omega$$

$$u_2(t) = \frac{1}{2\pi}\int_0^{\omega_m} \left(-\frac{U_m(j\omega)}{2}\right) e^{j(\omega+\omega_0)t} d\omega + \frac{1}{2\pi}\int_0^{\omega_m} \frac{U_m(j\omega)}{2} e^{j(\omega-\omega_0)t} d\omega +$$

$$+ \frac{1}{2\pi}\int_{-\omega_0}^{0} \frac{U_m(j\omega)}{2} e^{j(\omega_0+\omega)t} d\omega + \frac{1}{2\pi}\int_{-\omega_m}^{0} \left(-\frac{U_m(j\omega)}{2}\right) e^{j(\omega-\omega_0)} d\omega$$

$$u_2(t) = \frac{1}{2\pi}\left[\int_{\omega_0}^{\omega_0+\omega_m} \frac{-U_m(j(\omega-\omega_0))}{2} e^{j\omega t} d\omega + \int_{-\omega_0}^{-\omega_0+\omega_m} \frac{U_m(j(\omega+\omega_0))}{2} e^{j\omega t} d\omega + \right.$$

$$\left. + \int_{\omega_0-\omega_m}^{\omega_0} \frac{U_m(j(\omega-\omega_0))}{2} e^{j\omega t} d\omega + \int_{-\omega_m-\omega_0}^{-\omega_0} \frac{-U_m(j(\omega+\omega_0))}{2} e^{j\omega t} d\omega \right]$$

$$\boxed{u_1(t) - u_2(t) = \frac{1}{2\pi}\int_{\omega_0}^{\omega_0+\omega_m} U_m(j(\omega-\omega_0)) e^{j\omega t} d\omega + \frac{1}{2\pi}\int_{-\omega_0-\omega_m}^{-\omega_0} U_m(j(\omega+\omega_0)) e^{j\omega t} d\omega}$$

AM-USB

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ZADATAK 2.1. ZBORNIK

SIGNAL $u_m(t) = U_m \sin \omega_m t$ $\quad U_m = 2V \quad f_m = 1kHz$

NOSILAC $u_0(t) = U_0 \sin \omega_0 t$ $\quad U_0 = 5V \quad f_0 = 1MHz$

$$i(t) = a_1 u(t) + a_2 u^2(t) \qquad a_1 = 2\frac{mA}{V} \quad a_2 = 0{,}1\left(\frac{mA}{V}\right)^2$$

f_0, $Q = 100$, $R_0 = 10k\Omega$

a) $f_k = ?$ $\quad I_k = ?$

b) $\omega_0 = ?$ $\quad i_{AM} = ?$

c) $u_1(t) = u_{CAM}(t) = ?$

$\oplus + \triangleright = \otimes$

$$u_1 = Z(j\omega)\, i = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

a) $i(t) = ?$

$$i(t) = a_1(U_m \sin \omega_m t + U_0 \sin \omega_0 t) + a_2(U_m \sin \omega_m t + U_0 \sin \omega_0 t)^2$$

$$\frac{1}{2}\left[\cos(\omega_0 - \omega_m)t - \cos(\omega_0 + \omega_m)t\right]$$

$$i(t) = a_1 U_m \sin \omega_m t + a_1 U_0 \sin \omega_0 t + a_2 U_m^2 \sin^2 \omega_m t + 2a_2 U_m U_0 \sin \omega_m t \sin \omega_0 t$$

$$a_2 U_m^2 \frac{1}{2}(1 - \cos 2\omega_m t) + a_2 U_0^2 \sin^2 \omega_0 t$$

$$a_2 U_0^2 \frac{1}{2}(1 - \cos 2\omega_0 t)$$

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SVE UČESTANOSTI

f_k	0	f_m	$2f_m$	f_0-f_m	f_0	f_0+f_m	$2f_0$
I_k	$a_2U_0^2+\frac{1}{2}a_2U_m^2$	a_1U_m	$\frac{1}{2}a_2U_m^2$	$a_2U_0U_m$	a_1U_0	$a_2U_0U_m$	$\frac{1}{2}a_2U_0^2$
	1,45	4	0,2	1	10	1	1.25

CAM

e) $i_{CAM} = a_1U_0\sin\omega_0 t + 2a_2U_mU_0\sin\omega_m t\sin\omega_0 t$

$$i_{CAM}(t) = a_1U_0\underbrace{\left(1+\frac{2a_2U_m}{a_1}\sin\omega_m t\right)}_{U_0(t)}\sin\omega_0 t$$

m_0

$$m_0 = \frac{2a_2U_m}{a_1} = 0{,}2$$

c)

$$Z(j\omega) = \frac{R_0}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

$$Z(j\omega_k) = \frac{R_0}{\sqrt{1+Q^2\left(\frac{\omega_k}{\omega_0}-\frac{\omega_0}{\omega_k}\right)^2}} = \dots$$

$u_1(j\omega) = Z(j\omega)I(j\omega)$

$u_{1k} = Z(j\omega_k)I_k$

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f_k	0	f_m	$2f_m$	f_0-f_m	f_0	f_0+f_m	$2f_0$
u_k	0	$4\cdot10^{-4}$	$4\cdot10^{-5}$	9,8	100	9,8	$8\cdot10^{-2}$

$u_1(t) \cong u_{AM}(t)$!

ZADATAK 2.2. ZBORNIK

a) $\alpha_1, \alpha_2 = ?$ $\quad u(t) = u_{AMSSB}(t)$

b) $\alpha_2 - \alpha_1 = 2°$ $\quad \frac{u_L}{u_u} = ?$

a) $u(t) = y_1(t) - y_2(t) = x_1(t)\cos\omega_0 t - x_2(t)\underbrace{\cos(\omega_0 t - \pi/2)}_{\sin\omega_0 t}$

$\frac{1}{2}e^{j(\omega_m t-\alpha_1)} + \frac{1}{2}e^{-j(\omega_m t-\alpha_1)}$ $\quad \frac{1}{2}(e^{j\omega_0 t}+e^{-j\omega_0 t})$

$u(t) = U\cos(\omega_m t-\alpha_1)\cos\omega_0 t - U\underbrace{\cos(\omega_m t-\alpha_2-\pi/2)}_{\sin(\omega_m t-\alpha_2)}\sin\omega_0 t$

$$u(t) = U\left[\cos\frac{\alpha_2-\alpha_1}{2}\cos\left[(\omega_0+\omega_m)t-\frac{\alpha_1+\alpha_2}{2}\right] + \sin\frac{\alpha_2-\alpha_1}{2}\sin\left[(\omega_0-\omega_m)t+\frac{\alpha_1+\alpha_2}{2}\right]\right]$$

AM-USB $\Rightarrow \sin\frac{\alpha_2-\alpha_1}{2} = 0 \Rightarrow \alpha_1 = \alpha_2 \qquad \alpha_2 = \alpha_1 + 2k\pi$

$\frac{\alpha_2-\alpha_1}{2} = k\pi$

AM-LSB $\Rightarrow \cos\frac{\alpha_2-\alpha_1}{2} = 0 \qquad \frac{\alpha_2-\alpha_1}{2} = (2k+1)\pi/2 \qquad \alpha_2 = \alpha_1 + (2k+1)\pi$

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AM-DSB $\quad tg\,\frac{\alpha_2-\alpha_1}{2}=1$

$$\frac{\alpha_2-\alpha_1}{2}=\pi/4+k\pi-\ldots$$

AM-CAM – NE MOŽE DA SE POJAVI

b) $$\frac{U_L}{U_n}=\frac{\sin\frac{\alpha_2-\alpha_1}{2}}{\cos\frac{\alpha_2-\alpha_1}{2}}\approx\frac{\overbrace{\alpha_2-\alpha_1}^{2^\circ}}{2}=\frac{1}{180^\circ}\pi=\frac{\pi}{180^\circ}$$

ZADATAK 2.4 ZBORNIK

$u_{o1}=\cos\omega_1 t$, $u_{o2}=\cos\omega_2 t$, $f_2=f_1+120\,Hz$, $\Delta f=80\,Hz$

$P_k[dB]=P_p[dB]+40\,dB$, $a=?$ $f_0=?$

$T=40\,ms$ $\quad f_0=\frac{1}{T}=25\,Hz$

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$f_m = 0, \pm 25, \pm 75, \ldots$

$\frac{2\pi}{\tau} = \frac{2\pi}{T} \cdot \frac{T}{\tau} = 2\omega_0$

$f_g = 2f_0$

$P_k = U_0^2 + 2U_1^2$

$U_0 = \frac{U}{2}$

$U_1 = \frac{U}{2} \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{U}{\pi}$

$U_k = U \frac{\tau}{T} \frac{\sin\frac{k\omega_0 \tau}{2}}{\frac{k\omega_0 \tau}{2}}$

$\frac{k\omega_0 \tau}{2} = \frac{k}{2}\pi$

$P_k = \frac{U^2}{4} + \frac{2U^2}{\pi^2} = U^2\left(\frac{1}{4} + \frac{2}{\pi^2}\right)$

$P_p = U_k^2 \cdot 10^{-\frac{a}{10}}$

$A[dB] = \sim A = 20\log_{10} A$

$P[dB] \sim A^2 = 10\log_{10} P$

$P'[dB] = P[dB] - a[dB]$

$10\log P' = 10\log P - a \quad / \cdot 10$

$P' = P \cdot 10^{-\frac{a}{10}}$

$U_5 = \frac{U}{2} \frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}$

$P_p = \frac{U^2}{25\pi^2}[W]$

$P_k[dB] = P_p[dB] + 40dB$

$10\log U^2\left(\frac{1}{4} + \frac{2}{\pi^2}\right) = 10\log\frac{U^2}{25\pi^2} - a + 40$

$a = 40 + 10\log\frac{25(\pi^2+4)}{4}$

$a[dB] = 19{,}5dB$

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ZADATAK 2.5 ZBORNIK

AM-SSB , $f_1 = 300Hz$, $f_2 = 3400Hz$

$480{,}3kHz \div 483{,}4kHz$, $60dB$

$f_0 = 480kHz$

a)

$$f_{c1} - 0{,}955 f_{c1} \leq 2 f_1 \quad ?$$

$$\boxed{0{,}045 f_{c1} \leq 2 f_1}$$

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b) f_{o1}, f_{o2}

$0{,}045 f_{c1} = 2f_1 \rightarrow f_{c2} = \dots$

$f_{o1} = f_{c2} - f_1$

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ZADATAK 2.6 ZBORNIK

12 TELEFONSKIH SIGNALA

$f_{c3} = 60\,kHz$, $f_{c4} = 108\,kHz$, $f_{c2} - f_{c1} = 4\,kHz$

a) $f_{0min} = ?$

$f_{1,2,\dots 12} = ?$ F_2 – NIŽI BOČNI OPSEG

b) F_1 – NIŽI BOČNI OPSEG

a)

F_1 – VIŠI BOČNI OPSEG

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$k=1$ $\quad f_1-(f_0+f_m)=f_{c3}=60kHz$

$f_0+f_1\geq f_{c4}=108kHz$

$f_1-f_0-4kHz=60kHz \Rightarrow f_1=64kHz+f_0$

$2f_0+64kHz\geq 108kHz$

$2f_0\geq 44kHz$

$\boxed{f_0\geq 22kHz}$

NAJNIŽI UMNOŽAK $4kHz \Rightarrow f_{0min}=24kHz$

$f_1=f_0+f_m+f_{c3}=88kHz$

$f_k=f_1+(k-1)f_m \quad , k=2,3,\dots,12$

$f_2=92kHz$

$f_3=96kHz$

$\vdots$

$f_{12}=132kHz$

$\boxed{f_{c1}=f_0=24kHz \quad , \quad f_{c2}=f_0+f_m=28kHz}$

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e)

$$f_{12}+f_0=f_{c4}=108\,kHz$$

$$f_{12}-f_0+f_m \le f_{c3}=60\,kHz$$

$$\Downarrow$$

$$f_0 \ge 26\,kHz$$

$f_{0MIN}=28\,kHz$ (NAJMANJI UMNOŽAK 4kHz)

$$f_{12}=f_{c4}-f_0=80\,kHz$$

$$f_k=f_{12}-(12-k)f_m\,,\quad k=1,2,\dots 11$$

$$f_{11}=76\,kHz,\ f_{10}=72\,kHz,\ \dots,\ f_1=36\,kHz$$

$$f_{c1}=f_0-f_m=24\,kHz$$

$$f_{c2}=f_0=28\,kHz$$

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ZADATAK 2.8 ZBORNIK

AM-SSB $\quad u(t) = U_0 \cos \omega_0 t + k_u U_m \cos(\omega_0 + \omega_m) t$

$f_0 = 500 kHz \quad , f_m , f_m \in [0.3 \div 3,4] kHz$

$u_1(t) = U_1 \cos \omega_1 t \qquad f_1 = 488 kHz$

a) $u_i(t) = ?$

$$u_C(t) = u_A(t) \cdot u_1(t) = U_0 U_1 \cos\omega_0 t \cos\omega_1 t + k_u U_m U_1 \cos(\omega_0 + \omega_m) t \cos\omega_1 t$$

$$= \frac{1}{2} U_0 U_1 \left[\cos(\omega_0 - \omega_1) t + \cos(\omega_1 + \omega_0) t\right] + \frac{1}{2}\left[k_u U_m U_1 \left(\cos(\omega_0 - \omega_1 + \omega_m) t + \cos(\omega_0 + \omega_1 + \omega_m) t\right)\right]$$

12.3 ÷ 15.4 $\qquad$ 988.3 ÷ 991.4

$$u_D(t) = \frac{1}{2} U_0 U_1 \cos(\omega_0 - \omega_1) t$$

$$u_E(t) = \frac{1}{2} k_u U_m U_1 \cos(\omega_0 - \omega_1 + \omega_m) t$$

$$u_F(t) = u_D(t) \cdot u_E(t) = \frac{1}{4} U_0 U_m k_u U_1^2 \cos(\omega_0 - \omega_1 + \omega_m) t \cos(\omega_0 - \omega_1)$$

$$= \frac{1}{8} U_0 U_m k_u U_1^2 \left[\cos(\omega_0 - \omega_1 + \omega_m - \omega_0 + \omega_1) + \cos(2(\omega_0 - \omega_1) + \omega_m) t\right]$$

0,3 ÷ 3.4 $\qquad$ 24.3 ÷ 27.4

FILTAR PROPUSTA SAMO OVU

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$$u_i(t) = u_B(t) = \frac{1}{8} k_u U_m U_0 U_1^2 \cos\omega_m t$$

$$u_m(t) = U_{m} \cos\omega_m t$$

$$\boxed{u_i(t) = \frac{1}{8} k_u U_0 U_1^2 u_m(t)}$$

b) LO $\quad \Delta f = ? \quad f_1 = 488\,kHz$

$u_c(t)$

$$f_0 - f_1 \geq 11.98$$

$$f_0 - f_1 \leq 12.02$$

$$12.2 \leq f_0 - f_1 + m \leq 15.6$$

$$\Downarrow$$

$$f_1 \in [..]$$

$$f_m,\ 2(f_0 - f_1) + f_m$$

$$-200\,Hz \leq \Delta f \leq 100\,Hz$$

$$\boxed{487.9\,kHz \leq f_1 \leq 488.0\,kHz}$$

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ZADATAK 2.10 ZBORNIK

$u_1(t)$

$u_2(t)$ $\quad [0, f_m]$

P

$H(j\omega) = 1 \qquad \boxed{f_0 - f_m \le f \le f_0 + f_m}$

a) za $\theta = 0$, $u_{B1}(t) = ?$, $u_{B2} = ?$

b) $\theta = ?$ $\quad \Delta P[dB] = 26 dB$

a)

$$u_{C1}(t) = u_1(t) \cdot u_{01}(t) = u_1(t) \cos\omega_0 t$$

$$u_{C2}(t) = u_2(t) \cos(\omega_0 t - \pi/2) = u_2(t) \sin\omega_0 t$$

$$u_D = u_{C1}(t) + u_{C2}(t) = u_1(t) \cos\omega_0 t + u_2(t) \sin\omega_0 t$$

$f \in [f_0 - f_m, f_0 + f_m]$

$u_E(t) = u_D(t)$

$u_E(t) \longrightarrow \quad u_{F1}(t) = u_E(t) \cos(\omega_0 t + \theta)$

$$u_{F1}(t) = u_1(t) \cos\omega_0 t \cos(\omega_0 t + \theta) + u_2(t) \sin\omega_0 t \cos(\omega_0 t + \theta)$$

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$$u_{F1} = \frac{1}{2} u_1(t)\left[\cos(\underline{w_0 t - w_0 t - \theta}) + \cos(w_0 t + w_0 t + \theta)\right]$$
$$- \frac{1}{2} u_2(t)\left[\sin(2w_0 + \theta) \underline{- \sin\theta}\right]$$

$$u_{B1}(t) = \frac{1}{2} u_1(t)\cos\theta - \frac{1}{2} u_2(t)\sin\theta$$

za $\theta = 0$

$$\boxed{u_{B1} = \frac{1}{2} u_1(t)}$$

$$u_{F2} = u_E(t)\cdot\sin(w_0 t + \theta) = u_1(t)\cos(w_0 t)\sin(w_0 t + \theta) + u_2(t)\sin w_0 t \cdot \sin(w_0 t + \theta)$$

$$u_{F2} = u_E(t)\sin(w_0 t + \theta) = \frac{1}{2} u_1(t)\left[\sin(2w_0 t + \theta) + \sin\theta\right]$$
$$+ \frac{1}{2} u_2(t)\left[\cos\theta - \cos(2w_0 t + \theta)\right]$$

za $\theta = 0$

$$u_{B2}(t) = \frac{u_2(t)}{2}$$

b) $10\log\frac{P_1}{P_2} = 26\,dB$

$$\log\frac{P_1}{P_2} = 2{,}6$$

$$P_1 = P_2 \cdot 10^{2{,}6}$$

$$\frac{1}{4}\overline{u_1^2(t)}\cos^2\theta = \frac{1}{4}\overline{u_2(t)^2}\sin^2\theta$$

$$tg^2\theta = \frac{1}{10^{2{,}6}}$$

$$\theta^2 \approx \frac{1}{10^{2{,}6}} \rightarrow \theta \cong \ldots$$

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ZADATAK 2.16 ZBORNIK (JANVAR 2014)

10 SIGNALA, $B_m = f_v - f_n$

AM-SSB $\quad f_i - f_{i-1} = B_m + B$

$$H_i(jf) = e^{-\left(1.2\,\frac{f-f_{0i}}{B_m}\right)^2}, \quad i = 1,2,\dots 10$$

a) $B = ?$

$$20\log\left|\frac{|H_i(jf_{0i})|}{H_i\left\{j\left[f_{0i}\pm\left(\frac{B_m}{2}+B\right)\right]\right\}}\right| \geq 20\,dB, \quad i = 1,2,\dots,10$$

$$\log\frac{e^{-\left(1,2\,\frac{f_0-f_{0i}}{B_m}\right)^2}}{e^{-\left(1.2\cdot\frac{\cancel{f_{0i}}\pm\left(\frac{B_m}{2}+B\right)-\cancel{f_{0i}}}{B_m}\right)^2}} \geq 1\,dB$$

$$\log e^{1,44\left(\frac{1}{2}+\frac{B}{B_m}\right)^2} \geq 1\,dB$$

$$e^{1,44\left(\frac{1}{2}+\frac{B}{B_m}\right)^2} \geq 10 \quad /\ln$$

$$1,44\left(\frac{1}{2}+\frac{B}{B_m}\right)^2 \geq \ln 10 \qquad \left(\frac{1}{2}+\frac{B}{B_m}\right)^2 \geq 1,6$$

$$\frac{B}{B_m} \geq 1,26-0,5 \qquad B \geq 0,765\,B_m$$

$$B = 0,765\,B_m$$

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e) $N=10$

$B_0 = N B_m + (N-1) B = 10 B_m + 9 \cdot 0{,}765 B_m$

$$\boxed{B_0 = 16{,}885 B_m}$$

U ZBIRCI JE ZAOKRUŽENO $B \approx 0{,}8 B_m$

PA JE $B_0 = 10 B_m + 9 \cdot 0{,}8 B_m = 17{,}2 B_m$

ZADATAK 2.17 ZBORNIK

$u_1(t) = U_1 \cos\omega_1 t \quad u_2(t) = U_2 \cos\omega_2 t \qquad f_1 \geq 0\,kHz$

$f_2 \leq 4\,kHz$

$f_i \gg f_0$

a)

$u_C(t) = [u_1(t) + u_2(t)] \cos\omega_0 t$

$u_C(t) = \frac{U_1}{2}\cos(\omega_0 - \omega_1)t + \frac{U_1}{2}\cos(\omega_0 + \omega_1)t + \frac{U_2}{2}\cos(\omega_0 - \omega_2 t) + \frac{U_2}{2}\cos(\omega_0 + \omega_2)t$

$u_E(t) = \frac{U_1}{2}\cos(\omega_0 - \omega_1)t + \frac{U_2}{2}\cos(\omega_0 - \omega_2)t$

$u_0 = [u_1(t) - u_2(t)] \sin\omega_0 t$

$u_0 = \frac{U_1}{2}\sin(\omega_0 - \omega_1)t + \frac{U_1}{2}\sin(\omega_0 + \omega_1)t - \frac{U_2}{2}\sin(\omega_0 - \omega_2)t - \frac{U_2}{2}\sin(\omega_0 + \omega_1)t$

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$$u_F(t) = \frac{U_1}{2}\sin(\omega_0-\omega_1)t - \frac{U_2}{2}\sin(\omega_0-\omega_2)t$$

$$u_i(t) = u_E(t)\cos\omega_i t + u_F(t)\sin\omega_i t = \frac{U_1}{2}\cos(\omega_i-\omega_0+\omega_1)t + \frac{U_2}{2}\cos(\omega_i+\omega_0-\omega_2)t$$

$$|f_0-f_1| \le 4\,kHz \qquad |f_0-f_2| \le 4\,kHz$$

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b) AM-SSB SIGNAL

$u_2(t)$ – LSB

$u_1(t)$ – USB

c)

ZADATAK 2.18 ZBORNIK

a) $u_B(t) = ?$

$$u_A(t) = m(t)\cos\omega_0 t - \hat{m}(t)\sin\omega_0 t$$

$$u_B = U_0\cos(\omega_0 t+\theta) + m(t)\cos\omega_0 t - \hat{m}(t)\sin\omega_0 t$$

$$u_B(t) = \left(U_0\cos\theta + m(t)\right)\cos\omega_0 t - \left(U_0\sin\theta + \hat{m}(t)\right)\sin\omega_0 t$$

$$u_B(t) = \sqrt{\left(U_0\cos\theta + m(t)\right)^2 + \left(U_0\sin\theta + \hat{m}(t)\right)^2}$$

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$$u_B(t) = \sqrt{U_0^2 + 2U_0\left(u(t)\cos\theta + \hat{u}(t)\sin\theta\right) + u^2(t) + \hat{u}^2(t)}$$

$$u_B(t) = U_0\sqrt{1 + \frac{2u(t)\cos\theta}{U_0} + 2\frac{\hat{u}(t)\sin\theta}{U_0} + \frac{u^2(t)+\hat{u}^2(t)}{U_0^2}}$$

b) $U_0 \gg |u(t)|$ $\quad U_0 \gg |\hat{u}(t)|$

$$u_B(t) \approx U_0 + u(t)\cos\theta + \hat{u}(t)\sin\theta$$

$$u_B(t) \approx U_0 + u(t) \qquad u_i(t) = k_D u(t)$$

ZADATAK 2.20 ZBORNIK

a) $u_F(t) = ?$

$$u_E(t) = [u_1(t) + u_2(t)]\cos\omega_0 t$$

$$u_D(t) = [\hat{u}_2(t) - \hat{u}_1(t)]\sin\omega_0 t$$

$$u_F(t) = u_E(t) + u_D(t) = \overbrace{u_1(t)\cos\omega_0 t - \hat{u}_1(t)\sin\omega_0 t}^{\text{AM-USB}}$$

$$+ \underbrace{u_2(t)\cos\omega_0 t + \hat{u}_2(t)\sin\omega_0 t}_{\text{AM-LSB}}$$

$$B_a = f_{m1} + f_{m2} = 4 f_{m1}$$

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6)

$$\boxed{u_p(t) = u_1 \cos\omega_0 t + u_2 \sin\omega_0 t}$$

- UGAONE MODULACIJE

UM → PM – FAZNE MODULACIJE

UM → FM – FREKVENCIJSKE MODULACIJE

$u_0(t) = U_0 \cos(\omega_0 t + \varphi_0)$ $\qquad \omega_0 \gg \omega_m$

$u_m(t)$; $\omega \in [0, \omega_m]$ $\qquad \phi(t) = \omega_0 t + \varphi_0$

$\phi(t) = \omega_0 t + \varphi$ $\qquad \omega(t) = \omega_0 = \text{Const}$

$u_m \longrightarrow \phi(t)$

$\varphi(t) = k_\varphi \cdot u_m(t) + \varphi_0$ $\quad (\varphi_0 = 0)$

$\varphi(t) = k_\varphi u_m(t)$ – PM

$$\left.\begin{aligned} \omega(t) &= k_\omega u_m(t) \\ f(t) &= k_f u_m(t) \end{aligned}\right\} \text{– FM}$$

$f = \frac{\omega}{2\pi}$

$k_f = \frac{k_\omega}{2\pi}$

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PM: $u(t) = U_0 \cos(\omega_0 t + k_\varphi u_m(t))$

$$u_m(t) = U_m m(t) \quad , |m(t)| < 1$$

$$u(t) = U_0 \cos(\omega_0 t + \underbrace{k_\varphi U_m}_{\Delta\phi} m(t))$$

$\Delta\phi = k_\varphi \cdot U_m$

MAKSIMALNA DEVIJACIJA FAZE

$$u_{PM}(t) = U_0 \cos(\omega_0 t + \Delta\phi m(t))$$

FM: $\omega(t) = \dfrac{d\phi(t)}{dt} = k_\omega U_m(t) + \omega_0 = \omega_0 + \underbrace{k_\omega U_m}_{\Delta\omega} m(t)$

MAKSIMALNA DEVIJACIJA KRUŽNE UČESTANOSTI

$$\frac{d\phi}{dt} = \omega_0 + \Delta\omega\, m(t) \Big/ \int dt$$

$$\phi = \omega_0 t + \Delta\omega \int_{-\infty}^{t} m(t)\,dt$$

$$\boxed{u_{FM}(t) = U_0 \cos\left(\omega_0 t + \Delta\omega \int_{-\infty}^{x} m(\tau)\,d\tau\right)}$$

I) $m(t) = \cos\omega_m t \qquad (\omega_0 \gg \omega_m)$

$$u_{PM}(t) = U_0 \cos(\omega_0 t + \Delta\phi \cos\omega_m t)$$

$$m = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$$

$$u_{FM}(t) = U_0 \cos\left(\omega_0 t + \frac{\Delta\omega}{\omega_m} \sin\omega_m t\right)$$

$$u_F(t) = U_0 \cos(\omega_0 t + m \sin\omega_m t$$

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- SPEKTAR

$$u_{FM}(t) = U_0 \cos(\omega_0 t + m \sin \omega_m t) \qquad m \neq 0$$

$$= \mathrm{Re}\, U_0 e^{j(\omega_0 t + m \sin \omega_m t)}$$

$$e^{j0\phi},\ e^{\pm j\phi},\ e^{\pm j2\phi}$$

$$e^{jm\sin\phi} = \sum_{n=-\infty}^{\infty} J_n(m)\, e^{jn\phi}$$

$J_n(x)$ BESELOVE FUNKCIJE

n – RED, x – ARGUMENT

$$J_{-n}(x) = (-1)^n J_n(x)$$

$$\mathrm{Re}\, U_0 e^{j\omega_0 t} e^{jm\sin\omega_m t} = \mathrm{Re}\, U_0 \sum_{n=-\infty}^{\infty} J_n(m)\, e^{j(\omega_0 + n\omega_m)t}$$

SPEKTAR

$m \ll 1$

$B \simeq 2 f_m$

$B = 2 f_m (m+1)$

KARSONOV OBRAZAC

ETF.BEASTWEB.ORG

DEMODULACIJA

$$u_{PM}(t) = U_0 \cos(\overbrace{\omega_0 t + \Delta\phi m(t)}^{\phi(t)}) \rightarrow x(t)$$

$$u_{FM}(t) = U_0 \cos\left(\omega_0 t + \Delta\omega \int_{-\infty}^{t} m(\tau)d\tau\right)$$

LINEARNI DISKRIMINATOR

PM:

FM:

ETF.BEASTWEB.ORG

ZADATAK 2.21 ZBORNIK

$U_0 \cos\omega_0 t$

TRENUTNE DEVIJACIJE FAZE $\phi(t)$, $\delta f(t) = ?$

a) SIGNAL u_m PM, $U_m = 1V$, $\Delta\phi = 20\pi$ rad

b) FM, $U_m = 1V$, $\Delta f = 20kHz$

a) FAZNA MODULACIJA

$$\varphi(t) = k_\varphi \cdot U_m(t) = k_\varphi \cdot m(t) \quad [U_m = 1V]$$

$$\phi(t) = \omega_0 t + \varphi(t) = \omega_0 t + k_\varphi m(t)$$

$$f = \frac{1}{2\pi}\dot{\phi}(t) = \frac{1}{2\pi}(\omega_0 + k_\varphi \dot{m}(t))$$

$$\boxed{\frac{k_\varphi}{2\pi}\dot{m}(t) = \delta f(t)}$$

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b) FREKVENCIJSKA MODULACIJA

$$\delta f = k_f u_m(t) = k_f m(t) = 20\frac{kHz}{V} m(t)$$

$$f = f_0 + \delta f = \frac{1}{2\pi}\dot{\phi}(t) \qquad \delta f(t) = \begin{cases} k_f \cdot t & (0,1) \\ 40 - k_f t & (1,2 \end{cases}$$

$$\phi(t) = 2\pi \int_{-\infty}^{t} (f_0 + \delta f)\, d\tau$$

$$= \underbrace{2\pi f_0}_{\omega_0} \cdot t + \underbrace{\int_{-\infty}^{t} \delta f(t)\, d\tau}_{\delta\phi(t)}$$

ETF.BEASTWEB.ORG

ZADATAK 2.22 ZBORNIK

- NA ULAZ U MODULATOR – TEST SIGNAL

$u_m(t) = U_m \cos \omega_m t$ $\qquad \Delta f_0 = k_f U_m$

$u_0(t) = U_0 \cos \omega_0 t$

$U_m \rightarrow \Delta f_0 = 1kHz$

a) $f_m = 1kHz$, U_m

b) $f_m = 1kHz$, $\frac{U_m}{5}$

v) $f_m = 1{,}85kHz$, U_m

g) $f_m = 1{,}85kHz$, $\frac{U_m}{5}$

SPEKTAR FM SIGNALA?

$\omega_0 \pm n\omega_m \quad n = 0, 1, 2 \ldots$

a) $B = 2f_m(m+1) \Rightarrow m = \frac{\Delta f}{f_m} = 1$

$= 2(f_m + \Delta f)$

KARSONOV OBRAZAC $\qquad B = 2 \cdot (1kHz + 1kHz) = 4kHz = 2 \cdot 2kHz$

$J_n(1) \neq 0 \pm \frac{1}{2} \ldots$

$J_0(1) = 0{,}765 \quad J_1(1) = 0{,}44 \quad J_2(1) = 0{,}115 \ldots$

VREDNOSTI BESSELOVIH F-JA

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b) $m = \frac{\Delta f}{f_m'} = \frac{0,2}{1} = 0,2 \Rightarrow J_n(0,2) = \dots$

$n = 0, \pm 1, \pm 2 \dots$

c) $m = \frac{\Delta f}{f_m''} = \frac{1}{1,85}$

$J_n\left(\frac{1}{1,85}\right) = \dots$

$B = 2(f_m + \Delta f) = 2 \cdot 2,85$

d) NA ISTI NAČIN

$m = \frac{\Delta f}{f} = 0.108 \quad J_n(0.108) = \dots$

ZADATAK 2.23 ZBORNIK

$U_m(t) = U_1 \cos\omega_1 t + U_2 \cos\omega_2 t = U_m(\cos\omega_1 t + \cos\omega_2 t)$

$U_1, U_2 = U_m \quad , f_1 = 1kHz \quad f_2 = 1.85kHz$

$U_m \rightarrow \Delta f_0$

a) $\Delta f_0 = 1kHz$

b) $\Delta f_0 = 0,2kHz$

SPEKTAR?

ETF.BEASTWEB.ORG

$$u_{FM}(t) = U_0 \cos\left(\omega_0 t + \Delta\omega_0 \int_0^t u(\tau)\,d\tau\right)$$

$$u_{FM}(t) = U_0 \cos\left(\omega_0 t + \Delta\omega_0 \left(\frac{\sin\omega_1 t}{\omega_1} + \frac{\sin\omega_2 t}{\omega_2}\right)\right)$$

$$u_{FM}(t) = U_0 \cos\left(\omega_0 t + \underbrace{\frac{\Delta f_0}{f_1}}_{m_1} \sin\omega_1 t + \underbrace{\frac{\Delta f_0}{f_2}}_{m_2} \sin\omega_2 t\right)$$

$$u_{FM}(t) = U_0 \cos(\omega_0 t + m_1 \sin\omega_1 t + m_2 \sin\omega_2 t)$$

$$= \mathrm{Re}\, U_0 e^{j\omega_0 t} \cdot e^{j m_1 \sin\omega_1 t} e^{j m_2 \sin\omega_2 t}$$

$$= \mathrm{Re}\, U_0 e^{j\omega_0 t} \sum_{p=-\infty}^{\infty} J_p(m_1) e^{jp\omega_1 t} \cdot \sum_{q=-\infty}^{\infty} J_q(m_2) e^{jq\omega_2 t}$$

$$\boxed{= \mathrm{Re}\, U_0 \sum_{p,q=-\infty}^{\infty} J_p(m_1) J_q(m_2) e^{j(\omega_0 t + p\omega_1 + q\omega_2)t}}$$

b) $m_1 = \frac{\Delta f_0}{5 f_1} = 0{,}2 \qquad m_2 = \frac{\Delta f_0}{5 f_2} = 0.108$

$J_p(m_1), J_q(m_2) = \ldots$

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ZADATAK 2.27 ZBORNIK

$u_G \le 0{,}5$

$u_m(t) = U_m \cos\omega_m t$ $\qquad$ $50\,Hz \le f_m \le 15\,kHz$

$f_0 = 200\,kHz$

a) $\Delta f_{0G} = ?$

b) $\Delta f_{0FM} = 75\,kHz$, $n = ?$

c) $f_1 = ?$ $\quad f = 90\,MHz$

a) $u_B(t) = \int_{-\infty}^{t} u_m(t)\,dt = \frac{U_m}{\omega_m}\sin\omega_m t$

$u_E(t) = u_0(t)\cdot u_B(t) = \frac{U_0 U_m}{\omega_m}\sin\omega_m t\cos\omega_0 t$

$u_D(t) = U_0\cos(\omega t - \pi/2) = U_0\sin\omega_0 t$

$u_G(t) = u_E(t) + u_D(t) = \frac{U_0 U_m}{\omega_m}\sin\omega_m t\cos\omega_0 t + U_0\sin\omega_0 t$

$$a\cos x + b\sin x = \sqrt{a^2+b^2}\left(\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\cos\varphi}\cos x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\sin\varphi}\sin x\right)$$

$$= \sqrt{a^2+b^2}\cos(x-\varphi)$$

$$= \sqrt{a^2+b^2}\cos\left(x - \operatorname{arctg}\frac{b}{a}\right)$$

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$$u_G(t) = \sin(\omega_0 t - \phi(t)) \cdot \underline{\underline{V_G(t)}}$$

$$V_0\left(\sin\omega_0 t + \frac{U_m}{\omega_m}\sin\omega_m t + \cos\omega_0 t\right)$$

$$u_G(t) = \underbrace{V_0\sqrt{1+\frac{U_m^2}{\omega_m^2}}\cdot\sin\omega_m t}_{\substack{V_G(t)\\ \text{(AMPLITUDA)}}}\left(\underbrace{\frac{1}{\sqrt{1+\frac{U_m^2}{\omega_m^2}}}\cdot\cancel{\sin\omega_m t}}_{\cos\phi}\sin\omega_0 t + \underbrace{\frac{\frac{U_m}{\omega_m}\cancel{\sin\omega_m t}}{\sqrt{1+\frac{U_m^2}{\omega_m^2}}}}_{\sin\phi}\cancel{\sin\omega_m t}\cos\omega_0 t\right.$$

$$u_G(t) = V_G(t)\cdot\sin(\omega_0 t + \phi(t))$$

$$\phi(t) = \operatorname{arctg}(m\sin\omega_m t)\ ;\ m = \frac{U_m}{\omega_m}$$

$x \ll 1$

$\operatorname{arctg} \approx x$

$$\cancel{\sqrt{1+x} \approx x}$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x \approx 1$$

$$u_{FM}(t) = V_0\cos(\omega_0 t + m\sin\omega_m t)$$

$$u_G(t) \cong V_0\sin\left(\omega_0 t + \frac{U_m}{\omega_m}\sin\omega_m t\right)$$

$$m = \frac{U_m}{\omega_m} = \frac{\Delta\omega_{0G}}{\cancel{\Delta}\omega_m} = \frac{\Delta f_{0G}}{f_m} \le 0{,}5$$

$$\Delta f_{0G} \le 0{,}5 f_m$$

$$(\Delta f_{0G})_{MAX} = 0{,}5\underbrace{f_m}_{\text{MIN } f_m} = 25\,Hz\ !$$

$$\Delta f_{0G} \le 0{,}5$$

$$\forall f_m \in [50\,Hz,\ 15\,kHz]$$

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в) $u_H(t) = U_0 \sin(n\omega_0 t + nm\sin\omega_m t)$

! FAZA
! FREKVENCIJA
SE MNOŽE!

$u_J(t) = u_H(t) \cdot U_1 \sin(n\omega_0 t + nm\sin\omega_m t) \cdot \cos\omega_1 t$

$u_J(t) = \frac{1}{2} U_0 U_1 \left[\sin((\omega_1 + n\omega_0)t + nm\sin\omega_m t) + \sin((-\omega_1 - n\omega_0)t + nm\sin\omega_m t) \right]$

$\Delta f_{0FM} = n \cdot m \cdot f_{m,MIN} = n \cdot \Delta f_0$

$n \cdot m = \frac{\Delta f_0}{f_m} \cdot n$

$n \cdot \frac{\Delta f_0}{f_m} = \frac{\Delta f_{0FM}}{f_m}$

$\Delta f_{0FM} = n \cdot \Delta f_0 = 75\,kHz$

г) $u_{FM}(t) = \frac{1}{2} U_0 U_1 \sin(n(\omega_0 - \omega_1)t + nm\sin\omega_m t)$ $\boxed{n = 3000}$

$n f_0 - f_1 = 90\,MHz$

$f_1 = 510\,MHz$

$f_1 = n f_0 - 90\,MHz = 600\,MHz - 90\,MHz = \boxed{510\,MHz}$

$n f_0 + f_1 = 90\,MHz$

$f_1 < 0$ ⊥

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ZADATAK 2.31 ZBORNIK

$f_0 = 95.3MHz$, k_1

$u_m(t)$, $f_m \in 20 \div 53$ Hz kHz, $(\Delta f)_{MAX} = 1kHz$, k_2

F_2, f_0, $f_1 < f_0$, $k_1 \cdot k_2 = ?$ $(\Delta f)_{MAX} < 10Hz$

$$u_{FM}(t) = U_0 \cos\left[\omega_0 t + 2\pi k_1 \int_{-\infty}^{t} [u_m(\tau) - u_G(\tau)]d\tau + 2\pi \int_{-\infty}^{t} e(\tau)d\tau\right]$$

FLUKTUACIJA FREKVENCIJE ZA 1kHz (ODBIJANJE)

$\phi_0(t) = \omega_0 t$ OTSTO

$$\phi_0(t) = \int \underset{\omega_0 + \delta\omega_0}{\omega(t)} dt = \omega_0 t + 2\pi \int_{-\infty}^{t} e(\tau)d\tau$$

$u_C(t) = u_{FM}(t) \cdot 2\cos(\omega_0 - \omega_1)t$

$u_C(t) = \dots$ ADICIONA FORMULA

$$u_D(t) = U_0 \cos\left[\omega_1 t + 2\pi k_1 \int_{-\infty}^{t} [u_m(\tau) - u_G(\tau)]d\tau + 2\pi \int_{-\infty}^{t} e(\tau)d\tau\right]$$

$$u_E(t) = \frac{1}{2\pi} k_2 \frac{d\phi_0}{dt}$$

$u_E(t) = k_1 k_2 [u_m(t) + u_G(t)] + k_2 e(t)$ NEĆE PROĆI

ETF.BEASTWEB.ORG

$$u_G(t) = -k_1 k_2 u_G(t) + k_2 e(t)$$

$$u_G(t)(1 + k_1 k_2) = k_2 e(t)$$

$$u_G(t) = \frac{k_2 e(t)}{1 + k_1 k_2} \Rightarrow \text{UBACITI U INTEGRAL}$$

$$u_{FM}(t) = U_0 \cos\left[\omega_0 t + 2\pi k_1 \int_{-\infty}^{t} u_m(\tau)\, d\tau + 2\pi \int_{-\infty}^{t} \frac{e(\tau)}{1 + k_1 k_2}\, d\tau\right]$$

$$\frac{1\,kHz}{1 + k_1 k_2} < 10\,Hz$$

$$\cancel{(1 + k_1 k_2) \cdot 10 < 1}$$

$$1\,kHz \leq 10 + 10 k_1 k_2$$

$$990\,\cancel{Hz} \leq 10 k_1 k_2\,\cancel{Hz}$$

$$99\,\cancel{Hz} \leq k_1 k_2$$

$$\boxed{k_1 k_2 = 99\,\cancel{Hz}}$$

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ZADATAK 2.29 ZBORNIK

$$F: \left[f_1 - \frac{B_1}{2},\ f_1 + \frac{B_1}{2}\right],\ k_1,\ k_2,\ f_0 - f_1,\ f_0 \gg f_1$$

$$u_0(t) = U_0 \cos\left(\omega_0 t + \underbrace{\frac{\Delta f_0}{f_m}}_{m} \sin \omega_m t\right)$$

a) $u_i = ?$

b) $u_L = ?$

c) $k_1 k_2 = ?$ $\quad B = 20\% B_1,\ \frac{\Delta f_0}{f_m} = 9$

a) $u_i = k_1 \frac{1}{2\pi} \frac{d\phi(t)}{dt}$ FM DEMODULACIJA

$$u_F(t) = u_u(t) \cdot u_L(t)$$

$$u_L(t) = U_L \cos\left((\omega_0 - \omega_1)t + 2\pi k_2 \int_{-\infty}^{t} u_i(\tau)\,d\tau\right)$$

$$u_F(t) = U_0 U_L \cos\left[\omega_0 t + m \sin \omega_m t\right] \cos\left[(\omega_0 - \omega_1)t + 2\pi k_2 \int_{-\infty}^{t} u_i(\tau)\,d\tau\right] \cdot k_m$$

$$u_F(t) = \frac{1}{2} U_0 U_L \left[\cos\left(\omega_1 t + \frac{\Delta f_0}{f_m} \sin \omega_m t - 2\pi k_2 \int_{-\infty}^{t} u_i(\tau)\,d\tau\right)\right.$$

SAMO OVO KROZ F

$$\left. + \cos\left(2(\omega_0 - \omega_1)t + \frac{\Delta f_0}{f_m} \sin \omega_m t + 2\pi k_2 \int_{-\infty}^{t} u_i(\tau)\,d\tau\right)\right] \cdot k_m$$

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$$u_D(t) = \frac{1}{2} k_m U_0 U_L \cos\Big(\omega_1 t + \underbrace{\frac{\Delta f_0}{f_m}\sin\omega_m t - 2\pi k_2 \int_{-\infty}^{t} u_i(\tau)\,d\tau}_{\phi(t)}\Big)$$

$$u_i(t) = k_1 \Delta f_0 \cos\omega_m t - k_1 k_2 u_i(t)$$

$$\omega_0 = \frac{\Delta f_0}{f_m}$$

$$u_i(t)\,(1 + k_1 k_2) = k_1 \Delta f_0 \cos\omega_m t$$

$$u_i(t) = \frac{k_1 \Delta f_0}{1 + k_1 k_2}\cos\omega_m t$$

$$u_i(t) = \frac{k_1 k_f}{1 + k_1 k_2}\, u_m(t)$$ UBACI SE U INTEGRAL u_D

б) $$u_D = \frac{k_m U_0 U_L}{2}\cos\Big[\omega_1 t + \underbrace{\omega_1}_{\frac{\Delta f_0}{(1+k_1k_2) f_m}}\sin\omega_m t$$

$$B = 2 f_m(m_0 + 1)$$

$$2 f_m(m+1) = \cancel{2 f_m}\Big(\frac{1}{1+k_1k_2}\frac{\Delta f_0}{f_m} + 1\Big)$$

$$B = 2 \cancel{f_m}(m_1 + 1) = 2 \cancel{f_m}(m_0 + 1)\,0{,}2$$

$$= 0{,}2 \cdot \cancel{2 f_m}\Big(\frac{\Delta f_0}{f_m} + 1\Big)$$

$$\cancel{\frac{8 f_0}{(1+k_1k_2) f_m} + 1 = \frac{0{,}2 \Delta f_0}{f_m} + 0{,}2}$$

$$\frac{9}{1+k_1k_2} + 1 = 0{,}2 \cdot 9 + 0{,}2$$

$$\cancel{\frac{1}{1+k_1k_2} = 0{,}2}$$

$$\cancel{0{,}2 + 0{,}2 k_1 k_2 = 1}$$

$$\cancel{0{,}2 k_1 k_2 = 0{,}8}$$

$$\frac{9}{1+k_1k_2} = 2 - 1$$

$$\cancel{\frac{1}{1+k_1k_2} =}$$

$$\frac{9}{1+k_1k_2} = 1 \Rightarrow 1 + k_1 k_2 = 9$$

$$\underline{\underline{k_1 k_2 = 8}}$$

ETF.BEASTWEB.ORG

- IMPULSNE MODULACIJE
- PAM - IMPULSNA AMPLITUDSKA MODULACIJA
- PWM - IMPULSNA ŠIRINSKA MODULACIJA
- PAM - IMPULSNA POLOŽAJNA MODULACIJA

- PAM
- PRIRODNO / REGULARNO ODABIRANJE

$nT_s < t < nT_s + \tau,\ n \in Z$

AMPL. ZAVISI OD $U_m(t)$ U TOM TRENUTKU

$$U_n(t) = U_m(t)$$
$$nT < t < nT + \tau$$

AMPLITUDA ODBIRKA KONSTANTNA U INT. τ

$$U_n(t) = U_m(nT_s)$$
$$nT_s < t < nT_s + \tau$$

ETF.BEASTWEB.ORG

- PRIRODNO ODABIRANJE

$$u_s(t) = U_s \sum_{n=-\infty}^{\infty} \Pi(t - nT_s) = \frac{U_s \tau}{T} \sum_{n=-\infty}^{\infty} \frac{\sin(n\omega_s \tau/2)}{n\omega_s \tau/2} e^{jn\omega_s t}$$

$$\omega_s = \frac{2\pi}{T_s}$$

$$\Pi(t - nT_s) = \begin{cases} 1, & nT_s < t < nT_s + \tau \\ 0, & \text{ZA OSTALO} \end{cases}$$

$$u_{PAM}(t) = u_m(t) \cdot u_s(t) = U_s \frac{\tau}{T} \sum_{n=-\infty}^{\infty} u_m(t) \frac{\sin(n\omega_s \tau/2)}{n\omega_s \tau/2} e^{jn\omega_s (t - \tau/2)}$$

$$U_{PAM}(j\omega) = \mathcal{F}[u_{PAM}(t)] = \frac{U_s \tau}{T} \sum_{n=-\infty}^{\infty} \frac{\sin(n\omega_s \tau/2)}{n\omega_s \tau/2} U_m[j(\omega - \omega_s n)] e^{jn\omega_s}$$

$$= \frac{U_s \tau}{T} \sum_{n=-\infty}^{\infty} \frac{\sin(n\omega_s \tau/2)}{n\omega_s \tau/2} U_m[j(\omega - n\omega_s)] e^{-jn\omega_s \tau/2}$$

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- REGULARNO ODABIRANJE

S & H (SAMPLE AND HOLD)

- ODBIRCI IMAJU KONSTANTNU AMPLITUDU

$$u_{S\&H}(t) = \sum_k u_m(kT_s)\cdot \Pi(t-kT_s)$$

$$U_{S\&H}(j\omega) = \sum_k \int_{-\infty}^{\infty} u_m(kT_s)\,\Pi(t-kT_s)\,e^{-j\omega t}\,dt$$

$$= \frac{1}{2\pi}\sum_k \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_m(j\mu)\,e^{j\mu kT_s}\,\Pi(t-kT_s)\,e^{-j\omega t}\,dt\,d\mu$$

$$= \frac{1}{2\pi}\sum_k \Pi(j\omega)\int_{-\infty}^{\infty} U_m(j\mu)\,e^{j(\mu-\omega)kT_s}\,d\mu$$

$$\Pi(j\omega) = \int_{-\tau/2}^{\tau/2} e^{-j\omega t}\,dt = \tau\,\frac{\sin(\omega\tau/2)}{\omega\tau/2}\,e^{-j\omega\tau/2}$$

$$U_{S\&H}(j\omega) = \frac{1}{2\pi}\tau\omega_s\,\frac{\sin(\omega\tau/2)}{\omega\tau/2}\,e^{-j\omega\tau/2}\sum_k \int_{-\infty}^{\infty} U_m(j\mu)\,\delta(\mu-\omega-k\omega_s)\,d\mu$$

$$= \frac{\tau}{T_s}\,\frac{\sin(\omega\tau/2)}{\omega\tau/2}\sum_k U_m[j(\omega-k\omega_s)]\,e^{-j\omega\tau/2}$$

$$\sum_k e^{j(\mu-\omega)kT_s} = \omega_s\sum_k \delta(\mu-\omega-k\omega_s)$$

POASONOVA FORMULA
(SUMACIONA)

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ZADATAK 3.1 ZBORNIK

- TRI TELEFONSKA SIGNALA I JEDAN MUZIČKI

300 Hz ÷ 3.4 kHz 50 Hz ÷ 10 kHz

$f_T = 8\,kHz$ $f_M = 24\,kHz$

UČESTANOST ODABIRANDA TEL. SIGNALA UČESTANOST ODABIRANDA MUZ. SIGNALA

a) $f_{MIN} = ?$

$$T_T = \frac{1}{f_T} = 125\,\mu s$$
$$T_M = \frac{1}{f_M} = 41{,}66\,\mu s$$
$$T_M = \frac{T_T}{3}$$

$T_M/2$ – RASTOJANJE IZMEĐU IMPULSA $= \frac{T_M}{2}$

$$f_{MIN} = 2 \cdot \frac{1}{T_M} = \frac{2}{T_M} = \frac{6}{T_T} = \underline{48\,kHz}$$

b) $N = 3$ $N_2 = 1$

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ZADATAK 3.2 ZBORNIK

$N=24$

$f_0 = 8kHz$

$$y(t) = \begin{cases} 0 & , t<0 \\ Ue^{-2\pi f_c t} & , t \geq 0 \end{cases}$$

RC $f_c = \frac{1}{T_c}$

$u_1(t) \longrightarrow K_1 \longrightarrow u_1(t)$ $\quad n_1$

a) K_2, $n = 60dB$, $f_c = ?$

b) $n_2 = ?$

$$T_0 = \frac{1}{f_0} = 125\mu s$$

$$T_k = \frac{T_0}{N} = \frac{T_0}{24} \approx 5.2\mu s$$

$t = T_k$ SAMO SIGNAL IZ K_2

NA ULAZU U PRIJEMNI ODABIRAČ, SIGNAL ODZIVA LINIJE NA POBUDU U $t=0$ (SIGNAL K_1)

$t=0 \quad u_1(0) = U_1$

$t = T_k \quad u_1(T_k) = U_1 e^{-2\pi f_c T_k} = u_1(0) e^{-2\pi f_c T_k}$

$$10\log\left(\frac{\overline{u_1^2(0)}}{\overline{u_1^2(T_k)}}\right) = 10\log\left(\frac{1}{e^{-4\pi f_c T_k}}\right) = 10\log\left(e^{4\pi f_c T_k}\right) = 60dB$$

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$$10 \log\left(e^{4\pi f_c T_k}\right) = 60\,dB$$

$$\log\left(e^{4\pi f_c T_k}\right) = 6$$

$$\cancel{10}^{\log}\left(e^{4\pi f_c T_k}\right) = 10^6$$

$$e^{4\pi f_c T_k} = 10^6$$

$$\ln e^{4\pi f_c T_k} = 6 \ln 10$$

$$4\pi f_c T_k = 6 \ln 10 \simeq 13.81$$

$$f_c = \frac{13.81}{4\pi T_k} = \underline{\underline{211\,kHz}}$$

b)

$$10 \log\left(\frac{\overline{u_1^2(0)}}{\overline{(u_1(2T_k))^2}}\right) = 10 \log\left(e^{8\pi f_c T_k}\right) = 2 \cdot 60\,dB = \underline{\underline{120\,dB}}$$

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ZADATAK 3.3 ZBORNIK

$u_m(t) = U_m \cos \omega_m t \qquad U_m = 1V$

$t = k/4f_m \quad k = 0, \pm 1, \pm 2, \ldots, T_0$

a) $U_0, U_s = ?$ $\tau = 0$ DO $\tau = T_0$

b) SPEKTAR $u_i(t)$

$$u_i(t) = \begin{cases} U, & u_s(t) \geq U_0 \\ 0, & U_s(t) < U_0 \end{cases}$$

a) REGULARNO ODABIRANJE

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SIGNAL NA IZLAZU IZ KOMPARATORA

$$u_i(t) = \begin{cases} U, & u_s(t) \geq U_0 \\ 0, & u_s(t) < U_0 \end{cases}$$

MODULACIJA POLOŽAJA PREDNJE IVICE IMPULSA

TRAJANJE IMPULSA PROPORCIONALNO VREDNOSTI ODBIRKA MODULIŠUĆEG SIGNALA

$U_{MAX} = 1V \quad U_{MIN} = -1V$

$$U_{MAX} = U_0 \qquad \tau = T_0$$

$$U_{MIN} + U_s = U_0 \qquad \tau = 0$$

$$U_s = U_0 - U_{MIN} = U_{MAX} - U_{MIN} = \underline{\underline{2V}}$$

$$U_0 = \underline{\underline{1V}}$$

b)

$$U_{in} = S_{1n} + S_{2n} = \frac{U}{n\pi}\left[\sin(n3\pi/8) + \sin(n\pi/8)e^{-jn3\pi/4}\right]e^{-jn\pi/8}$$

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$$S_{1n} = 3\frac{U}{8}\frac{\sin(n3\pi/8)}{n3\pi/8}e^{-jn\pi/8} \qquad S_{2n} = \frac{U}{8}\frac{\sin(n\pi/8)}{n\pi/8}e^{-jn7\pi/8}$$

S_{1n} SPEKTAR OD $s_1(t)$

S_{2n} SPEKTAR OD $s_2(t)$

ZADATAK 3.4 ZBORNIK

N-TELEFONSKIH KANALA, VREMENSKA RASPODELA

PPM $\tau_0 = 0{,}5\,\mu s$ $T_0 = 125\,\mu s$, LPF $f_c = 165\,MHz$

ODZIV FILTRA NA ~~HEVISIDE-A~~ HEAVISIDE-A

$k = 1\,\mu s/V$

$|u_m(t)| \leq 4.65V$ NAJVEĆI BROJ KANALA U MULTIPLEKSU=?

$u(t) = x_H(t) - x_H(t-\tau_0)$ → PRAVOUGAONI IMPULS TRAJANJA τ_0

HEVISAJD

$y(t) = y_H(t) - y_H(t-t_0)$ – ODZIV FILTRA NA $u(t)$

IZOBLICENJE SIGNALA KROZ FILTAR (PREDNJE I ZADNJE IVICE IMPULSA)

$\tau_0' = \tau_0 + 1/f_c$

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$$\Delta t \leq \frac{T_0}{2N} - \frac{\tau_0'}{2} = \frac{T}{2N} - \frac{\tau_0}{2} - \frac{1}{2f_c}$$

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$$N \le \frac{T_0}{2\Delta t + \tau_0 + 1/f_c}$$

$$\Delta t_{MAX} = k\,|u_m(t)|_{MAX} = k \cdot U_m = \underline{\underline{4.65\,\mu s}}$$

$$N_{MAX} = \frac{T_0}{2\Delta t + \tau_0 + 1/f_c} = \frac{125\,\mu s}{2 \cdot 4.65\,\mu s + 0.5\,\mu s + 6.06\,\mu s} = 12.75$$

$$\Downarrow$$

$$\boxed{N_{MAX} = 12}$$

ZADATAK 3.5 ZBORNIK

$u_1(t), u_2(t), u_3(t)$, VREMENSKA RASPODELA, PAM

$f_1 = 5kHz \quad f_2 = f_3 = 2kHz$

LPF, $f_c = 20kHz$

U ODABIRAČA P

$u_1(t) \Rightarrow f_0 = 1/T_0 \ge f_1$

$u_2(t), u_3(t) \Rightarrow f_0 = 1/T_0 \ge 2f_2$

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$$\frac{T_0}{4} = \frac{k}{2f_c}, \quad k = 1, 2 \ldots$$

$$f_0 \geq f_1 = 5\,kHz \qquad f_0 \geq 2f_2 = 4\,kHz$$

$$f_0 \geq \frac{f_c}{2k} = \frac{10}{k}\,kHz$$

$$\underbrace{f_{01} = 10\,kHz \qquad f_{02} = 5\,kHz}$$

MOGUĆE VREDNOSTI KRUŽNE UČESTANOSTI OBRTANJA ODABIRAČA

$$\omega_{01} = 2\pi f_{01} \qquad \omega_{02} = 2\pi f_{02}$$

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ZADATAK 3.06. ZBORNIK

6 NEZAVISNIH SIGNALA U OOU, PAM

$f_1 = 3.4\,kHz$ – 4 SIGNALA

$f_2 = 10\,kHz$ – 2 SIGNALA

T_1/n, T_2/m, $n, m \in \mathbb{Z}$

$T_{1MAX} = 1/2f_1 = 147{,}06\,\mu s$ (ZA SIGNALE u_1, u_2, u_3 i u_4)

$T_{2MAX} = 1/2f_2 = 50\,\mu s$ (ZA SIGNALE u_5 i u_6)

T_{1MAX},

$$\frac{T_{1,MAX}}{T_{2MAX}} = 2{,}94$$

1 ODBIRAK $u_1 \div u_4$

BAR 3 u_5 i u_6

MIN BROJ JE 4 – ZBOG UNIFORMNOSTI I DA BUDE ZADOVOLJENA TEOREMA O ODABIRANJU

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ZA SLUČAJ DA JE $T = T_{2MAX}$

$$T_{2MAX} / T_{1MAX} = 0.34$$

$$T_a = \frac{T_{1MAX}}{12} = 12{,}25\,\mu s$$

$$T_b = \frac{T_{2MAX}}{6} = 8{,}33\,\mu s$$

b)

$$f_{s\,min} = \begin{cases} \frac{1}{2T_a} = 40.8\,kHz \\ \frac{1}{2T_b} \cong 60\,kHz \end{cases}$$

c) AM-SSB

$$B_{min} = 4f_1 + 2f_2 = 4 \cdot 3.4\,kHz + 2 \cdot 10\,kHz = \underline{\underline{33.6\,kHz}}$$

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TERMIČKI ŠUM

ŠUM $n(t)$ [NOISE] P_N - SNAGA ŠUMA (UKUPNA SNAGA ŠUMA)

$f \in [0, +\infty]$

$$p_N(f) \stackrel{def}{=} \frac{dP_N}{df}$$

dP_N - SNAGA ŠUMA U INTERVALU OD f DO $f+df$

SPEKTRALNA GUSTINA SNAGE ŠUMA

$$P_N = \int_0^{\infty} p_N(f)\,df = \dots$$

$$P_N = \int_{f_1}^{f_2} p_N(f)\,df = \dots$$

$p_N(f) = ?$

$$\boxed{p_N(f,T) = \frac{hf}{e^{\frac{hf}{kT}}-1}}$$

$$\xrightarrow{\text{LOPITAL}} \left.\frac{h}{\frac{h}{kT}e^{\frac{hf}{kT}}}\right|_{f=0} \Rightarrow \cancel{f_0 = kT} \quad f = 0,\ = kT$$

$p_N = p_N(T) \uparrow_T$

$$P_N = \int_0^{\infty} p_N(f)\,df = \sigma T^4$$

ŠTEFAN-BOLCMANOV ZAKON

$$P_N(f_1 f_2) = \int_{f_1}^{f_2} \underset{\substack{\uparrow \\ kT}}{p_N(f)}\,df = kT(f_2 - f_1) = \underline{\underline{kTB}}$$

$B = f_2 - f_1$

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$$P_N = kTB$$

$$P_N = \frac{N_0}{2} B \cdot k \cdot 2 = kTB$$

$$P_{N,i} = (P_{N,u} + P_{N,s})\, G$$

$$P_{Ni} = G \cdot (kT_0B + kT_eB)$$

$$P_{Ni} = G \cdot k (T_0 + T_e) \cdot B = G \cdot k \cdot T \cdot B \quad , \; T = T_0 + T_e$$

$$= F \cdot k \cdot T_0 \cdot B$$

$$F = \frac{T}{T_0} = \frac{T_0 + T_e}{T_0} = 1 + \frac{T_e}{T_0}$$

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KASKADNA VEZA

$G = ?$
$F = ?$
$T_e = ?$

$$P_{N1} = G_1 k (T_o + T_{e1}) B$$

$$P_{N2} = G_2 \left(G_1 k (T_o + T_{e1}) B + k T_{e2} B \right) = G_1 G_2 k \left(T_o + T_{e1} + \frac{T_{e2}}{G_1} \right) B$$

$$P_{Ni} = G_3 \left[G_2 G_1 k \left(T_o + T_{e1} + \frac{T_{e2}}{G_1} \right) B + k T_{e3} B \right]$$

$$P_{Ni} = G_1 G_2 \cdot G_3 k \left[T_o + T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} \right] B$$

$$\boxed{G = G_1 \cdot G_2 \cdot G_3}$$

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2}$$

$$F_1 = 1 + \frac{T_{e1}}{T_o}$$

$$F_2 = 1 + \frac{T_{e2}}{T_o}$$

$$F_3 = 1 + \frac{T_{e3}}{T_o}$$

$$F = 1 + \frac{T_e}{T_o} = 1 + \frac{T_{e1}}{T_o} + \frac{T_{e2}}{G_1 T_o} + \frac{T_{e3}}{G_1 G_2 T_o}$$

$$\boxed{F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}}$$

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n- UREĐAJA

$$G = G_1 \cdot G_2 \cdot \dots \cdot G_n$$

$$T_e = T_{e_1} + \frac{T_{e_2}}{G_1} + \dots + \frac{T_{e_n}}{G_1 \cdot G_2 \cdots G_{n-1}}$$

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 \cdot G_2 \cdots G_{n-1}}$$

$$P_N = FkT_0$$

ZADATAK 4.2 ZBORNIK

$T_L = ?$

$F_L = ?$

L, T

$$G = \frac{1}{L}$$

$$P_N = G(kT_L + kT)B = kTB$$

$$\frac{1}{L}(\cancel{k}T_L + \cancel{k}T)\cancel{B} = \cancel{k}T\cancel{B}$$

$$\frac{T_L}{L} + \frac{T}{L} = T \quad / \cdot L$$

$$T_L + T = L \cdot T$$

$$T_L = (L-1)T$$

$$F_L = 1 + \frac{T_L}{T_0}$$

$$\boxed{F_L = 1 + \frac{T}{T_0}(L-1)}$$

SPECIJALAN SLUČAJ

$T = T_0$

$F_L = \cancel{1} + L - \cancel{1} = L$!

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ZADATAK 4.3 ZBORNIK

$$g_{Ri}[dB],\ F_i$$

$$G_{Ri} = 10^{g_{Ri}/10}$$

KAKO TREBA VEZATI DA SE DOBIJE MINIMALNI FAKTOR ŠUMA?

$$F = F_1 + \frac{F_2 - 1}{G_{R1}} + \frac{F_3 - 1}{G_{R1} G_{R2}} = \dots$$

POJAČAVAČE TREBA POVEZATI TAKO DA SVAKI PRETHODNI FAKTOR ŠUMA NE BUDE VEĆI NI OD JEDNOG NAREDNOG

A – B – C $\qquad F = 1{,}998$

$$T_e = T_{e1} + \frac{T_{e2}}{G_{R1}} + \frac{T_{e3}}{G_{R1} G_{R2}} = \dots$$

$$F = 1 + \frac{T_e}{T_0} \Rightarrow T_e = (F-1)\,T_0$$

$$\underline{\underline{T_e = 289.42\,K}}$$

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ZADATAK 4.4 ZBORNIK

$T_a, T_p, d = ?$

α – PODUŽNO SLABLJENJE, $\alpha = 0{,}08 \frac{dB}{m}$, T_s

$$T_s = \frac{T_a}{L} + T\left(1 - \frac{1}{L}\right) + T_p$$

$$T_s = \frac{T_a}{L} + \frac{T_L}{L} + T_p \qquad T_L = T(L-1)$$

$$L = \frac{T - T_a}{T + T_p - T_s} = 1.02$$

$$L = 10^{0{,}1\alpha d} \qquad T = T_0 = 290\,K$$

PRETPOSTAVKA

$$d = \frac{10\log(L)}{\alpha} = \underline{\underline{1m}}$$

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- RIKIJEVA FORMA USKOPOJASNOG BELOG ŠUMA

$P_N \approx kT \approx const \rightarrow$ BELI ŠUM

$\left[f_0-\frac{B}{2}, f_0+\frac{B}{2}\right]$

$B \ll f_0$

$(f_m \ll f_0)$

$$n(t) = n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t \quad / HT$$

RIKIJEVA FORMULA

$$\omega_0 \quad X(t) = A\cos(\omega_0 t + \varphi_0)$$

$$= a\cos\omega_0 t + b\sin\omega_0 t$$

$a = const$

$b = const$

$\left[\omega_0 - \frac{B}{2}, \omega_0 + \frac{B}{2}\right]$

$\omega_0 \gg B$

$$\hat{n}(t) = n_c(t)\sin\omega_0 t + n_s(t)\cos\omega_0 t \quad / \cdot \sin\omega_0 t$$

$$n(t) = n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t \quad / \cdot \cos\omega_0 t \quad +$$

$$n_c(t) = n(t)\cos\omega_0 t + \hat{n}(t)\sin\omega_0 t$$

$$\hat{n}(t) = n_c(t)\sin\omega_0 t + n_s(t)\cos\omega_0 t \quad / \cdot \cos\omega_0 t$$

$$n(t) = n_c(t)\cos\omega_0 t - n_s(t)\sin\omega_0 t \quad / \cdot \sin\omega_0 t \quad -$$

$$n_s(t) = \hat{n}(t)\cos\omega_0 t - n(t)\sin\omega_0 t$$

$$P_N = \overline{n^2(t)_{sn}} = \overline{n^2(t)}$$

$$\boxed{\overline{n^2(t)} = \overline{\hat{n}^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}}$$

$\hat{n}(t) = HT(n(t))$ PROMENA FAZE

$$\hat{n}^2(t) = \overline{n^2(t)}$$

$$n_c^2(t) = n^2(t)\underbrace{\cos^2\omega_0 t}_{1/2} + \hat{n}^2(t)\underbrace{\sin^2\omega_0 t}_{1/2} + n(t)\hat{n}(t)\underbrace{\sin\omega_0 \cos\omega_0 t}_{0}$$

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ZADATAK 4.5 ZBORNIK

T_1, T_2

(1)

$$P_{Nu} = P_{Ns} + P_{N1} = k\,B(T_e + T_1)$$

RASPOLOŽIVA SREDNJA SNAGA ŠUMA (TEMP T_1)

SREDNJA SNAGA SOPSTVENOG ŠUMA SISTEMA

EFF. TEMPERATURA SOPSTV. ŠUMA SISTEMA

NA MS $\quad P_1 = G_R \cdot kB(T_e + T_1)$

U (2) $\quad P_2 = G_R \cdot kB(T_e + T_2)$

$$\boxed{T_e = \frac{T_1 - (P_1/P_2)\,T_2}{(P_1/P_2) - 1}}$$

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ZADATAK 4.6 ZBORNIK

$T_0 = 290K$

a) $F = ?$

$T_S = ?$

б) $U_{Nieff} = ?$

$$F_S = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = \cancel{10.26}\ \cancel{7.26}\ 10.26 \quad (10.1dB)$$

$$(7.26)$$

$$T_S = F_S \cdot T_0 = \cancel{2975.4K}\ 7.26 \cdot 290K = \cancel{2105.4K}$$

$$= 2975.4K$$

б)

$$P_{Nu} = FkT_0B = kT_S B_3 = k \cdot T_S \cdot 1MHz$$

$$P_{Ni} = G_1 G_2 G_3 \cdot P_{Nu} = G_1 \cdot G_2 \cdot G_3 \cdot kT_S B_3$$

$$U_{Nieff} = \sqrt{P_{Ni} R} = \sqrt{G_1 G_2 G_3 kT_S B_3 R} = \cancel{1.43mV}\ V$$

$$\boxed{\text{AKO JE } L < 1 \Rightarrow G = L,\ F = \frac{1}{L}}$$

$$F_S = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 6.31 + \frac{2-1}{2.0} + \frac{40-1}{0.5 \cdot 20}$$

$$= 10.26$$

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ZADATAK 4.7 ZBORNIK

$P_S = 5\,pW$

a) ODNOS $\frac{S}{N}$ NA UL. I IZL.

$$\left(\frac{S}{N}\right)_u = \frac{P_S}{kT_AB} = 402.8 \quad (26dB)$$

$$\left(\frac{S}{N}\right)_i = \frac{P_{Si}}{P_{Ni}} = \frac{A_2 \cdot P_S}{G_2 k(T_A+T_2)B} = 21.9 \quad (13.4dB)$$

$$T_2 = T_0(F_2-1) + \frac{T_0(F_2-1)}{G_1} = T_A + T_0 \cdot 0.05 G_1 + \frac{T_0(F_2-1)}{G_1}$$

$$\left.\frac{dT_S}{dG_1}\right|_{G_1=G_{1OPT}} = 0 \Rightarrow G_{1OPT} = 13.42$$

$$\left(\frac{S}{N}\right)_{ie} = \frac{P_{Si}}{P_{Ni}} = \frac{G_1 G_2 P_S}{G_1 G_2 k T_S B} = 112 \quad (20.5dB)$$

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UTICAJ ŠUMA NA PRENOS ANALOGNIH SIGNALA

ZADATAK 5.1 ZBORNIK

OOU, A_1, A_2 $n(t)$ NA ULAZU

$$n_{Ni} = n_{Nu} + a_2 = -44\,dBm$$

n_{Nu} – NIVO SREDNJE SNAGE ŠUMA NA ULAZU

$$a_2 = 10 \log A_2$$

$$n_{Si} = n_{Su} + a_1 + a_2 - a_L \cong 4\,dBm$$

NIVO SR. SNAGE SIGNALA NA ULAZU U PREDAJNIK

a_L – SLABLJENJE LINIJE VEZE [dB]

$$\left(\frac{S}{N}\right)_i [dB] = n_{Si} - n_{Ni} = n_{Su} - n_{Nu} + a_1 - a_L = 48\,dB$$

$$\left(\frac{S}{N}\right)_{i\,ZAHT.} [dB] = \left(\frac{S}{N}\right)_i [dB] + \Delta a_1 = 50\,dB$$

$$\Delta a_1 = \underline{\underline{2\,dB}}$$

$$n_{Si\,ZAHT.} = n_{Si} + \Delta a_1 + \Delta a_2 = 4\,dBm + 2\,dB + \Delta a_2 = 0\,dB$$

$$\underline{\underline{\Delta a_2 = -6\,dB}}$$

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ZADATAK 5.2 ZBORNIK

$u(t) = U\cos[(\omega_0+\omega_m)t]$, P , $0 \le f_m \le f_1$

AWGN $n(t)$ P_N

a) $\left(\frac{S}{N}\right)_i = ?$

b) f_0 , f_0+f_1 $\left(\frac{S}{N}\right)_i = ?$

c) $\Delta\left(\frac{S}{N}\right)_i [dB] = ?$

$$u_i(t) = \left(UA_u\cos[(\omega_0+\omega_m)t]\cos\omega_0 t\right)_{LPF} = \frac{UA_u}{2}\cos\omega_m t$$

$$P_{s,a} = \overline{\left(\frac{UA_u}{2}\cos\omega_m t\right)^2} = \frac{1}{4}PA_u^2 \quad , P = \frac{U^2}{2}$$

NA ULAZU U PRIJEMNIK POSTOJI USKOPOJASNI SUM

$$n(t) = n_c(t)\cos\omega_0 t + n_s(t)\sin\omega_0 t$$

$$n_i(t)_{i,a} = \left[A_u[n_c(t)\cos\omega_0 t + n_s(t)\sin\omega_0 t]\cos\omega_0 t\right]_{LPF} = \frac{1}{2}A_u n_c(t)$$

$$P_{N,a} = \int_{-f_1}^{f_1}\frac{1}{4}A^2 2P_N\,df = A^2 P_N f_1$$

$2P_N$ - SGSS SUMA $n_c(t)$ $|f| \le f_1$

$$\left(\frac{S}{N}\right)_{i,a} = \frac{P_{s,a}}{P_{N,a}} = \underline{\frac{P}{4P_N f_1}}$$

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b)

SGSS USKOPOJASNOG ŠUMA $n_1(t)$ NA IZLAZU IZ BPF JE P_N

U $f_0 \le |f| < f_0+f_1$

$$u(t)_{i,b} = \left[A_u\left[n_c(t)\cos\omega_0 t + n_s(t)\sin\omega_0 t\right]\cos\omega_0 t\right]_{LPF} = \frac{1}{2}A_u n_c(t)$$

$$P_{N_{i,a}} = \int_{-f_1}^{f_1} \frac{1}{4}A_u^2 P_N \, df = \frac{1}{2}A^2 P_N f_1$$

$$\left(\frac{S}{N}\right)_{i,b} = \frac{P_{S_{i,b}}}{P_{N_{i,b}}} = \frac{P}{2P_N f_1}$$

c)

$$\frac{\left(\frac{S}{N}\right)_{i,b}}{\left(\frac{S}{N}\right)_{i,a}} = \frac{\frac{\cancel{P}}{2\cancel{P_N}\cancel{f_1}}}{\frac{\cancel{P}}{4\cancel{P_N}\cancel{f_1}}} = \underline{\underline{2}}$$

DVA PUTA JE VEĆI U DRUGOM SLUČAJU

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ZADATAK 5.3 ZBORNIK

AM-DSB, AM-SSB, P_e, A, P_N

AM-DSB: F_1 $f_0-f_m \div f_0+f_m$

AM-SSB: F_1 $f_0 \div f_0+f_m$

AM-DSB:

$$P_{SU,AM-DSB} = \frac{P_e}{A}$$

$$P_{NU,AM-DSB} = \int_{f_0-f_m}^{f_0+f_m} P_N\,df = 2P_N f_m$$

$$\left(\frac{S}{N}\right)_{i,AM-DSB} = \frac{P_{Si,AM-DSB}}{P_{Ni,AM-DSB}} = 2\frac{P_{SU,AM-DSB}}{P_{NU,AM-DSB}} = G_{AM-DSB}\left(\frac{S}{N}\right)_{AM-DSB}$$

$$= \frac{P_e}{A P_N f_m}$$

$P_{Si,AM-DSB}$ i $P_{SU,AM-DSB}$

SREDNJE SNAGE KORISNOG SIGNALA

$P_{Ni,AM-DSB}$ i $P_{NU,AM-DSB}$

SREDNJE SNAGE ŠUMA

$G_{AM-DSB}=2$ → PROCESNO POJAČANJE SISTEMA

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AM-SSB :

$$P_{SU,AM-SSB} = \frac{P_e}{A}$$

$$P_{NU,AM-SSB} = \int_{f_0}^{f_0+f_m} P_N df = \underline{\underline{P_N f_m}}$$

$$\left(\frac{S}{N}\right)_{i,AM-SSB} = \frac{P_{Si,AM-SSB}}{P_{Ni,AM-SSB}} = \frac{P_{SU,AM-SSB}}{P_{NU,AM-SSB}} = G_{AM-SSB}\left(\frac{S}{N}\right)_{U,AM-SSB}$$

$$= \underline{\frac{P_e}{A P_N f_m}}$$

$P_{SU,AM-SSB}$, $P_{Si,AM-SSB}$

SREDNJE SNAGE KORISNOG SIGNALA

$P_{NU-AM-SSB}$, $P_{Ni,AM-SSB}$

SREDNJE SNAGE ŠUMA

$G_{AM-SSB} = 1$ → PROCESNO POJAČANJE SISTEMA

OSTVARUJE SE ISTI KVALITET PRENOSA

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ZADATAK 5.4 ZBORNIK

P, CAM, AM-DSB, AM-SSB, A, P_N

$u_m(t) = U_m \cos\omega_m t$

$f_1 \le f_m \le f_2 \qquad u_0(t) = U_0 \cos\omega_0 t$

$k_m = \frac{1}{U_0} \qquad U_m = U_0$

a) $\left(\frac{S}{N}\right)_i = ?$ KOHERENTNA DEMODULACIJA

b) $\left(\frac{S}{N}\right)_i$ NAJVEĆE OD NAVEDENIH MODULACIONIH POSTUPAKA?

a)

$$u_{CAM}(t) = \left[U_0 + k_m U_0 u_m(t)\right]\cos\omega_0 t = \left[U_0 + u_m(t)\right]\cos\omega_0 t$$

$$u_{AM\text{-}DSB}(t) = k_m U_0 u_m(t)\cos\omega_0 t = u_m(t)\cos\omega_0 t$$

$$u_{AM\text{-}SSB}(t) = k_m U_0 u_m(t)\cos\omega_0 t - k_m U_0 \hat{u}_m(t)\sin\omega_0 t$$

$$= u_m(t)\cos\omega_0 t - \hat{u}_m(t)\sin\omega_0 t$$

$$\left(\frac{S}{N}\right)_{i,CAM} = \frac{2P_{SU,CAM}}{P_{NU,CAM}} = 2\,\frac{\frac{1}{A}\frac{1}{2}\overline{u_m^2(t)}}{2P_N(f_2-f_1)} = \frac{U_m^2}{4AP_N(f_2-f_1)} = \frac{P}{8AP_N(f_2-f_1)}$$

$$\left(\frac{S}{N}\right)_{i,AM\text{-}DSB} = 2\,\frac{P_{SU,AM\text{-}DSB}}{P_{NU,AM\text{-}DSB}} = 2\,\frac{\frac{1}{A}\cdot\frac{1}{2}\overline{u_m^2(t)}}{2P_N(f_2-f_1)} = \frac{U_m^2}{4AP_N(f_2-f_1)} = \frac{P}{2AP_N(f_2-f_1)}$$

$$\left(\frac{S}{N}\right)_{i,AM\text{-}SSB} = \frac{P_{SU,AM\text{-}SSB}}{P_{NU,AM\text{-}SSB}} = \frac{\frac{1}{A}\overline{u_m^2(t)}}{P_N(f_2-f_1)} = \frac{U_m^2}{2AP_N(f_2-f_1)} = \frac{P}{AP_N(f_2-f_1)}$$

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$$P^{V}_{CAM} = \frac{1}{2}(U_0 + U_m)^2 = 2U_m^2 = P$$

$$P^{V}_{AM-DSB} = \frac{1}{2}U_m^2 = P$$

$$P^{V}_{AM-SSB} = \frac{1}{2}U_m^2 = P$$

6) NAJVEĆI ODNOS $\left(\frac{S}{N}\right)_i$ DOBIJA SE U SLUČAJU AM-SSB

$$10\log\left[\frac{(S/N)_{i,AM-SSB}}{(S/N)_{i,AM-CAM}}\right] \cong 9dB$$

$$10\log\left[\frac{(S/N)_{i,AM-SSB}}{(S/N)_{i,AM-DSB}}\right] \cong 3dB$$

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ZADATAK 5.5 ZBORNIK

CAM SIGNAL, P $\quad f_m = 1.5 f_1 \quad m_0 = 0.1$, f_0

$$u_{CAM}(t) = U_0\left[1 + m_0 m(t)\right]\cos\omega_0 t = U_0\left[1 + m_0 \cos\omega_m t\right]\cos\omega_0 t$$

$$P = \overline{u_{CAM}^2(t)} = \frac{U_0^2}{2}\left(1 + \frac{m_0^2}{4} + \frac{m_0^2}{4}\right) = P_0 + 2P_1$$

$P_0 = \frac{U_0^2}{2}$ – SREDNJA SNAGA NOSIOCA

$P_1 = \frac{m_0^2}{4} P_0 = \frac{m_0^2}{4} \frac{P}{1 + m_0^2/2}$

SREDNJA SNAGA SIGNALA U JEDNOM BOČNOM OPSEGU

$$|H(j(\omega_0 + \omega_m))| = |H(j(\omega_0 - \omega_m))| = 0.5$$

$$u_c(t) = \left[U_0 \cos\omega_0 t + \frac{m_0}{2} U_0 |H(j(\omega_0 - \omega_m))| \cdot \cos(\omega_0 - \omega_m)t + \right.$$
$$\left. + \frac{m_0}{2} U_0 |H(j(\omega_0 + \omega_m))| \cos(\omega_0 + \omega_m)t\right]\cos\omega_0 t$$

ETF BEASTWEB.ORG

$$u_i(t) = [u_c(t)]_{LPF} = 0.5\frac{w_0}{4}V_0\cos w_m t + 0.5\frac{w_0}{4}V_0\cos w_m t = \frac{w_0}{4}V_0\cos w_m t$$

$$P_{si} = \overline{u_i^2(t)} = \frac{1}{2}\left(\frac{w_0 V_0}{4}\right)^2 = \frac{V_0^2}{2}\frac{w_0^2}{16} = \frac{w_0^2}{16}\frac{P}{1+w_0^2/2}$$

$$P_{Nic} = \begin{cases} \frac{1}{4}P_N, & |w| \le w_1,\ 2w_0 - w_1 < w < 2w_0 + w_1 \\ \frac{1}{16}P_N, & w_1 < |w| < 2w_1 \\ \frac{1}{16}P_N, & 2w_0 - 2w_1 < w < 2w_0 - w_1,\ 2w_0 + w_1 < w < 2w_0 + 2w_1 \\ 0, & \text{ZA OSTALO } w \end{cases}$$

$$P_{Ni} = \begin{cases} \frac{1}{4}P_N, & |w| \le w_1 \\ \frac{1}{16}P_N, & w_1 < |w| < 2w_1 \end{cases}$$

SREDNJA SNAGA ŠUMA NA IZLAZU IZ PRIJEMNIKA:

$$P_{Ni} = 2\int_0^{f_1}\frac{1}{4}P_N\,df + 2\int_{f_1}^{f_2}\frac{1}{16}P_N\,df = \frac{5}{8}P_N f_1$$

$$\left(\frac{S}{N}\right)_i = \frac{P_{si}}{P_{Ni}} = \frac{w_0^2}{10(1+w_0^2/2)}\frac{P}{P_N f_1} \cong 10^{-3}\frac{P}{P_N f_1}$$

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ZADATAK 5.7

A - B 600 TEL. SIGNALA 60kHz ÷ 2540kHz

$L = 9km$, F

$\alpha = 0.27 \cdot 10^{-3} \sqrt{f[Hz]}$ [N/km]

a)

SREDNJA SNAGA SIGNALA U JEDNOM KANALU U TAČKI A

$P_{SB} = P_{SA} (A_P/A_L)^3$ – SREDNJA SNAGA SIGNALA U JEDNOM KANALU NA IZLAZU LINIJE VEZE U TAČKI B

A_P – POJAČAVA

A_L – SLABI

$P_N = FkT$

$P_N = P_N B_k = FkTB_k$

$$P_{NB} = P_N A_P \left[1 + \frac{A_P}{A_L} + \left(\frac{A_P}{A_L}\right)^2\right]$$

$$\left(\frac{S}{N}\right)_i = \frac{P_{SB}}{P_{NB}} = \frac{P_{SA}}{FkTB_k} \frac{\left(\frac{A_P}{A_L}\right)^3}{A_P\left[1 + \frac{A_P}{A_L} + \left(\frac{A_P}{A_L}\right)^2\right]}$$

$$A_P = A_{LV} = e^{2\alpha_{LV}} = e^{2\alpha_V L} = e^{0.27 \cdot 10^{-3}\sqrt{2540 \cdot 10^3} \cdot 9km} \cong 2321.57$$

$$\left(\frac{S}{N}\right)_{i,V} = \frac{P_{SA}}{FkTB_k} \cdot \frac{1}{3A_{LV}}$$ – NAJVIŠI KANAL

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$$\left(\frac{S}{N}\right)_{i,N} = \frac{P_{SA}}{FKTB_K} \frac{\left(\frac{A_P}{A_{LN}}\right)^3}{A_P\left[1+\frac{A_P}{A_{LN}}+\left(\frac{A_P}{A_{LN}}\right)^2\right]} = \frac{P_{SA}}{FKTB_K} \cdot \frac{\left(\frac{A_{LV}}{A_{LN}}\right)^3}{A_{LV}\left[1+\frac{A_{LV}}{A_{LN}}+\left(\frac{A_{LV}}{A_{LN}}\right)^2\right]}$$

$$A_{LN} = e^{2\alpha_N L} = 3.29$$

$$\left(\frac{S}{N}\right)_{i,N} = \frac{P_{SA}}{FKTB_K} \cdot \frac{1}{A_{LN}} \qquad \frac{A_{LV}}{A_{LN}} = e^{2L(\alpha_V-\alpha_N)} = 7028 \gg 1$$

$$10\log\left(\frac{S}{N}\right)_{i,N} - 10\log\left(\frac{S}{N}\right)_{i,V} = 10\log\left(3\frac{A_{LV}}{A_{LN}}\right) = \underline{\underline{33.2\,dB}}$$

b) POJAČANJE ZAVISI OD UČESTANOSTI

NA SVAKOJ UČESTANOSTI JEDNAKO SLABLJENJU KABLA IZMEĐU DVA POJAČAVAČA

$$\left(\frac{S}{N}\right)_{i,N} = \frac{P_{SA}}{FKTB_K} \cdot \frac{1}{3A_{LN}}$$

$$10\log\left(\frac{S}{N}\right) - 10\log\left(\frac{S}{N}\right)_{i,V} = 10\log\left(\frac{A_{LV}}{A_{LN}}\right) = \underline{\underline{28.5\,dB}}$$

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ZADATAK 5.9 ZBORNIK

AM-SSB $u(t)$, P-SREDNJA SNAGA

$B \ll f_0$

AWGN $n(t)$, P_N

$\left(\frac{S}{N}\right)_i = ?$

$P_{Si} = k_1 k_2 P$

$P_{NC} = P_N |H(jf)|^2$

$$P_{ND} = \begin{cases} P_N k_1 \left(2 - \frac{f}{B}\right), & 0 < f < B \\ P_N k_1 \left(3 - \frac{2f}{B}\right), & B < f < 3B/2 \end{cases}$$

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$$P_{NE}(f) = P_{ND}(f)\,|H_N(jf)|^2 = \begin{cases} P_N k_1 k_2 (2 - f/B), & 0 < f < B \\ P_N k_1 k_2 \left(3 - \frac{2f}{B}\right)^2, & B < f < 3/2\,B \end{cases}$$

$$P_{Ni} = \int_0^B P_N k_1 k_2 (2 - f/B)\,df + \int_B^{3B/2} P_N k_1 k_2 \left(3 - \frac{2f}{B}\right)^2 df = \frac{5}{3} P_N k_1 k_2 B$$

$$\left(\frac{S}{N}\right)_i = \frac{P_{Si}}{P_{Ni}} = \frac{3}{5}\frac{P}{P_N B}$$

$$(S/N)_i = \frac{\cancel{k_1 k_2} P}{\frac{5}{3} P_N \cancel{k_1 k_2} B} = \underline{\underline{\frac{3}{5}\frac{P}{P_N B}}}$$

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ZADATAK 5.10 ZBORNIK

AM-DSB SIGNAL

$u(t) = U_0 m(t) \cos\omega_0 t$

$m(t) = \cos\omega_m t \quad 0 \le f_m < f_0$

P – SREDNJA SNAGA SIGNALA $u(t)$

P_N, $n(t)$

– SIMETRIČAN FILTAR PROPUSNIK OPSEGA UČESTANOSTI

$$|H_e(jf)|^2 = |H_{1e}(jf)|^2 \cdot |H_{2e}(jf)|^2 = \begin{cases} 10f/B, & 0 < f < B/5 \\ 2, & B/5 \le f \le B/2 \\ -2f/B + 3, & B/2 < f \le B \\ (-2f/B+3)^2, & B \le f \le 3B/2 \end{cases}$$

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$$x_c(t) = U_0 m(t) = \sqrt{2P}\cos\omega_m t \qquad 0 \le f_m \le B$$

$$U_0\sqrt{f_m \frac{10}{B}}\cos\omega_m t\ ,\ P_i = f_m \frac{10}{B} P\ ,\ 0 < f_m \le B/5$$

$$U_0\sqrt{2}\cos\omega_m t\ ,\ P_i = 2P\ ,\ B/5 \le f_m \le B/2$$

$$U_0\sqrt{\left(3 - \frac{2}{B} f_m\right)}\cos\omega_m t\ ,\ P_i = \left(3 - \frac{2}{B} f_m\right)P\ ,\ B/2 \le f_m < B$$

$$\left(\frac{S}{N}\right)_i = \begin{cases} 5.82\, f_m / P_N B^2 P\ , & 0 < f_m \le B/5 \\ 1.165\, P / P_N B\ , & B/5 \le f_m \le B/2 \\ 0.583(3 - 2f_m/B)\, P / P_N B\ , & B/2 \le f_m \le B \end{cases}$$

ZADATAK 5.11 ZBORNIK

CAM-SIGNAL, f_m, $m_0 = 0.8$, P_N, $\left(\frac{S}{N}\right)_i = 30\,dB$

$\left(\frac{S}{N}\right)_m = ?$

$$u(t) = u_{CAM}(t) + n(t) = U_0(1 + m_0 \sin\omega_m t)\cos\omega_0 t + n_c(t)\cos\omega_m t - n_s(t)\sin\omega_m t$$

$$= A(t)\cos[\omega_0 t + \varphi(t)]$$

ANVELOPA

TRENUTNA DEVIJACIJA FAZE UKUPNOG SIGNALA

$$A(t) = U_0\sqrt{\left[1 + m_0\sin\omega_m t + \frac{n_c(t)}{U_0}\right]^2 + \left(\frac{n_s(t)}{U_0}\right)^2}$$

$$\cong U_0\left[1 + m_0\sin\omega_m t + \frac{n_c(t)}{U_0}\right]$$

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APROKSIMACIJA POD PRETPOSTAVKOM $\left[\frac{u_s(t)}{U_0}\right]^2 \ll 1$

$$u_i(t) = k_{DA}\left[U_0 m_0 \sin\omega_m t + n_c(t)\right]$$

↑ KONSTANTA DETEKTORA ANVELOPE

$$\left(\frac{S}{N}\right)_i = \frac{\overline{(k_{DA} U_0 m_0 \sin\omega_m t)^2}}{\overline{(k_{DA} n_c(t))^2}} = \frac{1}{2}\frac{(U_0 m_0)^2}{\overline{n_c^2(t)}} = \frac{P_0 m_0^2}{2P_N f_m} = 2\left(\frac{S}{N}\right)_u$$

$P_0 = \frac{U_0^2}{2}$ → SREDNJA SNAGA NOSIOCA

$$\left(\frac{S}{N}\right)_u = \frac{\overline{(U_0 m_0 \sin\omega_m t \cdot \cos\omega_0 t)^2}}{\overline{n^2(t)}} = \frac{1}{4}\frac{(U_0 m_0)^2}{\overline{n^2(t)}} = \frac{P_0 m_0^2}{4 P_N f_m}$$

a) $$10\log\left(\frac{C}{N}\right)_u = 10\log\frac{P_0}{\int_{f_0-f_m}^{f_0+f_m} P_N\,df} = 10\log\left[\frac{(S/N)_i}{m_0^2}\right] \cong 31.94\,dB$$

b)

$$P(u_N \geq U_0) = \int_{U_0}^{\infty}\frac{u_N}{\sigma^2}e^{-\frac{u_N^2}{2\sigma_N^2}}du = e^{-P_0/\sigma_N^2} = e^{-\left(\frac{C}{N}\right)_u}$$

$\sigma_N^2 = 2P_N f_m$ – SREDNJA SNAGA SUMA NA ULAZU U PRIJEMNIK

$P(u_N \geq U_0) = 0.01$

$$\left(\frac{C}{N}\right)_u = 6.63\,dB$$

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ZADATAK 5.12 ZBORNIK

$f_1 = 30\,Hz$

$f_2 = 15\,kHz$

$P_e = 1\,kW$

FM

$\Delta f_0 = 75\,kHz$

$F = 20$

$\left(\frac{S}{N}\right)_i = ?$

$a = 140\,dB$

$$\left(\frac{S}{N}\right)_i = \frac{P_{Si}}{P_{Ni}} = \frac{\Delta f_0^2\,\overline{m^2(t)}}{\int_{f_1}^{f_2} f^2 df} \cdot \frac{P_0}{FkT}$$

$$\left(\frac{S}{N}\right)_i = \frac{\Delta f_0^2 \cdot \overline{m^2(t)}}{\int_{f_1}^{f_2} f^2 df} \frac{P_e \cdot 10^{-a/10}}{FkT}$$

P_0

SREDNJA SNAGA NEMODULISANOG NOSIOCA

$|m(t)| \leq 1$

$kT \cong 4 \cdot 10^{-21}\,W/Hz$

$f_1 \ll f_2 \qquad \overline{m^2(t)} = 1/2$

$$10\log\left(\frac{S}{N}\right)_i = 10\log\left(\frac{3}{2}\,\frac{\Delta f_0^2 \cdot P_e \cdot 10^{-a/10}}{f_2^3\;FkT}\right)$$

$$= 10\log\left(\frac{3}{2}\,\frac{(75\,kHz)^2 \cdot 1W \cdot 10^{-145/10}}{(15\,kHz)^2 \cdot 20 \cdot 4 \cdot 10^{-21}\,W/Hz}\right) = \underline{\underline{8.45\,dB}}$$

ZADATAK 5.13 ZBORNIK

$u_1(t), u_2(t)$

FM, $\quad f_1 = 0\,Hz \div f_2 = 15\,Hz$

$\qquad f_3 = 15\,Hz \div f_4 = 30\,Hz$

$m_1(t), m_2(t)$ – NORMALIZOVANI MUZIČKI SIGNALI

$$\left(\frac{S}{N}\right)_{i,1} = \frac{(\Delta f_0)^2\,\overline{m_1^2(t)}}{\int_{f_1}^{f_2} f\,df}\,\frac{P_0}{FkT} \qquad \left(\frac{S}{N}\right)_{i,2} = \frac{(\Delta f_0)^2\,\overline{m_2(t)^2}}{\int_{f_3}^{f_4} f\,df}\,\frac{P_0}{FkT}$$

$$10\log\left[\frac{(S/N)_{i,1}}{(S/N)_{i,2}}\right] = 10\log\left(\frac{\int_{f_3}^{f_4} f^2 df}{\int_{f_1}^{f_2} f^2 df}\right) = \underline{8.45\,dB}$$

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ZADATAK S.17 ZBORNIK

$f_1 = 30\,Hz \quad f_2 = 15\,kHz$

$kT = 4 \cdot 10^{-21}\,\frac{W}{Hz}$

FM

$\Delta f_0 = 75\,kHz$

$F = 10$

a) $P_{0,MIN} = 10FkTB = 20FkT(\Delta f_0 + f_2)$

$B = 2(\Delta f_0 + f_2)$

$P_{0,MIN} = \underline{0.072\,pW}$

b) $P_0 = P_{0,MIN}$

$10\log\left(\frac{S}{N}\right)_{i,MIN} \simeq 10\log\left[3\frac{(\Delta f_0)^2}{f_2^3}\frac{P_{0MIN}}{2FkT}\right] = \underline{36.5\,dB}$

ZADATAK S.18 ZBORNIK

$u_{FM}(t), n(t), f_0, P_0$

$f_m, n(t)\ P_N, F_1\ f_0 - B/2 \div f_0 + B/2$

$B = 2(\Delta f_0 + f_m)$

$$\left(\frac{S}{N}\right)_i = \frac{P_{Si}}{P_{Ni}} = \frac{(\Delta f_0)^2\,\overline{m^2(t)}}{\int_{f_1}^{f_2} f^2 df}\frac{P_0}{P_N} = \frac{3\Delta f_0^2}{f_m^2}\frac{P_0}{2P_N f_m}$$

$f_1 = 0 \quad f_2 = f_m$

$|m(t)| \le 1$

$P_0 \ge 10P_N B = 20P_N f_m\left(\frac{\Delta f_0}{f_m} + 1\right)$

$P_0 \ge \frac{1}{3}\,10^{4.5}\,\frac{2P_N f_m}{\left(\frac{\Delta f_0}{f_m}\right)^2}$

a) $\frac{\Delta f_0}{f_m} = 5 \Rightarrow P_0 \ge 120P_N f_m \quad P_0 \ge 843.27\,P_N f_m$

$\underline{P_{0MIN} = 843.27\,P_N f_m}$

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$$\frac{\Delta f_0}{f_m} = 20 \quad (1)$$

$$P_0 \geq 420 P_N f_m \quad \wedge \quad P_0 \geq 52.7 P_N f_m \quad (2) \qquad P_{0,MIN} = MAX(1,2)$$

$$P_{0,MIN} = 420 P_N f_m$$

b)

$$20 P_N f_m \left(\frac{\Delta f_0}{f_m} + 1\right) = \frac{1}{3} 10^{4.5} \frac{2 P_N f_m}{\left(\frac{\Delta f_0}{f_m}\right)^2} \Rightarrow \underline{\frac{\Delta f_0}{f_m} = 10}$$

$$P_{0,MIN} = P_0\left(\frac{\Delta f_0}{f_m} = 10\right) = \underline{\underline{220 P_N f_m}}$$

NAJMANJA MOGUĆA VREDNOST SREDNJE SNAGE NOSIOCA NA ULAZU PRIJEMNIKA

ZADATAK 5.20 ZBORNIK

$$u(t) = U_0(\sin(\omega_0 t + m \sin \omega_m t)) \quad , P_0 \quad , n(t) \to P_N$$

a) D-FAZNI DEMODULATOR

b) D-FREKV. —"—

c) ~~D~~ U KOM SLUČAJU JE BOLJI $\left(\frac{S}{N}\right)_i = ?$

$$\left(\frac{S}{N}\right)_{i,PM} = (\Delta\phi_0)^2 \frac{P_0}{2 P_N f_m}$$

$$\left(\frac{S}{N}\right)_{i,FM} = \frac{3}{2} \frac{\Delta f_0^2}{f_m^3} \frac{P_0}{P_N}$$

a) $\Delta\phi_0 = m$

$$\left(\frac{S}{N}\right)_{i,PM} = m^2 \frac{P_0}{2 P_N f_m}$$

b) $(\Delta f_0 = m f_m)$

$$\delta f_i = \frac{d\varepsilon(t)}{dt} = m f_m \cos(2\pi f_m t)$$

$$\left(\frac{S}{N}\right)_{i,FM} = \frac{3}{2} m^2 \frac{P_0}{P_N f_m}$$

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c) $10\log\left[\frac{(S/N)_{iFM}}{(S/N)_{iPM}}\right] = 10\log\left[\frac{\frac{3}{2}\cancel{k^2}\frac{P_0}{\cancel{P_N f_m}}}{\cancel{k^2}\frac{P_0}{\cancel{2P_N f_m}}}\right] = 10\log 3 = \underline{\underline{4.77\,dB}}$

ZADATAK S21 ZBORNIK

$u_0(t) = U_0 \sin\omega_0 t \;,\; A \;,\; 2AB \ll U_0$

$u(t) = U_0 \sin\omega_0 t + A\,\delta(t)$ DELTA IMPULS KOJI PREDSTAVLJA SMETNJE

$$u_1(t) = U_0 \sin\omega_0 t + \frac{1}{2\pi}\int_{-\omega_0-\pi B}^{-\omega_0+\pi B} A e^{j\omega t}\,d\omega + \frac{1}{2\pi}\int_{\omega_0-\pi B}^{\omega_0+\pi B} A e^{j\omega t}\,d\omega$$

$$u_1(t) = U_0 \sin\omega_0 t + 2AB\,\frac{\sin\pi B t}{\pi B t}\cos\omega_0 t$$

$$= U(t)\sin[\omega_0 t + \varphi(t)]$$

$$U(t) = U_0\sqrt{1+tg^2\varphi(t)} \qquad \varphi(t) = \operatorname{arctg}\left[\frac{2AB}{U_0}\,\frac{\sin\pi B t}{\pi B t}\right]$$

$\sin x \cong \frac{2AB}{U_0}\frac{\sin\pi B t}{\pi B t}$

$$u_{0i} = \frac{d\varphi(t)}{dt} = \frac{d}{dt}\left[\frac{2AB}{U_0}\,\frac{\sin\pi B t}{\pi B t}\right] \qquad 2AB \ll U_0$$

$$U_{0i}(j\omega) = j\omega\int_{-\infty}^{\infty}\frac{2AB}{U_0}\,\frac{\sin\pi B t}{\pi B t}\,e^{-j\omega t}\,dt = \begin{cases} j\omega\frac{2A}{U_0}, & |\omega|<\pi B \\ 0, & |\omega|>\pi B \end{cases}$$

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$$U_i(j\omega) = \begin{cases} j\omega \frac{2A}{U_0}, & |\omega| < \pi B/4 \\ 0, & |\omega| > \pi B/4 \end{cases}$$

$$E = \frac{1}{2\pi} \int_{-\pi B/4}^{\pi B/4} |U_i(j\omega)|^2 d\omega = \frac{A^2 B^3 \pi^2}{48 U_0^2}$$

ZADATAK 5.22 ZBORNIK

0 – 5MHz

$U_{ni,eff} = 35mV$

600 TELEFONSKIH SIGNALA
60 kHz ÷ 2.54 MHz

a)

$$\left(\frac{S}{N}\right)_i = \frac{U_{si,eff}^2}{U_{ni,eff}^2} = \frac{\Delta f_0^2}{\int_{f_1}^{f_2} f^2 df} \frac{P_0}{2FkT}$$

EFEKTIVNE VREDNOSTI NAPONA

$$U_{ni,eff}^2 = \left(\frac{U_{si,eff}}{\Delta f_0}\right)^2 \frac{2FkT}{P_0} \int_{f_1}^{f_2} f^2 df = C \int_{f_1}^{f_2} f^2 df \quad , C = const$$

$U_{ni,eff} = 35mV$

$$C = \frac{(35mV)^2}{\int_0^{5MHz} f^2 df} = 29.4 \frac{mV^2}{MHz^3}$$

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NAJVIŠI TELEFONSKI KANAL $2.536\,MHz \div 2.54\,MHz$

- SPEKTAR SIGNALA GOVORA $\rightarrow 2.5363\,MHz \div$ ~~2.539 MHz~~ $2539.4\,kHz$ $(2.5394\,MHz)$

$$U_{Ni\,eff} = \sqrt{C \cdot \int_{f_1}^{f_2} f^2 df} = 0.766\,mV$$

6)

$$\left(\frac{S}{N}\right)_i = 10\log\left(\frac{U_{Ti\,eff}^2}{U_{Ni\,eff}^2}\right) = 30\,dB$$

$$U_{Ti\,eff} = 10^{1.5} \cdot U_{Ni\,eff} = 24.22\,mV$$

$$U_{Ti\,MAX} = \sqrt{2}\,U_{Ti\,eff} = 34.25\,mV$$

$$\Delta f_{0,T} = \frac{4\,MHz}{0.5\,V} \cdot U_{Ti,MAX} = \underline{\underline{274\,kHz}}$$

ZADATAK $\boxed{5.24}$ ZBORNIK

$u_m(t)$, $f_m = 15\,kHz$

$k_{LD} = 1\,[mV/kHz]$ $B = 180\,kHz$, P_0, $\Delta B = 100\,kHz$

a) $$P_{Na}(f) = k_{LD}^2 \frac{P_N}{P_0} f^2$$

$$P_{ND} = U_{eff}^2 = \int_{f_m}^{f_m+\Delta B} P_{Na}(f)\,df = k_{LD}^2 \frac{P_N}{P_0} \frac{(f_m+\Delta B)^3 - f_m^3}{3}$$

$$\cong k_{LD}^2 \frac{P_N}{P_{N0}} f_m^2 \Delta B$$

$$\underbrace{f_m \gg \Delta B}_{USLOV}$$

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$P_{op} = 10 P_N B$ – NA ULAZU U FM PRIJEMNIK, NA PRAGU PRIJEMA

$$U_{eff} = k_{LO} f_m \sqrt{\frac{\Delta B}{10B}} = \underline{\underline{0{,}158\,mV}}$$

b) $$10\log\left(\frac{S}{N}\right)_{i,p} = 10\log\left[\frac{(\Delta f_0)^2}{f_m^2}\frac{3P_{op}}{2P_N f_m}\right] = 10\log\left[15\frac{B}{f_m}\left(\frac{B}{2f_m}-1\right)\right] = \underline{\underline{36{,}5\,dB}}$$

$$\frac{\Delta f_0}{f_m} = \left(\frac{B}{2f_m}-1\right)$$

ZADATAK 5.25 ZBORNIK

$B = f_2 - f_1$, N, $u_i(t) = U_0\cos\left[\omega_i t + \Delta\omega\int_{-\infty}^{t} m_i(t)\,dt\right]$, $i = 1, 2 \dots N$

$|m_i(t)| \le 1$ $\quad \Delta f_0 = 5 f_m \ll B$

a) $$N_{MAX} = \frac{B}{B_1} = \frac{f_2 - f_1}{12 f_m}, \quad B_1 = 12 f_m = 2(\Delta f_0 + f_m) = 2\cdot 6 f_m = \underline{12 f_m}$$

b) $2N_{MAX}$ FM SIGNALA

$$B_{1,2} = \frac{B}{2N_{MAX}} = 2(\Delta f_{0,b} + f_m) = 6 f_m$$

$$\Delta f_{0,b} = \underline{\underline{2 f_m}}$$

c) $$\left(\frac{S}{N}\right)_i = \frac{(\Delta f_0)^2 \overline{m^2(t)}}{\int_0^{f_m} f^2 df}\frac{P_0}{FkT}$$

$$(\Delta f_0)^2 P_{0,a} = (\Delta f_{0,b})^2 P_{0b}$$

$$10\log\left(\frac{P_{0b}}{P_{0a}}\right) = 20\log\left(\frac{\Delta f_0}{\Delta f_{0b}}\right) = \underline{\underline{8\,dB}}$$

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ZADATAK 5.26 ZBORNIK

P_0, $A_{N_i} = \frac{(\Delta f_0)^2}{\int_0^{f_m} f^2 df} \frac{P_0}{2FkT}$ $\quad \Delta f_0 = k_f U_m$, $U_m = 1V$

$$P_0 = P_M \cdot 10^{-\frac{a}{10}} \geq 10FkTB_{FM} = 20Fk\pi(\Delta f_0 + f_m)$$

$$B_{FM} = 2(\Delta f_0 + f_m)$$

$$\Delta f_{0MAX} = \frac{P_M \cdot 10^{-\frac{a}{10}}}{2FkT} - f_m = \underline{235kHz}$$

$$k_f = \frac{\Delta f_{0,MAX}}{U_m} = 235\left[\frac{kHz}{V}\right]$$

$$10\log\left(\frac{S}{N}\right)_{i,MAX} = 10\log\left[\frac{3}{2}\left(\frac{\Delta f_{0,MAX}}{f_m}\right)^2 \frac{P_M \cdot 10^{-\frac{a}{10}}}{FkTf_m}\right] \cong \underline{\underline{51dB}}$$

$$B_{FM} = 2(\Delta f_{0,MAX} + f_m) = \underline{500kHz}$$

b) AM-DSB

$$10\log\left(\frac{S}{N}\right)_{i,AM-DSB} = 10\log\left(2\frac{P_{SU}}{P_{NU}}\right) = 10\log\left[\frac{P_M \cdot 10^{-\frac{a}{10}}}{FkTf_m}\right]$$

$$\cong \underline{\underline{25dB}}$$

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ZADATAK 5.27 ZBORNIK

a)

$$P_{u,CAM} = \frac{1}{A}\cdot P_e = \frac{1}{A}P_0 + \frac{1}{A}\cdot\frac{1}{2}\underbrace{m_0^2}_{\text{STEPEN MODULACIJE}}\underbrace{P_0}_{\text{SREDNJA SNAGA NEMODULISANOG NOSIOCA}}$$

$$P_{su,CAM} = \frac{1}{A}\cdot\frac{1}{2}\frac{m_0^2}{1+\frac{m_0^2}{2}}P_e$$

$$P_{su,CAM\,MAX} = \lim_{m_0\to 1} P_{su,CAM} = \frac{P_e}{3A}$$

$$P_{nu,CAM} = P_N B_{CAM} = 2\underline{P_N f_m}, \quad B_{CAM} = 2f_m$$

$$\left(\frac{S}{N}\right)_{i,CAM\,MAX} = 2\frac{P_{su,CAM\,MAX}}{P_{nu,CAM}} = \frac{2}{3}\frac{P_e}{A P_N B_{CAM}}$$

b) FM

$$P_{su,FM} = \frac{P_e}{A} = \frac{P_0}{A}$$

$$\left(\frac{S}{N}\right)_{i,FM} = 3m^2\frac{P_e}{2A f_m P_N} \qquad B_{FM} = 2f_m(m+1)$$

c)

$$\left(\frac{S}{N}\right)_{i,FM} = 3\left(\frac{B_{FM}}{B_{CAM}}-1\right)^2\frac{P_e}{A P_N B_{CAM}}$$

d)

$$\frac{B_{FM}}{B_{CAM}} = 1+\frac{\sqrt{3}}{3} \cong 1{,}47$$

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ZADATAK 5.28 ZBORNIK

12-TELEFONSKIH SIGNALA

AM-SSB $u_k(t) = U_k \cos(2\pi f_k t)$ $f_k = (56+4k)\,kHz$, $1 \le k \le 12$

$f_0 \gg f_{12}$, P_N

a) BLOK ŠEMA SISTEMA

U TAČKI A

$B_c = 4kHz$

U TAČKI B

$$\frac{U_k}{U_1} = \frac{f_k}{f_1} = \frac{56+4k}{60} = 1 + \frac{k-1}{15}$$

b) $$\left(\frac{S}{N}\right)_i^{(k)} \cong \frac{(\Delta f_{0,k})^2 \overline{m_k^2(t)}\, P_0}{f_k^2 P_N B_c} \qquad f_k = (56+4k)\,kHz$$

$$\left(\frac{S}{N}\right)_i^{(1)} = \dots = \left(\frac{S}{N}\right)_i^{(12)} = \frac{(\Delta f_k)^2 \overline{m_k^2(t)}\, P_0}{f_k^2 P_N B_c} = \frac{(\Delta f_{0,k})^2 \overline{m_k^2(t)}\, U_k^2}{2 f_k^2 P_N B_c} = C \cdot \frac{U_k^2}{f_k^2}$$

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- ANALOGNO - DIGITALNA KONVERZIJA

ZADATAK 6.1 ZBORNIK

PCM

$t = kT_0$, $T_0 = 125\mu s$

$|u_1(kT_0)| \leq 0{,}8V$ $\quad q = 8$

$u_1(t) = U\sin(\omega_m t + \pi/4)$, $U = 0.8V$, $f_m = 2kHz$

a) VREMENSKI OBLICI NA IZLAZU IZ ODABIRAČA

$u_0(t) = u_0(T_0) = 0{,}565V$ $\quad u_0(2T_0) = u_0(3T_0) = -0{,}565V$

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SLEDEĆI KORAK KVANTIZACIJA

$$\Delta u = \frac{[u_1(t)]_{max} - [u_1(t)]_{min}}{2} = \frac{2U}{2} = \underline{\underline{0{,}2V}}$$ KORAK KVANTIZACIJE

$$2 \le 2^n, \quad n_{min} = 3$$

$$u_g = \begin{cases} 0{,}5V, & u_0(0) = u_0(T_0) = 0{,}565V \\ -0{,}5V, & u_0(2T_0) = u_0(3T_0) = -0{,}565V \end{cases}$$

a)

NA IZLAZU IZ KVANTIZATORA

NA IZLAZU IZ KODERA

b) $u_1'(t) = U' \sin(\omega_m t + \pi/4)$

$t = 0, T_0, 2T_0, 3T_0 \quad u_1'(2T_0) = \pm 0{,}5V$

$u_1'(0) = U' \sin(\pi/4) = 0{,}5V \Rightarrow U' = 0{,}707V$

$u_2(t) = U' \sin(\omega_m t + \pi/4)$

$\Delta u(t) = (U - U') \sin(\omega_m t + \pi/4) = 0{,}093 \cdot \sin(\omega_m t + \pi/4)$

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ZADATAK 6.2 ZBORNIK

$u(t)$ – PCM, 4 NIVOA

$$p(u)=\begin{cases} ke^{-|u(t)|}, & |u(t)|\le 4V \\ 0, & \text{ZA OSTALE } u(t) \end{cases}$$

a)
$$\Delta u=\frac{[4-(-4)]}{2}=\underline{\underline{2V}}$$

b)
$$P=\int_{-4}^{4} ke^{-|u|}du=1 \Rightarrow k\cong \underline{\underline{0{,}509}}$$

$$\cancel{u^2(t)}=\overline{u^2}=\int_{-4}^{4}u^2p(u)du=1.55V^2$$

c)
$$\overline{u_N^2}=\sum_{i=0}^{3}\overline{u_{Ni}^2}=\int_{-4}^{-2}(u-u_{g0})^2p(u)du+\int_{-2}^{0}(u-u_{g1})^2p(u)du$$
$$+\int_{0}^{2}(u-u_{g2})^2p(u)du+\int_{2}^{4}(u-u_{g3})^2p(u)du=0{,}374V^2$$

$u_{g0}=-3V$

$u_{g1}=-1V \quad u_{g2}=1V \quad u_{g3}=3V$

d)
$$\left(\frac{S}{N}\right)_q=\frac{\overline{u^2}}{\overline{u_N^2}}=4.144$$

e) $\overline{u^2}=16/3 \quad \overline{u_N^2}\ 1/3$

$$p(u)=\begin{cases} 1/8, & |u(t)|\le 4V \\ 0, & \text{OSTALO} \end{cases}$$

$$\left(\frac{S}{N}\right)_q=\frac{\overline{u^2}}{\overline{u_N^2}}=2^2\cong\underline{\underline{16}}$$

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ZADATAK 6.3 ZBORNIK

$q = 2^n$

p_u UNIF $[-U/2, U/2]$, 44dB

$$\overline{u_u^2} = \int_{-U/2}^{U/2} u_u^2 p_u \, du_u = \int_{-U/2}^{U/2} u_u^2 \frac{1}{U} du_u = \underline{\underline{U^2/12}}$$

$$\overline{u_N^2} = 2\left(\overline{u_{N1}^2} + \overline{u_{N2}^2} + \overline{u_{N3}^2}\right)$$

$$= \sum_{i=1}^{q_1} \int_{x_i - \Delta x_1/2}^{x_i + \Delta x_1/2} (x - x_i)^2 p(x) dx + \sum_{i=1}^{q_2} \int_{x_i - \Delta x_2/2}^{x_i + \Delta x_2/2} (x - x_i)^2 p(x) dx$$

$$+ \sum_{i=1}^{q_3} \int_{x_i - \Delta x_3/2}^{x_i + \Delta x_3/2} (x - x_i)^2 p(x) dx$$

$$= \sum_{i=1}^{q_1} \frac{(\Delta x_1)^3}{12} p_u + \sum_{i=1}^{q_2} \frac{(\Delta x_2)^3}{12} p_u + \sum_{i=1}^{q_3} \frac{(\Delta x_3)^3}{12} p_u$$

$\overline{u_N^2}, \ldots$

$p(x) = p_u = 1/U$ $\qquad \Delta x_1 = \frac{U/8}{q/4} = \frac{U}{2q}$, $\Delta x_2 = \frac{U/8}{q/8} = \frac{U}{q}$

$$\Delta x_3 = \frac{U/4}{q/8} = \frac{2U}{q}$$

$$\overline{u_N^2} = 2\frac{p_u}{12} \cdot \left[\sum_{i=1}^{q_1} (\Delta x_1)^3 + \sum_{i=1}^{q_2} (\Delta x_2)^3 + \sum_{i=1}^{q_3} (\Delta x_3)^3\right] = \frac{37}{192} \frac{U^2}{q^2}$$

$$10 \log\left(\frac{\overline{u_u^2}}{\overline{u_{N\Sigma}^2}}\right) = 10 \log\left(\frac{16}{27} q^2\right) \geq 44\text{dB} \Rightarrow q = 2^n \geq 241.01$$

$q = 2^n$ $\underline{\underline{n_{min} = 8}}$

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ZADATAK 6.5 ZBORNIK

a) $f(t) = U\frac{t}{T}, \quad -T/2 \le t \le T/2$

$$\tau = \frac{T}{2} - t_0$$

$$u_s(t_0) = u_0 + f(t_0) = u_0 + U\frac{t_0}{T} = 0 \Rightarrow t_0 = -\frac{u_0}{U}T$$

$$\tau = T/2\left(1 + \frac{2u_0}{U}\right)$$

$$n\Delta T \le \tau < (n+1)\Delta T$$

$$0 \le \tau - n\Delta T < \Delta T$$

b) $\tau_{MAX} = T = 2\cdot\Delta T$

$$\Delta f = \frac{1}{\Delta T} = \frac{2}{T} = \underline{2048\,kHz}$$

c) $u_0 - \frac{U}{2\mu}\left[e^{-2(t_0/T)\ln(1+\mu)} - 1\right] = 0 \quad 0 \le u_0 \le U/2$

$u_0 + \frac{U}{2\mu}\left[e^{2(t_0/T)\ln(1+\mu)} - 1\right] = 0 \quad -U/2 \le u_0 \le 0$

$$\Downarrow$$

$$t_0 = -T\frac{\ln(1+2\mu u_0/U)}{2\ln(1+\mu)}, \quad 0 \le u_0 \le U/2$$

$$t_0 = T\frac{\ln(1-2\mu u_0/U)}{2\ln(1+\mu)}, \quad -U/2 \le u_0 \le 0$$

$$U(t) = F[u(t)] = \begin{cases} U\dfrac{\ln(1+2\mu u_0/U)}{2\ln(1+\mu)}, \\ -U\dfrac{\ln(1-2\mu u_0/U)}{2\ln(1+\mu)}, \end{cases}$$

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ZADATAK 6.6 ZBORNIK

a) $$\left(\frac{S}{N}\right)_{gc} = \frac{P_1+P_2}{P_{Ng_1}+P_{Ng_2}} = (1+W)\frac{2^{2(n_1+n_2)}}{2^{2n_1}+W2^{2n_2}}$$

$$V = \frac{n_1+n_2}{T_0} = 64\ kb/s \qquad W = \frac{P_1}{P_2}$$

$$T_0 = 1/f_0 = 250\mu s$$

$$\left(\frac{S}{N}\right)_{gc} = (1+W)2^{2VT_0}\frac{2^{2n_1}}{2^{4n_1}+W2^{2VT_0}}$$

$$\frac{d(S/N)_{gc}}{dn_1} = 0 \Rightarrow n_1 = \frac{1}{4}\frac{\ln(W2^{2VT_0})}{\ln 2}$$

$$\left(\frac{S}{N}\right)_{gc,MAX} = \frac{1+W}{2\sqrt{W}}2^{VT_0}$$

$$n_1 = 10,\ n_2 = 6 \quad 10\log\left[\left(\frac{S}{N}\right)_{gc\,MAX}\right] \cong 57\,dB$$

b) $$f_0' = 2f_0 = 8\,kHz$$

$$n' = \frac{V}{f_0'} = 8 \qquad \left(\frac{S}{N}\right)_{g'} = 2^{2n'} = 2^{16}$$

ZADATAK 6.7 ZBORNIK

a) MAX UČESTANOST U SPEKTRU TONSKOG SIGNALA $\Rightarrow f_T = 5\,kHz$

$$f_0 \geq 2f_T = 10\,kHz$$

$$f_L = 25\cdot 625 = 15625\,Hz$$

$$f_{IMAX} = \frac{f_L}{2} = 7812.5\,Hz$$ UČESTANOST POJAVLJIVANJA IMPULSA ZA ZAMRAČENJE ZADNJEG DELA LINIJE

b) $$\frac{1}{T_B} \leq B = 5\,MHz \quad T_2 \geq mNT_B \quad N_{MAX} = \frac{T_2 B}{m} = 4$$

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ZADATAK 6.9 ZBORNIK

$f_u \div f_v = 4f_u$

$n = 8$ $\qquad f_{0MIN} = 2f_v = 8f_u \qquad f_u' = 0 \quad f_0' = 3f_u$

$f_{0,MIN} = 6f_u$

a)

$$u_B(t) = u(t)s_1(t) = \frac{u(t)}{2} + \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n\pi/2} u(t)\cos\omega_1 t$$

$f_1 - 4f_u \geq 4f_u$ $\qquad$ SIGNAL U TAČKI B

$f_{1,MIN} = 8f_u$

$f_1 - 4f_u \div f_1 - f_u$ NAJNIŽI MOGUĆI PROPUSNI OP. F_1

$f_{c1min} = f_{1MIN} - 4f_u = 4f_u$, $f_{c2,MIN} = f_{1MIN} - f_u = 7f_u$

$f_2 - 7f_u \geq 0 \qquad f_2 - 4f_u \leq 4f_u$

$f_{2MIN} = 7f_u \qquad f_{c3MIN} = f_{2MIN} - 4f_u = 3f_u$

b) $V = nf_{0,MIN} = 8nf_u$

$V' = nf_{0MIN}' = 6nf_u \qquad V/V' = 4/3$

c)

ZADATAK 6.10 ZBORNIK

a) $f_{0,M} = \frac{1}{T_{0M}} \geq 2f_M = \underline{30kHz}$

$f_{0,M,min} = 32kHz = 4f_{0,T}$

b) $10\log\left(\frac{S}{N}\right)_{q,M} = 10\log(q_M^2) = 10\log(2^{2M}) = \underline{72dB}$

$M_{min} = 12$

c) $N_M = \frac{M \cdot 48}{N} = 24$

$N_T = 30 - N_M = 6$

d) $T_0 = 125\mu s$

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ZADATAK 6.12 ZBORNIK

a) $u_k = \tilde{u}_{k+1} + e_{q,k}$ ← GREŠKA UNETA POSTUPKOM KVANTIZACIJE SIGNALA KAZI SE $\Delta_k = u_k - \tilde{u}_k$

↑ KVANTIZIRANI ODBIRCI SIGNALA $u(t-T_0)$

$$|\Delta_k| = |u_k - \tilde{u}_k| = |u_k - u_{k-1} + e_{q,k-1}| \leq q_{DPCM} \frac{\Delta u}{2}$$

$$|u_k - u_{k-1}|_{max} + |e_{q,k-1}|_{max} = \frac{U_1}{2} + \frac{\Delta u}{2} \leq \underset{\uparrow\, q_{DPCM}}{q_{PCM}} \frac{\Delta u}{2}$$

$(\forall k)\ |u_k - u_{k-1}|_{max} = U_1/2 \quad |e_{q,k-1}|_{max} = \frac{\Delta u}{2}$

$$q_{DPCM} \geq 1 + \frac{U_1}{\Delta u}$$

b) $q_{PCM} \geq \frac{U}{\Delta u}$

c) $q_{DPCM} = 32 = 2^5 \quad q_{PCM} = 256 = 2^8 \quad V_{DPCM} = \frac{5}{T_0} \Rightarrow V_{PCM} = \frac{8}{T_0}$

$$\frac{B_{PCM}}{B_{DPCM}} = \frac{V_{PCM}}{V_{DPCM}} = 1.6$$

ZADATAK 6.13 ZBORNIK

$$\rho(n) = \begin{cases} 1 - \frac{|n|}{4}, & n = 0, \dots, 4 \\ 0, & n \text{ ZA OSTALE VR.} \end{cases}$$

N-TRENUCI ODABIRANJA

$$\boxed{\rho(i) - \sum_{k=1}^{N} a_k \rho(i-k) - \sum_{k=1}^{N} a_k \frac{1}{A_{NQ}} \delta_k = 0}$$

$\rho(i) = \frac{R_u(i)}{R_u(0)}$ → normalizovana vrednost ODBIRKA AUTOKORELACIONE F-JE $R_u(i)$ SIGNALA $u(t)$

N-JE RED FILTRA

A_{NQ} - ODNOS $\frac{S}{N}$ KVANTIZACIJE

$A_{NQ} = 2^7 \gg 1$

$\delta_{i,k}$ → KRONEKEROVA DELTA F-JA

$$a_1 \rho(0) + a_2 \rho(-1) = \rho(1)$$
$$a_1 \rho(1) + a_2 \rho(0) = \rho(2)$$

$$a_1 = \frac{6}{7} \quad a_2 = -\frac{1}{7}$$

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ZADATAK 6.14 ZBORNIK

a) $k = \Delta \cdot f_0/2$

$$u_m(t) = \frac{\Delta}{2T_0} \cdot t$$

b) $u_m(t) = \frac{2\Delta}{T_0} t$

c) $u_m(t) = \frac{\Delta}{T_0} t$

ZADATAK 6.15 ZBORNIK

$u_L(t) = \Delta \cdot \mathrm{sgn}[e(t)]$ 1Vs $f_0 = 1/T_0$

a) $t = 9T_0/2$ $u_m(t) < \hat{u}_m(t)$

b) $\left|\frac{du_m(t)}{dt}\right|_{MAX} = U_m \omega_m \leq \Delta f_0' \Rightarrow f_0' \geq \frac{U_m \omega_m}{\Delta} = 20\pi f_m = \pi f_0$

ZADATAK 6.17 ZBORNIK

$A_{NG} = 3 \cdot \frac{\overline{u_m^2}}{\Delta^2} \frac{f_s}{f_c}$ $u_m(t) = U \sin \omega_c t$ $f_c = 4kHz$

a) $\left|\frac{du_m(t)}{dt}\right|_{MAX} = U\omega_c \leq \Delta f_s$

$10 \log A_{NG} \geq 48dB$; $\overline{u_m^2} = \frac{1}{2} U^2$

$f_{s\,MIN} = \frac{1}{T_{s\,MAX}} = 473.68 kHz \Rightarrow V_{DM} = \frac{1}{T_{s\,MAX}} = 473{,}68 \left[\frac{kbit}{s}\right]$

b) $V_{PCM} = \frac{1}{T_b} = \frac{n}{T_0} = f_0 n$ $f_{0,min} = 2f_c$

$10 \log(2^{2n}) \geq 48dB \Rightarrow n_{min} = 8$

$V_{PCM,min} = f_{0,min} \cdot n_{min} = \underline{\underline{64 \frac{kbit}{s}}}$

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ZADATAK 6.19 ZBORNIK

$$A_N = \frac{\overline{u_k^2}}{\overline{u_{Ng}^2} + \overline{u_e^2}} \qquad \overline{u_e^2} = P_e \sum_{j=1}^{n} e_j^2$$

$$\overline{u_e^2} = P_e\left[(2^0\Delta u)^2 + (2^1\Delta u)^2 + \dots + (2^{n-1}\Delta u)^2\right] = \frac{2^{2n}}{3} P_e (\Delta u)^2$$

$$\overline{u^2} = \frac{U^2}{12} \qquad \overline{u_{Ng}^2} = \frac{(\Delta u)^2}{12} = \frac{U^2}{12 g^2}$$

$$A_N = \frac{\overline{u^2}}{\overline{u_{Ng}^2} + \overline{u_e^2}} = \frac{2^{2n}}{1 + 4\cdot 2^{2n} P_e} = \frac{2^{2n}}{1 + 2^{n+1}\,\mathrm{erfc}\sqrt{P_e T/P_N}}$$

$$10\log\left(\frac{A_{Ng}}{A_N}\right) = 10\log\left[n + 2^{2n+1}\,\mathrm{erfc}\left(\sqrt{P_e T/P_N}\right)\right] = 1\,dB$$

$$P_{Rp} = 0.72\,pW \qquad P_{ep} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{P_{Rp} T}{P_N}}\right) = \underline{\underline{10^{-6}}}$$

– RAZNI ZADACI

$\boxed{\underline{\underline{10.1}}}$

$$\sum_{n=1}^{N} U_n \cos(n\omega_0 t + \theta_n) \qquad u_2(t) = \sum_{n=1}^{N} U_n \cos(n\omega_1 t + \theta_n)$$

$$\Delta f = f_0 - f_1 \le f_0/2N$$

a) $$R_{11}(\tau) = \frac{1}{T_0}\left[\int_{-T_0/2}^{T_0/2} \sum_{n=1}^{N} U_n \cos(n\omega_0 t + \theta_n) \sum_{m=1}^{N} U_m \cos[m\omega_0(t+\tau) + \theta_m]\,dt\right]$$

$$= \frac{1}{2}\sum_{n=1}^{N} U_n^2 \cos n\omega_0 \tau$$

b) $$x(t) = u_1(t - t_0)\,u_2(t) = \sum_{n=1}^{N}\sum_{m=1}^{N} U_n U_m \cos(n\omega_0(t - t_0) + \theta_n)\cos(m\omega_1 t + \theta_m)$$

$$y(t) = \frac{1}{2}\sum_{n=1}^{N} U_n^2 \cos n\omega_0\left(\frac{\Delta\omega}{\omega_0} t - t_0\right)$$

$$f_g = N\Delta f = N(f_0 - f_1) \qquad f_g \le f_0/2$$

c) $$y(t) = R_{11}\left(\frac{\Delta\omega}{\omega_0} t - t_0\right)$$

$$\underline{(t_m)_{min} = 2Nt_0}$$

d) $$R_{11}(0) \ge R_{11}(\tau) \qquad t_m = \frac{\omega_0}{\Delta\omega} t_0 \Rightarrow t_0 = \frac{\Delta f}{f_0} t_m$$

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10.2

$$H(j\omega) = \begin{cases} 1, & |\omega| \le \omega_c \\ \frac{j\omega_c}{\omega}, & \omega_c \le |\omega| < \infty \end{cases}$$

$$P_{NA} = \frac{1}{2\pi} \int_0^{\omega_c} P_N \, d\omega = P_N f_c$$

$$S_{ND}(\omega) = \frac{P_N}{2} |H(j\omega)|^2, \quad |\omega| < \infty$$

$$P_{N,TRANSL} = \frac{1}{2\pi} \left[\int_{\omega_0-\omega_c}^{\omega_0+\omega_c} P_N \left(\frac{\omega_c}{\omega}\right)^2 d\omega + \int_{2\omega_0-\omega_c}^{2\omega_0+\omega_c} P_N \left(\frac{\omega_c}{\omega}\right)^2 d\omega + \dots \right]$$

$$= 2 P_N f_c \omega_c^2 \left(\frac{1}{\omega_0^2 - \omega_c^2} + \frac{1}{4\omega_0^2 - \omega_c^2} + \frac{1}{9\omega_0^2 - \omega_c^2} + \dots \right)$$

$$\frac{P_{NI}}{P_{NA}} = \frac{P_{NA} + P_{N,TRANSL}}{P_{NA}} = 1 + 2\omega_c^2 \left(\frac{1}{\omega_0^2 - \omega_c^2} + \dots \right)$$

a) $f_0 = 2f_c$

$P_{NI}/P_{NA} = 1{,}67$

b) $f_0 = 3f_c$

$P_{NI}/P_{NA} = 1.25$

c) $f_0 = 4f_c$

$P_{NI}/P_{NA} = 1.13$

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JANUAR 2013.

1. $E=2\pi V$ $T=2ms$ $u_c(t)=U_c\cos\omega_c t$ $U_c=1V$ $f_c=20kHz$

$k_{FM}=3\sqrt{10}\,\frac{kHz}{V}$

$B=?$

$f_g=1kHz \Rightarrow f_{c1}=500Hz$

$$u_1(t)=2\frac{E\tau}{T}\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}\cos(n\omega_0 t)$$

$$=\cancel{2}\cdot\frac{2\pi}{\cancel{2}}\sin\left(\frac{\cancel{2}\pi}{T}\cdot\frac{\tau}{\cancel{2}}\right)\cdot\frac{1}{\frac{\cancel{2}\pi}{T}\frac{\tau}{\cancel{2}}}\cos\omega_0 t=\underline{\underline{4\cos\omega_0 t}}$$

$$u_m(t)=u_1(t)\cdot u_c(t)=4\cos(\omega_0 t)\cos(\omega_c t)$$

$$=2(\cos((\omega_0-\omega_c)t)+\cos((\omega_0+\omega_c)t))$$

$$u_{FM}=U_0\cos(\omega_0 t+\Delta\omega_0\int u_m(t)dt)\qquad \Delta f_0=\frac{1}{2\pi}\frac{d[\varphi(t)]}{dt}=\frac{1}{2\pi}k_{FM}U_1U_c=6{,}04kHz$$

$f_c-f_{01}\div f_c+f_{01}\Leftrightarrow 19{,}5kHz\div 20{,}5kHz \Rightarrow \underline{\underline{f_m=20{,}5kHz}}$

$B=2(\Delta f_0+f_m)=2(6{,}04+20{,}5)=\boxed{53{,}08kHz}$

2. CRNO-BELA TV SLIKA

$N=625$ $b:h=4:3$ $f_s=25Hz$

$N_{eff}=N^2\frac{b}{h}=520833$ $\qquad H=ld\frac{1}{\frac{1}{10}}=ld\,10=\frac{\log 10}{\log 2}=3{,}322\frac{bit}{sim}$

$f_{MAX}=\frac{1}{2}N^2 f_s\frac{b}{h}=6.51MHz$

$I_{TV}=4N_{eff}=2{,}083Mb$

$U_{TV}=I_{TV}\cdot f_s=\cancel{26{,}03f_s}\ 26{,}0375MHz$

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FEBRUAR 2013.

1. ZADATAK 5.26 IZ ZBIRKE

a) $\left(\frac{S}{N}\right)_i = \frac{(\Delta f_0)^2 \overline{m^2(t)}}{\int_0^{f_m} f^2 df} B_0 = \frac{(\Delta f_0)^2}{\int_0^{f_m} f^2 df} \cdot \frac{P_0}{2FkT}$, $\Delta f_0 = k_f U_M$

$P_0 = P_N \cdot 10^{-\frac{a}{10}} \geq 10FkTB_{FM} = 10FkT2(\Delta f_0 + f_m)$ $\quad \Delta f_{0MAX} = \frac{P_M \cdot 10^{-\frac{a}{10}}}{20FkT} - f_m = \underline{\underline{235\ kHz}}$

$k_f = \frac{\Delta f_0}{U_M} = \underline{\underline{235\ \frac{kHz}{V}}}$ $(U_M = 1V)$

$10\log\left(\frac{S}{N}\right)_{i\,MAX} = 10\log\left(\frac{3}{2}\left(\frac{\Delta f_{0,MAX}}{f_m}\right)^2 \cdot \frac{P_M 10^{-\frac{a}{10}}}{FkTf_m}\right) \cong \underline{\underline{51dB}}$

b) $10\log\left(\frac{S}{N}\right)_{i,AM-DSB} = 10\log\left(2\frac{P_{SV}}{P_{NV}}\right) = 10\log\left(\frac{P_M \cdot 10^{-\frac{a}{10}}}{FkTf_m}\right) \cong \underline{\underline{25dB}}$

2. ZADATAK 2.16 ZBIRKA

a) $20\log\left|\frac{|H_i(jf_{0i})|}{H_i\left[j(f_{0i} \pm (\frac{B_M}{2} + B))\right]}\right| \geq 20dB$

$\log \frac{e^{-\left(1,2\frac{f_0 - f_{0i}}{B_M}\right)^2}}{e^{-\left(1,2\frac{\cancel{f_{0i}} \pm (\frac{B_M}{2}+B) - \cancel{f_{0i}}}{B_M}\right)^2}} \geq 1dB$

$\log e^{1,44\left(\frac{1}{2} + \frac{B}{B_M}\right)^2} \geq 1dB$ $\qquad \frac{B}{B_M} \geq 1,26 - 0,5$

$e^{1,44\left(\frac{1}{2} + \frac{B}{B_M}\right)^2} \geq 10 \,/\ln$ $\qquad B \geq 0,765 B_M$

$\underline{\underline{B = 0,765 B_M \approx 0,8 B_M}}$

$1,44\left(\frac{1}{2} + \frac{B}{B_M}\right)^2 \geq \ln 10$ $\quad \left(\frac{1}{2} + \frac{B}{B_M}\right)^2 \geq 1,6$

b) $N = 10$ $\quad B_0 = NB_M + (N-1)B_i = 10B_M + 9 \cdot 0,765 B_M = \underline{\underline{16,885 B_M}}$

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JUN 2013.

(1) 6.17 IZ ZBIRKE

$$A_{Ng} = 3 \cdot \frac{\overline{U_m^2}}{\Delta^2} \frac{f_s}{f_c}$$

a) $\left|\frac{dU_m(t)}{dt}\right|_{MAX} = U\omega_c \leq \Delta f_s$

$10 \log A_{Ng} \geq 48 dB$

$\overline{U_m^2} = \frac{1}{2} U^2$

$V_{DM} = \frac{1}{T_{s,MAX}} = 473{,}68 \frac{kBit}{s}$

$f_{s,MIN} = \frac{1}{T_{s,MAX}} = 473{,}68 kHz$

$f_{s,MIN} = \frac{1}{T_{s,MAX}} = 473{,}68 kHz$

b) $V_{PCM} = \frac{1}{T_b} = \frac{n}{T_0} = f_0 \cdot n$; $f_{0MIN} = 2f_c$; $10 \log(2^n) \geq 48 dB$

$V_{PCM,MIN} = f_{0,MIN} \cdot n_{MIN} = 64 \frac{kBit}{s}$ $\quad n_{MIN} = 8$

(2) 2.16. ZBIRKA, ISTO KAO FEB 2013.

JUL 2013.

(1) ZADATAK 10.11 ZBIRKA

a) $u_A(t) = U_0 \cos\left[\omega_{01} t + k_\omega \cdot \int_{-\infty}^{t} u_m(\tau) d\tau\right]$

$u_B(t) = U_0 \cos[\omega_0 t + \phi(t)][1 + u_A(t)]$

$u_C(t) = u_B^2(t) = \frac{U_0^2}{2}[1 + u_A(t)]^2 + \frac{U_0^2}{2}[1 + u_A(t)]^2 \cos[2\omega_0 t + 2\phi(t)]$

b) $f_{c1} = f_{01} - 6f_m$, $f_{c2} = f_0 + 6f_m$

$u_D(t) = U_0^2 u_A(t) = U_0^2 \cos(\omega_0 t + k_\omega \int_{-\infty}^{t} u_m(\tau) d\tau)$

$2f_0 - 2\Delta f_0 - f_m \geq f_{01} + \Delta f_0 + f_m \Rightarrow f_{01} \geq 3f + 2f_m = 17 f_m$

(2) 10.19

a) $m=1 \Rightarrow U_{D1} = U_B$; $m=2 \Rightarrow U_{D2} = U_B + AU_B$; $m=3 \Rightarrow U_{Dm} = U_b \sum_{i=1}^{m} A^{i-1} = U_b \frac{1-A^m}{1-A}$

$P_S = U^2 \left(\frac{1-A^m}{1-A}\right)^2$; $m=1 \Rightarrow n_{D1} = n_1$ $\quad U_b = \pm U$

$m=2$ $\quad n_{D2} = n_2 + An_1$; $n_{DM} = \sum_{i=1}^{m} n_i A^{m-i}$

$P_N = \overline{n_{DM}^2} = \sigma_n^2 \sum_{i=1}^{m} A^{2(m-i)} = \sigma_n^2 \frac{1-A^{2m}}{1-A^2}$; $\sigma_n^2 = \overline{n_i^2}$

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$A_{n,D} = \frac{P_S}{P_N} = \frac{U^2}{\sigma_n^2} \cdot \frac{1+A}{1-A} \cdot \frac{1-A^m}{1+A^m}$ b) $10\log\left(\frac{A_{n,D}}{A_n}\right) = 10\log\left(\frac{1+A}{1-A}\,\frac{1-A^m}{1+A^m}\right) = \underline{\underline{6dB}}$

v) $\left[10\log\left(\frac{A_{n,D}}{A_n}\right)\right]_{MAX} = 10\log\left(\lim_{n\to\infty}\left(\frac{1+A}{1-A}\cdot\frac{1-A^m}{1+A^m}\right)\right) = \underline{\underline{12.8dB}}$ $\Rightarrow m=4$

SEPTEMBAR 2013.

1. 10.21 ZBIRKA

25 NEZAVISNIH SIGNALA $f_0 = 8kHz$ $f_m = 3.3kHz$

a) $V_{PAM} = \frac{1}{T_{PAM}} = N \cdot f_0 = 25 \cdot 8 = \underline{\underline{200kHz}}$ $B_{PAM} = \frac{1}{2} \cdot V_{PAM} = \underline{\underline{100kHz}}$

b) $B_{PPM} = 20 B_{PAM} = 2MHz$ (ZA REZOLUCIJU OD 5% POTREBNO JE DA U OKVIRU JEDNOG SIGNALIZACIONOG INTERVALA T_{PAM} POSTOJI 20 POZICIJA IMPULSA BEZ PREKLAPANJA)

c) 0,5%

$n \geq \log(200) = 7.6 \Rightarrow n = 8$ $V_{PCM} = n \cdot N \cdot f_0 = 8 \cdot 25 \cdot 8 = 1600kHz$

$B_{PCM} = \frac{1}{2} V_{PCM} = \underline{\underline{800kHz}}$

2. ZADATAK 10.2 ZBIRKA URADEN PAR STRANA RANIJE

OKTOBAR 2013.

1. 10.21 ZBIRKA
2. 10.2 ZBIRKA

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