$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{+\infty} x(t)\cdot e^{-st}\,dt$$

1. $\mathcal{L}\{x(t-t_0)\} = e^{-st_0}\cdot X(s)$
2. $\mathcal{L}\{e^{-at}\cdot x(t)\} = X(s+a)$
3. $\mathcal{L}\{x(at)\} = \frac{1}{|a|}\cdot X\left(\frac{s}{a}\right)$
4. $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = s\cdot X(s)$
5. $\mathcal{L}\left\{\int_{-\infty}^{t} x(\tau)\,d\tau\right\} = \frac{1}{s}\cdot X(s)$
6. $\delta(t) \rightarrow 1$ ROC: $s \in \mathbb{C}$
7. $u(t) \rightarrow \frac{1}{s}$ ROC: $Re\{s\} > 0$
8. $-u(-t) \rightarrow \frac{1}{s}$ ROC: $Re\{s\} < 0$
9. $e^{-at}\cdot u(t) \rightarrow \frac{1}{s+a}$ ROC: $Re\{s\} > -a$
10. $-e^{-at}\cdot u(-t) \rightarrow \frac{1}{s+a}$ ROC: $Re\{s\} < -a$
11. $t\cdot u(t) \rightarrow \frac{1}{s^2}$ ROC: $Re\{s\} > 0$
12. $t\cdot e^{-at}\cdot u(t) \rightarrow \frac{1}{(s+a)^2}$ ROC: $Re\{s\} > -a$
13. $\sin(at)\cdot u(t) \rightarrow \frac{a}{s^2+a^2}$ ROC: $Re\{s\} > 0$
14. $\cos(at)\cdot u(t) \rightarrow \frac{s}{s^2+a^2}$ ROC: $Re\{s\} > 0$
15. $e^{-bt}\cdot \sin(at)\cdot u(t) \rightarrow \frac{a}{(s+b)^2+a^2}$ ROC: $Re\{s\} > -b$
16. $e^{-bt}\cdot \cos(at)\cdot u(t) \rightarrow \frac{s+b}{(s+b)^2+a^2}$ ROC: $Re\{s\} > -b$
17. $x(t)\cdot e^{-ast} = \frac{1}{(s+a)} = X(s+a)$
18. $x(t)\cdot e^{ast} = \frac{1}{(s-a)} = X(s-a)$

FURIJEOVE TRANSFORMACIJE

① SINTETIČKI OBLIK $x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$

② ANALITIČKI OBLIK $X(j\omega) = \int_{-\infty}^{+\infty} x(t)\cdot e^{-j\omega t}dt$

③ KONVERGENCIJA $E = \int_{-\infty}^{+\infty} |e(t)|^2 dt = 0;\ e(t) = x(t) - \tilde{x}(t)$

$\tilde{x}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}dt$

④ SIGNAL ČIJA FUR. TRANS. NE KONVERGIRA $x(t) = \sin(t)$

⑤ FUR. TRANSFORMACIJA PARNOG SIGNALA JE ČISTO REALNA

⑥ $x(t)$ NEPARAN TADA ZA TRANS. $X(j\omega)$ VAŽI $Re\{X(j\omega)\} = 0$

⑦ KONVOLUCIJA $x(t) * y(t) \overset{F}{\longleftrightarrow} ?$

$F\{x(t)*y(t)\} = \int_{-\infty}^{+\infty} x(t)*y(t)\cdot e^{-j\omega t}dt = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} x(\tau)y(t-\tau)d\tau\right]e^{-j\omega t}dt =$

$= \int_{-\infty}^{+\infty} x(\tau)e^{-j\omega\tau}\cdot\int_{-\infty}^{+\infty} y(t-\tau)e^{-j\omega t}\cdot e^{+j\omega\tau}d\tau dt = X(j\omega)\cdot Y(j\omega)$

⑧ F-JA PRENOSA $H(j\omega) = F\{h(t)\}$

LAPLASOVE TRANSFORMACIJE

1. SINTETIČKI OBLIK $x(t) = \frac{1}{j2\pi} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) \cdot e^{st} ds$

2. ANALITIČKI OBLIK $\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-st} dt$

3. GRANIČNE TEOREME (1) $x(0) = \lim_{s \to +\infty} s \cdot X(s)$ (2) $x(\infty) = \lim_{s \to 0} s \cdot X(s)$

4. FUNKCIJA PRENOSA KONT. LTI SISTEMA JE LAPLASOVA TRANSFORMACIJA NJEGOVOG IMPULSNOG ODZIVA

5. F-JA PRENOSA $H(s) = \mathcal{L}\{h(t)\}$

6. PR. SIGNALA ČIJA JE OBLAST KONV. PRAZAN SKUP $x(t) = 1$

ZED TRANSFORMACIJA

① ANALITIČKI SIGNAL $X(z) = \sum_{n=-\infty}^{+\infty} x[n] \cdot z^{-n}$

② GRANIČNE TEOR: ① $\lim_{z \to +\infty} X(z) = x[0]$ ⑪ $x[\infty] = \lim_{z \to 1} (1 - z^{-1}) \cdot X(z)$

③ $H(z) = \frac{1}{1 - z^{-1}}$ SE NAZIVA AKUMULATOR

④ OBLAST KONVERG. PRIMER: ① PRAZAN SKUP $x[n] = 1$
⑪ SKUP KOMPLEKS. BR. $\delta[n]$

$\delta[n]$	1	$z \in \mathbb{C}$
$-u[-n-1]$	$\frac{z}{z-1}$	$\lvert z \rvert < 1$
$a^n u[n]$	$\frac{z}{z-a}$	$\lvert z \rvert > \lvert a \rvert$
$-a^n u[-n-1]$	$\frac{z}{z-a}$	$\lvert z \rvert < \lvert a \rvert$
$u[n]$	$\frac{z}{z-1}$	$\lvert z \rvert > 1$

⑤ F-JA PRENOSA

$$H(z) = Z\{h(t)\}$$

⑥ KONVOLUCIJA

$$Z\{x[n] * y[n]\} = \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k] y[n-k] \right) z^{-n} =$$

$$= \sum_{k=-\infty}^{\infty} x[k] z^{-k} \sum_{n=-\infty}^{\infty} y[n-k] z^{-(n-k)} = X(z) Y(z)$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

① $e^{j\omega_0 t} \longrightarrow 2\pi\delta(\omega-\omega_0)$

② $x(t-t_0) \longrightarrow e^{-j\omega t_0} \cdot X(j\omega)$

③ $x(t)\cdot e^{j\omega_0 t} \longrightarrow X(j(\omega-\omega_0))$

④ $x(at) \longrightarrow \frac{1}{|a|}\cdot X(j\frac{\omega}{a})$

⑤ $\frac{dx}{dt} \longrightarrow j\omega \cdot X(j\omega)$

⑥ $\int_{-\infty}^{t} x(t)\,dt \longrightarrow \frac{1}{j\omega}\cdot X(j\omega) + \pi\cdot X(j\cdot 0)\cdot\delta(\omega)$

⑦ $u(t) \longrightarrow \frac{1}{j\omega} + \pi\delta(\omega)$

⑧ $x(t)\cdot y(t) \longrightarrow \frac{1}{2\pi}\left[X(j\omega) * Y(j\omega)\right]$

⑨ $x^*(t) \longrightarrow X^*(-j\omega)$

$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$

⑩ $e^{-at}u(t) \longrightarrow \frac{1}{a+j\omega}$

⑪ $e^{at}u(-t) \longrightarrow \frac{1}{a-j\omega}$

⑫ $\delta(\omega) \longrightarrow \frac{1}{2\pi}$

⑬ $\sin(at) \longrightarrow \frac{1}{2j}\left(e^{jat} - e^{jat}\right)$

⑭ $\cos(at) \longrightarrow \frac{1}{2}\left(e^{jat} + e^{-jat}\right)$

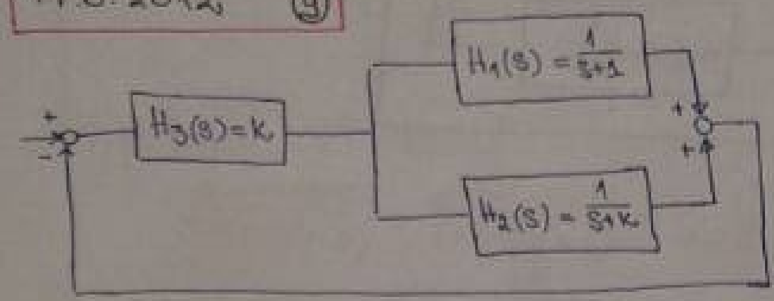
⑮ $E_x = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$ → FREKVENCIJSKI DOMEN

[illegible]

⑯ $E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt$ → DEFINICIJA ZA ENERGIJU U VREMENSKOM DOMENU

STRUKTURNI BLOK DIJAGRAM

17.6.2012. (9)



$H_4 = H_1 + H_2$ $\qquad$ $H_e = \frac{H_5}{1+H_5}$

$H_5 = H_3 \cdot H_4$

$$H_4 = \frac{1}{s+1} + \frac{1}{s+K_1} = \frac{s+1+s+K_1}{(s+1)(s+K)} = \frac{2s+K_1+1}{(s+1)(s+K_1)}$$

$$H_5 = \frac{K_1(2s+K_1+1)}{(s+1)(s+K_1)}$$

$$H_e = \frac{\frac{K_1(2s+K_1+1)}{(s+1)(s+K_1)}}{1 + \frac{K_1(2s+K+1)}{(s+1)(s+K)}} = \frac{\frac{K_1(2s+K+1)}{(s+1)(s+K)}}{\frac{(s+1)(s+K)+K_1(2s+K+1)}{(s+1)(s+K_1)}}$$

$$H_e = \frac{K_1(2s+K_1+1)}{s^2+sK+s+K_1+2Ks+K^2+K_1} = \frac{K_1(2s+K+1)}{s^2+s(K_1+1)+K_1+2Ks+K_1(1+K_1)}$$

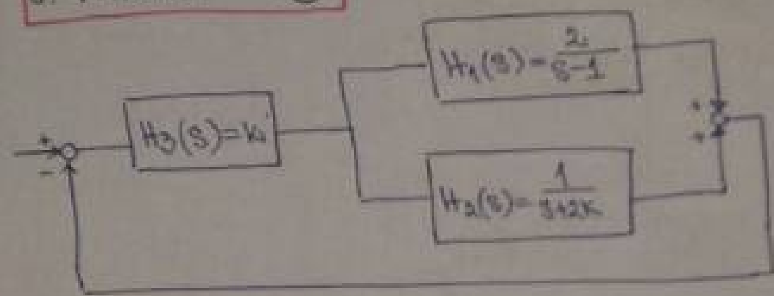
$K_1+1>0 \Rightarrow \boxed{K_1>-1}$

$\boxed{K_1>0}$

$K_1(1+K_1)>0 \Rightarrow \boxed{K=0} \vee \begin{matrix} 1+K_1>0 \\ \boxed{K>-1} \end{matrix}$

$$\boxed{K \in (0, +\infty)}$$

8.7.2012. 12



$$H_4 = H_1 + H_2 = \frac{2}{s-1} + \frac{1}{s+2k} = \frac{2(s+2k)+s-1}{(s-1)(s+2k)} = \frac{2s+4k+s-1}{(s-1)(s+2k)}$$

$$H_4 = \frac{3s+4k-1}{(s-1)(s+2k)}$$

$$H_e = \frac{H_5}{1+H_5}$$

$$H_5 = \frac{k(3s+4k-1)}{(s-1)(s+2k)}$$

$$H_e = \frac{\frac{k(3s+4k-1)}{\cancel{(s-1)(s+2k)}}}{\frac{(s-1)(s+2k)+k(3s+4k-1)}{\cancel{(s-1)(s+2k)}}} = \frac{k(3s+4k-1)}{s^2+2sk-s-2k+3sk+4k^2-k}$$

$$H_e = \frac{k(3s+4k-1)}{s^2+5sk-s-3k+4k^2} = \frac{k(3s+4k-1)}{s^2+s(5k-1)+k(4k-3)}$$

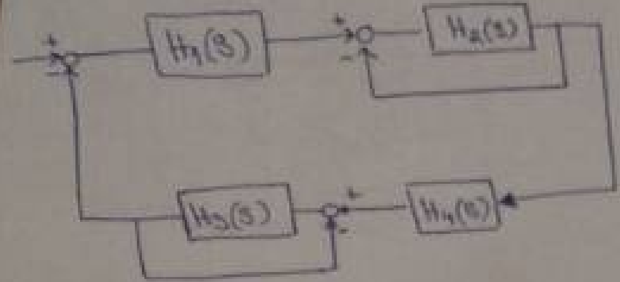
$$5k-1>0 \Rightarrow 5k>1$$
$$\boxed{k>\frac{1}{5}}$$

$\boxed{k=0}$

$$4k-3>0 \Rightarrow 4k>3$$
$$\boxed{k>\frac{3}{4}}$$

$$\boxed{k \in \left(\frac{3}{4}, +\infty\right)}$$

.6. 2011. 7

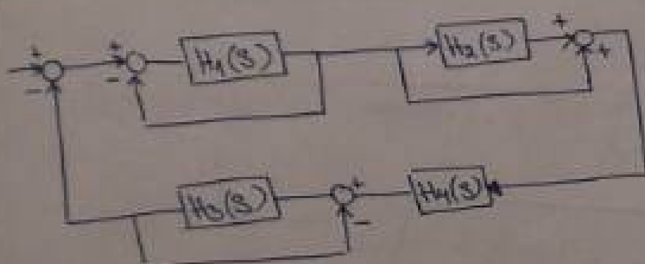


$$H_5 = \frac{H_2}{1+H_2} \qquad H_7 = \frac{H_3}{1+H_3}$$

$$H_6 = H_1 \cdot H_5 \qquad H_8 = H_7 \cdot H_4$$

$$\boxed{H_e = \frac{H_6}{1+H_6 \cdot H_8}}$$

17. 6. 2010. 5



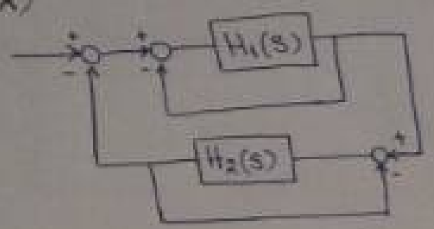
$$H_5 = \frac{H_1}{1+H_1} \qquad H_6 = 1+H_2$$

$$H_7 = \frac{H_3}{1+H_3} \qquad H_8 = H_4 \cdot H_3$$

$$\boxed{H_e = \frac{H_5 \cdot H_6}{1+H_5 H_6 \cdot H_8}}$$

7.6.2009 ⑥

A)



$$H_3 = \frac{H_1}{1+H_1} \qquad H_4 = \frac{H_2}{1+H_2}$$

$$\boxed{H_e = \frac{H_3}{1+H_3 \cdot H_4}}$$

B) $H_1 = \dfrac{k_1}{s+1} \qquad H_2 = \dfrac{1}{s-1}$

$$H_3 = \frac{\frac{k_1}{s+1}}{1+\frac{k_1}{s+1}} = \frac{\frac{k_1}{s+1}}{\frac{s+1+k_1}{s+1}} = \frac{k_1}{s+1+k_1}$$

$$H_4 = \frac{\frac{1}{s-1}}{1+\frac{1}{s-1}} = \frac{\frac{1}{s-1}}{\frac{s-1+1}{s-1}} = \frac{1}{s}$$

$$H_e = \frac{\frac{k_1}{s+1+k_1}}{1+\frac{k_1}{s(s+1+k)}} = \frac{\frac{k_1}{s+1+k_1}}{\frac{s(s+1+k_1)+k}{s(s+1+k)}} = \frac{k_1 s}{s^2+s+sk+k_1} = \frac{k_1 s}{s^2+s(1+k)+k_1}$$

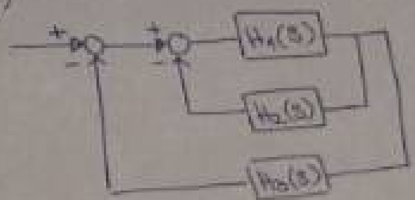
$\boxed{k_1 > 0}$

$k_1+1 > 0 \Rightarrow \boxed{k_1 > -1}$

$$\boxed{k_1 \in (0, +\infty)}$$

1.7.2008 5

A)



$$H_4 = \frac{H_1}{1+H_1H_2} \qquad H_5 = \frac{H_4}{1+H_3H_4}$$

$$\boxed{H_E = \frac{H_1}{1+H_1H_2+H_1H_3}}$$

B) $H_1 = \frac{k_1}{s+1} \quad H_2 = \frac{1}{s+2} \quad H_3 = 1$

$$H_E = \frac{\frac{k_1}{s+1}}{1+\frac{k_1}{(s+1)(s+2)}+\frac{k_1}{s+1}} = \frac{\frac{k_1}{s+1}}{\frac{(s+1)(s+2)+k_1+k_1(s+2)}{(s+1)(s+2)}} = \frac{k_1(s+2)}{s^2+3s+2+k_1+k_1s+2k_1}$$

$$H_E = \frac{k_1(s+2)}{s^2+3s+k_1+k_1s+2+2k_1} = \frac{k_1(s+2)}{s^2+s(3+k_1)+(3k_1+2)}$$

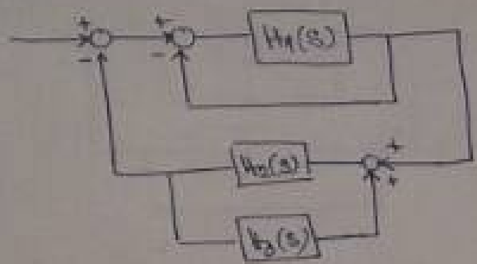
$3+k_1>0 \Rightarrow \boxed{k_1>-3}$

$3k_1+2>0 \Rightarrow 3k_1>-2$

$\boxed{k_1>-\frac{2}{3}}$

$$\boxed{k_1 \in \left(-\frac{2}{3}, +\infty\right)}$$

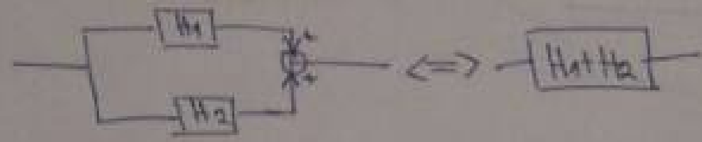
27.6.2007. 6

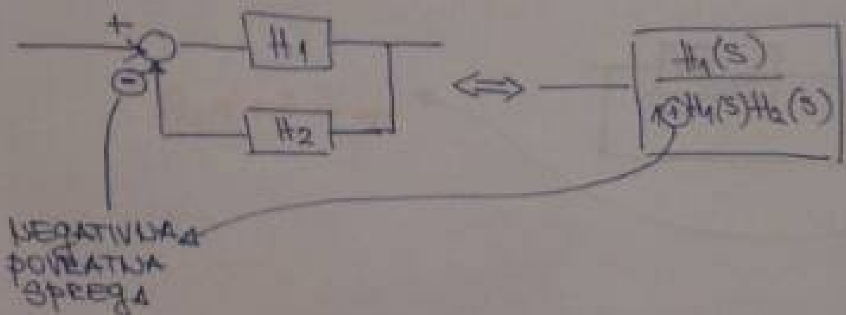


$$H_4 = \frac{H_1}{1 + H_1} \qquad H_5 = \frac{H_2}{1 - H_2 H_3}$$

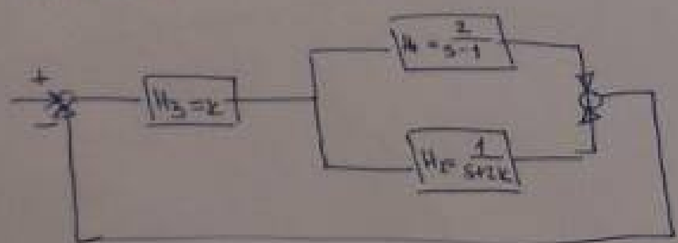
STRUKTURNI BLOK DIJAGRAM

① 

② 

③ 

8.7.2012.



$H_A = H_1 + H_2$

$H_B = H_3 \cdot H_A$

$He = \frac{H_B}{1+H_B}$

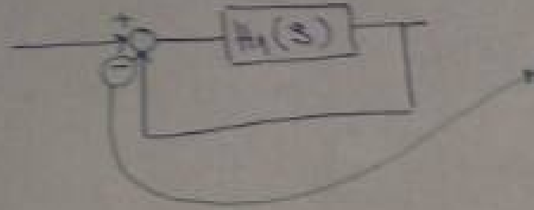
$$He = \frac{k(3s+4k-1)}{1 \cdot s^2 + (5k-1)s + k(4k-3)}$$

↓

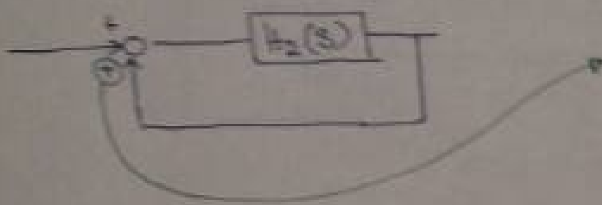
AKO JE UZ s^2 POZITIVAN SVI TREBAJU DA BUDU U ⊕

$5k-1>0 \quad \wedge \quad 4k^2-3k>0$

$$\boxed{k \in \left(\frac{3}{4}, +\infty\right)}$$



$$H_e = \frac{H_1(s)}{1+H_1(s)}$$



$$H_e = 1 + H_2(s)$$

BODEOVE KARAKTERISTIKE

8.7.2012

$$H(j\omega)=250\,\frac{(1+j\omega)}{(0{,}5+j\omega)(5+j\omega)(10+j\omega)}$$

$$x(t)=5\sin(2t)$$

SKICIRANJE BOD. KARAKT.: (FREKVENC. KARAKT.)

$$H(j\omega)=250\,\frac{(1+j\omega)}{0{,}5\left(1+\frac{j\omega}{0{,}5}\right)\cdot 5\left(1+\frac{j\omega}{5}\right)\cdot 10\left(1+\frac{j\omega}{10}\right)}$$

$$\boxed{|H(j\omega)|_{dB}=20\log|H(j\omega)|}$$

$$|H(j\omega)|_{dB}=20\log 10+20\log\sqrt{1+\left(\frac{\omega}{1}\right)^2}-20\log\sqrt{1+\left(\frac{\omega}{0{,}5}\right)^2}-20\log\sqrt{1+\left(\frac{\omega}{5}\right)^2}-$$
$$-20\log\sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

$\omega<0{,}5$ $\quad |H(j\omega)|_{dB}=20\log 10=20$

$$20\log\frac{\omega}{1}-20\log\frac{\omega}{0{,}5}=\frac{\cancel{20}\log\frac{\omega}{1}}{\cancel{20}\log\frac{\omega}{0{,}5}}=\log\frac{\frac{\omega}{1}}{\frac{\omega}{0{,}5}}=\log\frac{0{,}5}{1}$$

$\omega\in(0{,}5,1)$ $\quad |H(j\omega)|_{dB}=20-20\log\frac{\omega}{0{,}5}$

$$|H(j\cdot 1)|_{dB}=20-20\log\frac{1}{0{,}5}\cong 14dB$$

$\omega\in(1,5)$ $\quad |H(j\omega)|_{dB}=20-20\log\frac{\omega}{0{,}5}+20\log\frac{\omega}{1}=$

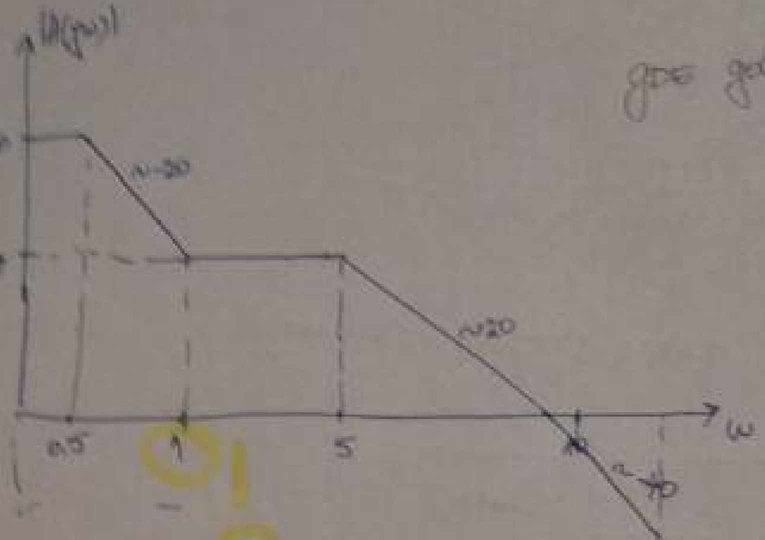
$$=20-20\log\frac{1}{0{,}5}=14dB$$

$\omega\in(5,10)$ $\quad |H(j\omega)|_{dB}=14dB-20\log\frac{\omega}{5}$

$$|H(j\cdot 10)|_{dB}=14dB-20\log 2\approx 8$$

$\omega>10$ $\quad |H(j\omega)|_{dB}=14dB-20\log\frac{\omega}{5}-20\log\frac{\omega}{10}=$

$$=14dB-40\log\omega+20\log 50$$



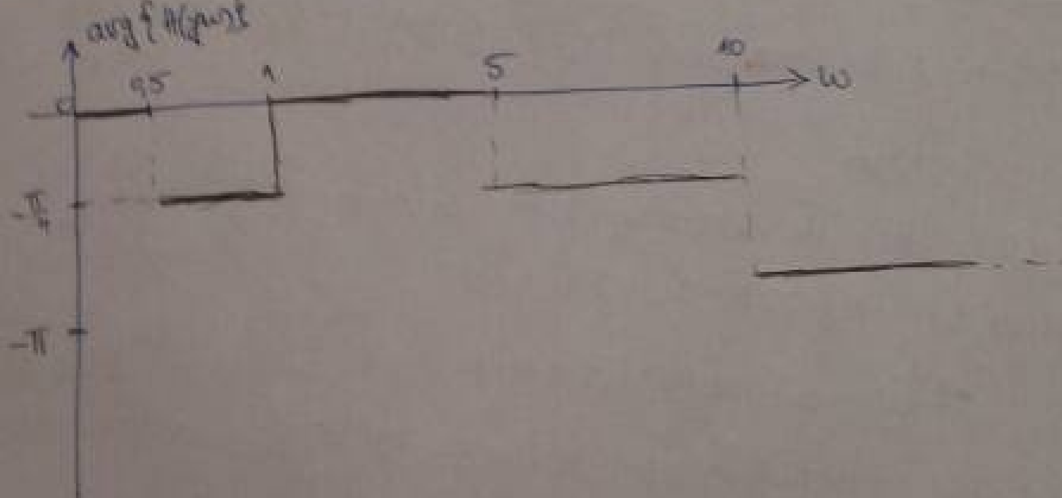
gde god ye $-20 = -\frac{\pi}{2}$

$-40 = -\pi$

$-60 = -\frac{3\pi}{2}$

$20 = \frac{\pi}{2}$ ...

② FAZNA ASIMPTOTSKA KARAKTER:



$$\arg\{H(j\omega)\} = \arg\frac{\omega}{1} - \arg\frac{\omega}{0{,}5} - \arg\frac{\omega}{5} - \arg\frac{\omega}{10}$$

(KAD GOD YE CONST NA GORNJOJ SLICI UVEK YE $\underline{\underline{0}}$!)

$\omega < 0{,}5$	$\arg\{H(j\omega)\} = 0$
$\omega \in (0{,}5;1)$	$= -\frac{\pi}{2}$
$\omega \in (1,5)$	$= -\frac{\pi}{2} + \frac{\pi}{2} = 0$
$\omega \in (5,10)$	$= -\frac{\pi}{2}$
$\omega > 10$	$= -\pi/2 - \frac{\pi}{2} = -\pi$

ODZIV NA POBUDU $x(t) = 5\sin(2t)$

$$\boxed{y(t) = 5 \cdot A \sin(2t + \varphi)}$$

$\omega = 2$

$A_{dB} = 20 \log A$

jer lijeva para

$A_{dB} = 14 - \cancel{0} \cdot \log \frac{\omega}{7} = 14$

$A = 10^{\frac{14}{20}} \times 5$

$\varphi = \operatorname{arctg} \frac{2}{7} - \operatorname{arctg} \frac{2}{0{,}5} - \operatorname{arctg} \frac{2}{5} - \operatorname{arctg} \frac{2}{10} = -0{,}8 \text{ rad}$

$y = 25 \sin(2t - 0{,}8)$

BODEOVE KARAKTERISTIKE

$\boxed{17.6.2012. \quad ⑧}$

$$H(j\omega) = 250\,\frac{(1+j\omega)^2}{(0{,}5+j\omega)(5+j\omega)(10+j\omega)} \quad ; x(t) = 5\sin(2t)$$

a)

$$H(j\omega) = \overset{10}{250}\,\frac{(1+j\omega)^2}{0{,}5\left(1+\frac{j\omega}{0{,}5}\right)\cdot 5\left(1+\frac{j\omega}{5}\right)\cdot 10\left(1+\frac{j\omega}{10}\right)}$$

$$H(j\omega) = 10\,\frac{(1+j\omega)^2}{\left(1+\frac{j\omega}{0{,}5}\right)\left(1+\frac{j\omega}{5}\right)\left(1+\frac{j\omega}{10}\right)}$$

$$|H(j\omega)|_{dB} = 20\log|H(j\omega)|$$

$$|H(j\omega)|_{dB} = 20\log 10 + 40\log\sqrt{1+\left(\tfrac{\omega}{1}\right)^2} - 20\log\sqrt{1+\left(\tfrac{\omega}{0{,}5}\right)^2} - 20\log\sqrt{1+\left(\tfrac{\omega}{5}\right)^2} - 20\log\sqrt{1+\left(\tfrac{\omega}{10}\right)^2}$$

$\omega < 0{,}5 \quad |H(j\omega)|_{dB} = 20\,dB$

$\omega \in (0{,}5, 1) \quad |H(j\omega)|_{dB} = 20 - 20\log\frac{\omega}{0{,}5}$

$$|H(j\cdot 1)|_{dB} = 20 - 20\log\frac{1}{0{,}5} \cong 14\,dB$$

$\omega \in (1, 5) \quad |H(j\omega)|_{dB} = 20 - 20\log\frac{\omega}{0{,}5} + 40\log\frac{\omega}{1} =$

$= 20 - 20(\log\omega - \log 0{,}5) + 40(\log\omega - \log 1) =$

$= 20 - 20\log\omega + 20\log 0{,}5 + 40\log\omega - 40\log 1 =$

$= 20 + 20\log\omega + 20\log 0{,}5 - 20\log 1^2 = 20 + 20\log\omega + 20\log\frac{0{,}5}{1}$

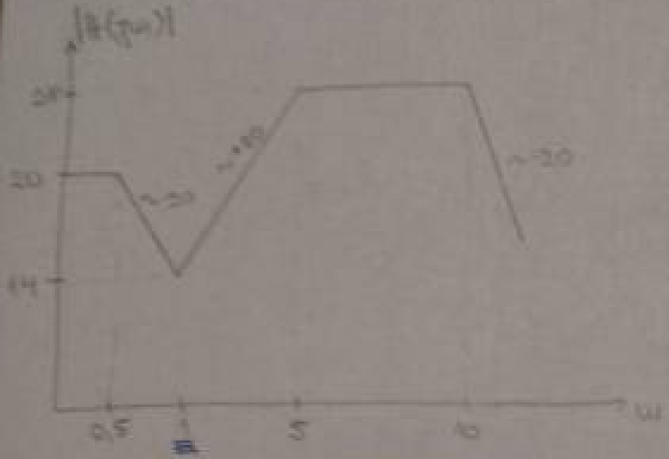
$$|H(j\cdot 5)|_{dB} = 20 + 20\log 5 + 20\log\frac{0{,}5}{1} = 14\,dB + 20\log 5 \cong 28\,dB$$

$\omega \in (5, 10) = 20 + 20\log\omega + 20\log\frac{0{,}5}{1} - 20\log\frac{\omega}{5} =$

$= 20 + 20\cancel{\log\omega} + 20\log\frac{0{,}5}{1} - 20\cancel{\log\omega} + 20\log 5 =$

$= 20 + 20\log\frac{0{,}5}{1} + 20\log 5 \cong 28\,dB$

$\omega > 10 \quad |H(j\omega)|_{dB} = 28{,}018 - 20\log\frac{\omega}{10}$



argg KVADRATA

$\arg\{H(j\omega)\} = 2\arg\frac{\omega}{1} - \arg\frac{\omega}{0{,}5} - \arg\frac{\omega}{5} - \arg\frac{\omega}{10}$

$0 \Rightarrow 0$

$-20 = -\frac{\pi}{2} \qquad +20 = \frac{\pi}{2}$

$-40 = -\pi \qquad +40 = \pi$

$-60 = -\frac{3\pi}{2} \qquad +60 = \frac{3\pi}{2}$

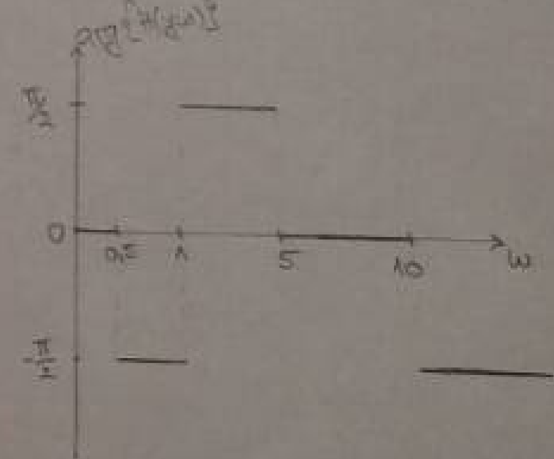
$\omega < 0{,}5 \quad \arg\{H(j\omega)\} = 0$

$\omega \in (0{,}5\,,1) \quad \arg\{H(j\omega)\} = -\frac{\pi}{2}$

$\omega \in (1,5) \quad \arg\{H(j\omega)\} = \frac{\pi}{2}$

$\omega \in (5,10) \quad \arg\{H(j\omega)\} = 0$

$\omega > 10 \quad \arg\{H(j\omega)\} = -\frac{\pi}{2}$



$(t) = 5 \sin(2t)$

$w = 2$

$y(t) = 5 \cdot A \cdot \sin(2t \cdot \varphi)$

$A_{dB} = 20 \log A$

$A_{dB} = 14 + 20 \cdot \log \frac{w}{1} = 14 + 20 \log \frac{2}{1} \approx 20\,dB$

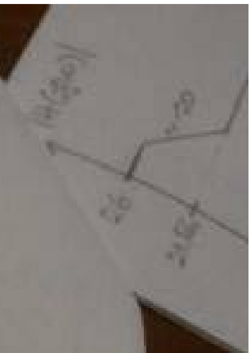
$20 \log A \approx 20\,dB$

$A \approx 10^{1}$

$\varphi = 2 \operatorname{arctg} \frac{2}{1} - \operatorname{arctg} \frac{2}{0{,}5} - \operatorname{arctg} \frac{2}{5} - \operatorname{arctg} \frac{2}{10} \approx 0{,}31\,rad$

$\Rightarrow \boxed{y(t) = 50 \sin(2t + 0{,}31)}$

19.6.2011. ⑤



$$H(j\omega) = \frac{1200(j\omega+1)}{(j\omega+0{,}6)(j\omega+10)^2}$$

$$H(j\omega) = 1200\,\frac{(j\omega+1)}{0{,}6\left(1+\frac{j\omega}{0{,}6}\right)100\left(1+\frac{j\omega}{10}\right)^2}$$

$$H(j\omega) = 20\,\frac{(j\omega+1)}{\left(1+\frac{j\omega}{0{,}6}\right)\left(1+\frac{j\omega}{10}\right)^2}$$

$$|H(j\omega)|_{dB} = 20\log 20 + 20\log\sqrt{1+\left(\frac{\omega}{1}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{0{,}6}\right)^2} - 40\log\sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

$\omega < 0{,}6$ $\quad |H(j\omega)|_{dB} = 20\log 20 \approx 26\,dB$

$\omega \in (0{,}6, 1)$ $\quad |H(j\omega)|_{dB} = 26 - 20\log\frac{\omega}{0{,}6}$

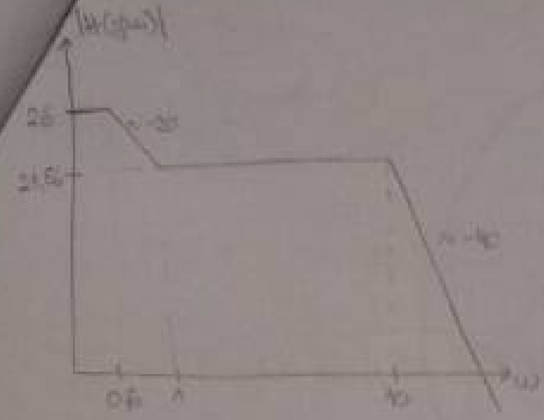
$$|H(j1)|_{dB} = 26 - 20\log\frac{1}{0{,}6} \cong 21{,}56\,dB$$

$\omega \in (1, 10)$ $\quad |H(j\omega)|_{dB} = 26 - 20\log\frac{\omega}{0{,}6} + 20\log\frac{\omega}{1} =$

$= 26 - 20(\log\omega - \log 0{,}6) + 20(\log\omega - \log 1) =$

$= 26 - 20\log\omega + 20\log 0{,}6 + 20\log\omega - 20\log 1 \cong 21{,}56\,dB$

$\omega > 10$ $\quad |H(j\omega)|_{dB} = 21{,}56 - 40\log\frac{\omega}{10}$



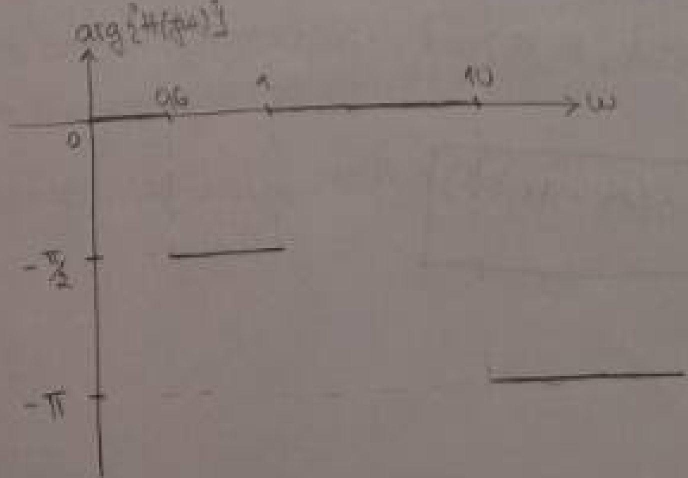
$$\arg\{H(j\omega)\} = \arg\frac{\omega}{1} - \arg\frac{\omega}{0{,}6} - \arg\frac{\omega}{10}$$

$\omega < 0{,}6 \quad \arg\{H(j\omega)\} = 0$

$\omega \in (0{,}6, 1) \quad \arg\{H(j\omega)\} = -\frac{\pi}{2}$

$\omega \in (1, 10) \quad \arg\{H(j\omega)\} = 0$

$\omega > 10 \quad \arg\{H(j\omega)\} = -\pi$



B) $x(t) = 7\sin(8t + \frac{\pi}{2})$

$y(t) = 7 \cdot A \sin(8t + \theta)$

$\theta = \frac{\pi}{2} + \varphi$

$A_{dB} = 20\log A$

$A_{dB} = 21{,}56 + 0 \cdot \log\frac{\omega}{1} \cong 21{,}56\,dB$

$20\log A \cong 21{,}56$

$A = 10^{\frac{21{,}56}{20}} \cong 11{,}97$

$\varphi = \operatorname{arctg}\frac{\omega}{1} - \operatorname{arctg}\frac{\omega}{0{,}6} - 2\operatorname{arctg}\frac{\omega}{10}$

$\varphi = \operatorname{arctg}\frac{8}{1} - \operatorname{arctg}\frac{8}{0{,}6} - 2\operatorname{arctg}\frac{8}{10}$

$\varphi = 82{,}87 - 85{,}71 - 77{,}31 = -80{,}15\,rad$

$\theta = \frac{\pi}{2} - 80{,}15\,rad = (1{,}57 - 80{,}15)\,rad = -78{,}58\,rad$

$$\boxed{y(t) = 83{,}96\sin(8t - 78{,}58)}$$

.7.2011. 6

$$H(j\omega) = \frac{800(j\omega+1)}{(j\omega+0{,}8)(j\omega+10)^2}$$

$$H(j\omega) = 800\,\frac{(j\omega+1)}{0{,}8\left(1+\frac{j\omega}{0{,}8}\right)100\left(1+\frac{j\omega}{10}\right)}$$

$$H(j\omega) = 10\,\frac{(j\omega+1)}{\left(1+\frac{j\omega}{0{,}8}\right)\left(1+\frac{j\omega}{10}\right)}$$

$$|H(j\omega)|_{dB} = 20\log 10 + 20\log\sqrt{1+\left(\frac{\omega}{1}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{0{,}8}\right)^2} - 40\log\sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

$\omega < 0{,}8 \quad |H(j\omega)|_{dB} = 20\log 10 = 20\,dB$

$\omega \in (0{,}8, 1) \quad |H(j\omega)|_{dB} = 20 - 20\log\frac{\omega}{0{,}8}$

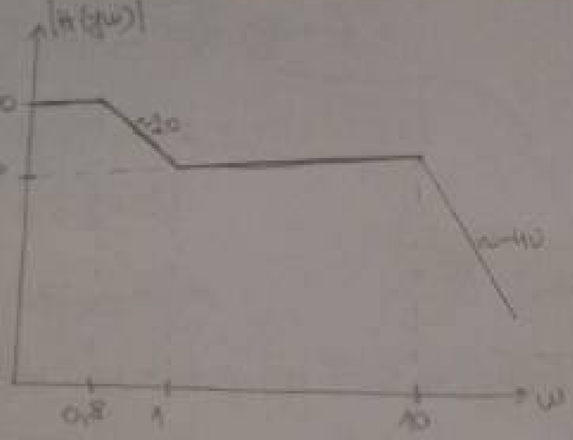
$\quad |H(j1)|_{dB} = 20 - 20\log\frac{1}{0{,}8} \cong 18\,dB$

$\omega \in (1, 10) \quad |H(j\omega)|_{dB} = 20 - 20\log\frac{\omega}{0{,}8} + 20\log\frac{\omega}{1} =$

$= 20 - 20(\log\omega - \log 0{,}8) + 20(\log\omega - \log 1) =$

$= 20 - \cancel{20\log\omega} + 20\log 0{,}8 + \cancel{20\log\omega} - 20\log 1 \cong 18\,dB$

$\omega > 10 \quad |H(j\omega)|_{dB} = 18\,dB - 40\log\frac{\omega}{10}$



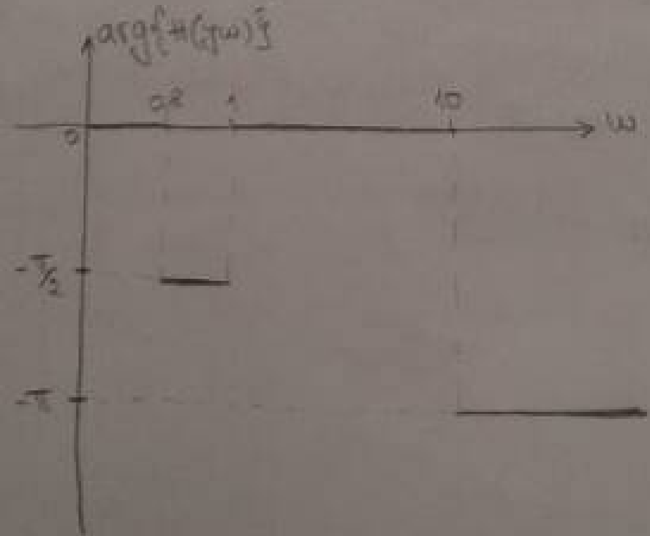
$$\arg\{H(j\omega)\} = \arg\frac{\omega}{1} - \arg\frac{\omega}{0{,}8} - 2\arg\frac{\omega}{10}$$

$\omega < 0{,}8 \quad \arg\{H(j\omega)\} = 0$

$\omega \in (0{,}8, 1) \quad \arg\{H(j\omega)\} = -\frac{\pi}{2}$

$\omega \in (1, 10) \quad \arg\{H(j\omega)\} = 0$

$\omega > 10 \quad \arg\{H(j\omega)\} = -\pi$



6) $x(t) = 7\sin(8t + \frac{\pi}{2})$

$y(t) = 7 \cdot A \sin(8t + \theta)$

$\theta = \frac{\pi}{2} + \varphi$

$A_{db} = 20 \log A$

$\varphi = \operatorname{arctg}\frac{\omega}{1} - \operatorname{arctg}\frac{\omega}{0,8} - 2\operatorname{arctg}\frac{\omega}{10}$

$\varphi = \operatorname{arctg}\frac{8}{1} - \operatorname{arctg}\frac{8}{0,8} - 2\operatorname{arctg}\frac{8}{10}$

$\varphi = 82,87 - 84,28 - 77,31 = -78,72 \text{ rad}$

$\theta = \frac{\pi}{2} - 78,72 = (1,57 - 78,72) \text{ rad} = -77,15 \text{ rad}$

$A_{db} = 12db + 0 \cdot \log\frac{\omega}{1} = 12db$

$12 = 20 \log A$

$A = 10^{\frac{12}{20}} \doteq 7,95$

$$\boxed{y(t) = 55,6 \sin(8t - 77,15)}$$

17.6.2010. ②

$$H(j\omega) = \frac{1200(j\omega+1)}{(j\omega+0{,}4)(j\omega+12)^2}$$

$$H(j\omega) = 1200\frac{(j\omega+1)}{0{,}4\left(1+\frac{j\omega}{0{,}4}\right)\cdot 144\left(1+\frac{j\omega}{12}\right)^2}$$

$$H(j\omega) = 20{,}83\frac{(j\omega+1)}{\left(1+\frac{j\omega}{0{,}4}\right)\left(1+\frac{j\omega}{12}\right)^2}$$

$$|H(j\omega)|_{dB} = 20\log 20{,}83 + 20\log\sqrt{1+\left(\frac{\omega}{1}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{0{,}4}\right)^2} - 40\log\sqrt{1+\left(\frac{\omega}{12}\right)^2}$$

$\omega < 0{,}4$ $\quad |H(j\omega)|_{dB} = 20\log 20{,}83 \cong 26{,}37\,dB$

$\omega \in (0{,}4,1)$ $\quad |H(j\omega)|_{dB} = 26{,}37 - 20\log\frac{\omega}{0{,}4}$

$$|H(j\cdot 1)| = 26{,}37 - 20\log\frac{1}{0{,}4} \cong 18{,}41\,dB$$

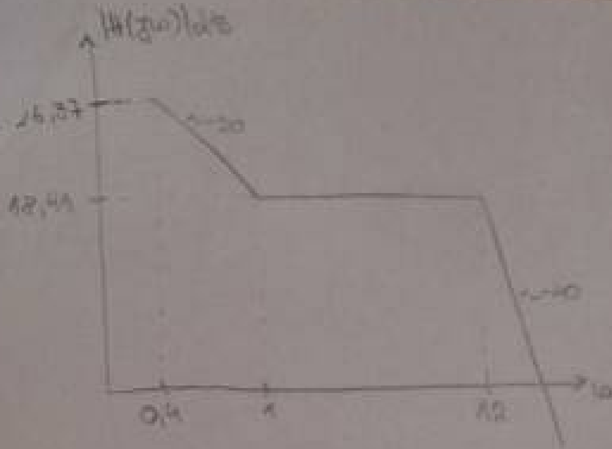
$\omega \in (1,12)$ $\quad |H(j\omega)|_{dB} = 26{,}37 - 20\log\frac{\omega}{0{,}4} + 20\log\frac{\omega}{1} =$

$= 26{,}37 - 20(\log\omega - \log 0{,}4) + 20(\log\omega - \log 1) =$

$= 26{,}37 - 20\cancel{\log\omega} + 20\log 0{,}4 + 20\cancel{\log\omega} - 20\log 1 =$

$= 26{,}37 + 20\log 0{,}4 - 20\log 1 \cong 18{,}41\,dB$

$\omega > 12$ $\quad |H(j\omega)|_{dB} = 18{,}41 - 40\log\frac{\omega}{12}$



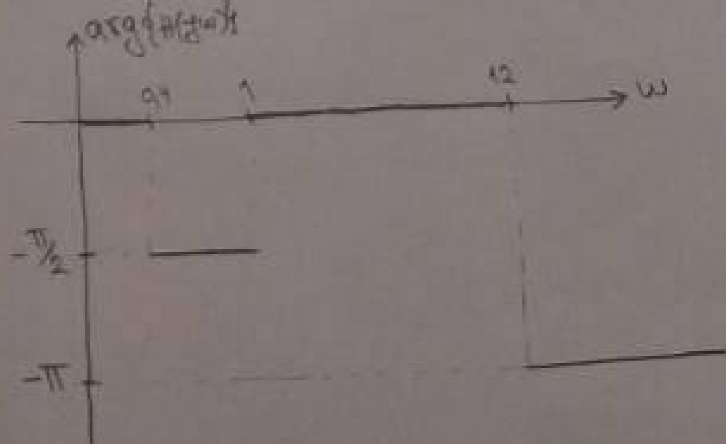
$$\arg\{H(j\omega)\} = \arg\frac{\omega}{1} - \arg\frac{\omega}{0{,}4} - 2\arg\frac{\omega}{12}$$

$\omega < 0{,}4 \quad \arg\{H(j\omega)\} = 0$

$\omega \in (0{,}4,1) \quad \arg\{H(j\omega)\} = -\frac{\pi}{2}$

$\omega \in (1,12) \quad \arg\{H(j\omega)\} = 0$

$\omega > 12 \quad \arg\{H(j\omega)\} = -\pi$



ZED TRANSFORMACIJE

$\boxed{8.7.2012. \quad 5}$

A) $\mathcal{Z}\{0{,}8^n \cdot u[n]\} = \boxed{\dfrac{z}{z-0{,}8}}$ $\quad |z| > 0{,}8 \to ROC$

B) $\mathcal{Z}\{0{,}5\delta[n+2] + 2\delta[n] + 0{,}5\delta[n-2]\} = 0{,}5\,z^2 + 2 + 0{,}5\,z^{-2} =$

$\boxed{= 0{,}5\,z^2 + 2 + 0{,}5\,\dfrac{1}{z^2}}$ $\quad |z| \neq 0 \to ROC$

c) $\mathcal{Z}^{-1}\left\{\dfrac{10z}{(z-1)(z-0{,}5)}\right\} = X(z) \qquad |z| \in (0{,}5, 1) \to ROC$

$$\frac{X(z)}{z} = \frac{10}{(z-1)(z-0{,}5)} = \frac{A}{(z-1)} + \frac{B}{(z-0{,}5)} =$$

$$= \frac{A(z-0{,}5) + B(z-1)}{(z-1)(z-0{,}5)} = \frac{Az - 0{,}5A + Bz - B}{(z-1)(z-0{,}5)}$$

$-0{,}5A - B = 10 \Rightarrow B = -10 - 0{,}5A \to B - 0{,}5B = -10$

$A + B = 0 \Rightarrow A = -B$

$0{,}5B = -10$

$\boxed{B = 20}$

$\boxed{A = -20}$

$$\frac{X(z)}{z} = -20 \cdot \frac{1}{z-1} + 20\,\frac{1}{z-0{,}5}$$

$$X(z) = -20\,\boxed{\frac{z}{z-1}} + 20\,\boxed{\frac{z}{z-0{,}5}} - \boxed{+20\,u[-n-1] + 20 \cdot 0{,}5^n \cdot u[n]}$$

$\dfrac{z}{z-1} \to -u[-n-1]$, $\quad |z| < 1$

$\dfrac{z}{z-0{,}5} \to a^n u[n]$, $\quad |z| > |a|$

8.7.2012. (15)

a) $h[n] = (-0,5)^n \cdot u[n]$ → $x[n] = u[n]$

b) JEDINIČNI ODSKOČNI ODZIV?

A) $H(z) = \mathcal{Z}\{(-0,5)^n u[n]\} = \boxed{\frac{z}{z+0,5}}$ $\quad |z| > 0,5 \to \text{ROC}$

B)

$Y(z) = X(z) \cdot H(z)$

$X(z) = \mathcal{Z}\{x[n]\} = \frac{z}{z-1}$ $\quad |z| > 1 \to \text{ROC}$

PRESEK $X(z)$ i $H(z)$

$Y(z) = X(z) \cdot H(z) = \frac{z}{z-1} \cdot \frac{z}{z+0,5} = \frac{z^2}{(z-1)(z+0,5)}$ $\quad |z| > 1 \to \text{ROC}$

$\frac{Y(z)}{z} = \frac{z}{(z-1)(z+0,5)} = \frac{A}{(z-1)} + \frac{B}{(z+0,5)} = \frac{A(z+0,5) + B(z-1)}{(z-1)(z+0,5)} =$

$= \frac{Az + 0,5A + Bz - B}{(z-1)(z+0,5)} \Rightarrow$ $0,5A - B = 0$, $A + B = 1$

$B = 1 - A$

$0,5A - 1 + A = 0$

$1,5A = 1$

$A = \frac{1}{\frac{3}{2}} \Rightarrow \boxed{A = \frac{2}{3}}$

$B = 1 - \frac{2}{3}$ $\quad \boxed{B = \frac{1}{3}}$

$\frac{Y(z)}{z} = \frac{2}{3} \cdot \frac{1}{z-1} + \frac{1}{3} \cdot \frac{1}{z+0,5}$

$Y(z) = \frac{2}{3} \underbrace{\boxed{\frac{z}{z-1}}}_{u[n],\ |z|>1} + \frac{1}{3} \underbrace{\boxed{\frac{z}{z+0,5}}}_{a^n u[n],\ |z|>|a|} = \boxed{\frac{2}{3} \cdot u[n] - \frac{1}{3} \cdot 0,5^n \cdot u[n]}$

6.2009. 7

A) $h[n] = (-1)^n u[n] = \boxed{\frac{z}{z+1}}$ $|z|>1 \rightarrow ROC$

c) ODZIV, $x[n] = 0.5^n \cdot u[n] = \frac{z}{z-0,5}$ $|z|>0,5 \rightarrow ROC$

$Y(z) = X(z) \cdot H(z)$

$Y(z) = \frac{z}{z-0,5} \cdot \frac{z}{z+1}$ $|z|>1 \rightarrow ROC$

$Y(z) = \frac{z^2}{(z-0,5)(z+1)}$

$\frac{Y(z)}{z} = \frac{z}{(z-0,5)(z+1)} = \frac{A}{(z-0,5)} + \frac{B}{(z+1)} =$

$= \frac{A(z+1)+B(z-0,5)}{(z-0,5)(z+1)} = \frac{Az+A+Bz-0,5B}{(z-0,5)(z+1)}$

$A+B=1$ $\quad A-0,5B=0$

$\downarrow$ $\quad 1-B-0,5B=0$

$A=1-B$ $\quad 1=1,5B$

$A=1-\frac{2}{3}$ $\quad \boxed{B=\frac{2}{3}}$

$\boxed{A=\frac{1}{3}}$

$$Y(z) = \frac{1}{3} \cdot \boxed{\frac{z}{z-0,5}} + \frac{2}{3} \cdot \boxed{\frac{z}{z+1}} = \boxed{\frac{1}{3} \cdot 0,5^n u[n] + \frac{2}{3}(-1)^n \cdot u[n]}$$

$a^n u[n]$ $\quad a^n u[n]$

$|z|>|a|$ $\quad |z|>|a|$

17.6.2010. 6

a) $h[n] = (-0{,}5)^n u[n]$

$H(z) = \mathcal{Z}\{h[n]\} = \boxed{\frac{z}{z+0{,}5}}$ $\quad |z| > 0{,}5 \rightarrow$ ROC

b) $x[n] = (-1)^n \cdot u^n$

$X(z) = \mathcal{Z}\{x[n]\} = \boxed{\frac{z}{z+1}}$ $\quad |z| > 1 \rightarrow$ ROC

$Y(z) = H(z) \cdot X(z) = \frac{z}{z+0{,}5} \cdot \frac{z}{z+1}$ $\quad |z| > 1 \rightarrow$ ROC

$Y(z) = \frac{z^2}{(z+0{,}5)(z+1)}$

$\frac{Y(z)}{z} = \frac{z}{(z+0{,}5)(z+1)} = \frac{A}{(z+0{,}5)} + \frac{B}{(z+1)} = \frac{A(z+1)+B(z+0{,}5)}{(z+0{,}5)(z+1)} =$

$= \frac{Az+A+Bz+0{,}5B}{(z+0{,}5)(z+1)} \Rightarrow A+0{,}5B=0 \quad \wedge \quad A+B=1$

$A = 1-B$

$1-B+0{,}5B=0$

$1 = 0{,}5B$

$\boxed{B=2}$

$A = 1-2$

$\boxed{A=-1}$

$Y(z) = 2\boxed{\frac{z}{z+1}} - \boxed{\frac{z}{z+0{,}5}} \rightarrow \boxed{2 \cdot (-1)^n \cdot u[n] - (-0{,}5)^n \cdot u[n]}$

$\downarrow$ $a^n u[n]$, $|z| > |a|$ $\qquad$ $\downarrow$ $a^n u[n]$, $|z| > |a|$

DVOSTRANA ZED TRANSF. $X(z) = \dfrac{z+1}{z^4(z-1)(z-0{,}5)}$; $|z|>1$

$$X(z) = z^{-5} \cdot X_1(z)$$

$$X_1(z) = X(z) \cdot z^5 = \frac{z(z+1)}{(z-1)(z-0{,}5)}$$

$$\frac{X_1(z)}{z} = \frac{(z+1)}{(z-1)(z-0{,}5)} = \frac{A}{z-1} + \frac{B}{z-0{,}5} = \frac{A(z-0{,}5)+B(z-1)}{(z-1)(z-0{,}5)}$$

$$= \frac{Az - 0{,}5A + Bz - B}{(z-1)(z-0{,}5)} \Rightarrow A+B = 1 \ \wedge \ -0{,}5A - B = 1$$

$$B = 1 - A \longrightarrow -0{,}5A - 1 + A = 1$$

$$0{,}5A = 2$$

$$\boxed{A = 4}$$

$$B = 1 - 4$$

$$\boxed{B = -3}$$

$$x_1[n] = 4\,\boxed{\frac{z}{z-1}} - 3\,\boxed{\frac{z}{z-0{,}5}} = 4u[n] - 3 \cdot 0{,}5^n \cdot u[n]$$

$$\frac{z}{z-1} \to u[n], \quad |z|>1 \qquad \frac{z}{z-0{,}5} \to a^n \cdot u[n], \quad |z|>0{,}5$$

$$x[n] = x_1[n-5] = \boxed{4u[n-5] - 3 \cdot 0{,}5^{n-5} \cdot u[n-5]}$$

7.6.2009. (8)

A) DVOSTRANA ZED TRANSF. $X(z)=\dfrac{z+1}{z^4(z-1)(z-0{,}5)}$ $|z|>1$

↓

ISTI KAO NA PRETHODNOJ STRANI

B) $$\mathcal{Z}\{x[n]*y[n]\}=\sum_{n=-\infty}^{+\infty}\left(\sum_{k=-\infty}^{+\infty}x[k]\,y[n-k]\right)z^{-n}=$$
$$=\sum_{k=-\infty}^{+\infty}x[k]\cdot z^{-k}\cdot\sum_{n=-\infty}^{+\infty}y[n-k]\,z^{-(n-k)}=X(z)\,Y(z)$$

→ ZED TRANSFORMACIJA KONVOLUCIJE DVA SIGNALA PROIZVOD ODGOVARAJUĆIH ZED TRANSF.

C) POZNAT $x[n]$, NEPARNI DEO?

$$\mathcal{Z}\left\{\frac{x[n]-x[-n]}{2}\right\}=\frac{\mathcal{Z}\{x[n]\}-\mathcal{Z}\{x[-n]\}}{2}=\frac{X[z]-X(z^{-1})}{2}$$

D)

.7.2011. 8

A) $h[n] = 0.8^{n+1} \cdot u[n-1]$

$0.8^n \cdot u[n] \longleftrightarrow \frac{z}{z-0.8}$

$0.8^{n-1} \cdot u[n-1] = z^{-1} \cdot \frac{z}{z-0.8} = \frac{1}{z-0.8}$

$H(z) = 0.8^2 \cdot 0.8^{n-1} \cdot u[n-1] = \boxed{\frac{0.64}{z-0.8}}$ $|z|$

B) $x[n] = (-1)^n \cdot u[n]$

$X(z) = \mathcal{Z}\{x[n]\} = \frac{z}{z+1}$ $|z|>1 \rightarrow$ ROC

$Y(z) = H(z)\,X(z) = \frac{0.64}{z-0.8} \cdot \frac{z}{z+1} = \frac{0.64z}{(z-0.8)(z+1)} =$

$\frac{Y(z)}{z} = \frac{0.64}{(z-0.8)(z+1)} = \frac{A}{(z-0.8)} + \frac{B}{(z+1)} = \frac{Az+A+Bz-0.8B}{(z-0.8)(z+1)}$

$A+B=0 \wedge A-0.8B=0.64$

$A=-B$ $\quad -B-0.8B=0.64$

$\boxed{A=+0.36}$ $\quad -1.8B=0.64$

$\boxed{B=-0.36}$

$Y(z) = 0.36\left(\frac{z}{z-0.8} - \frac{z}{z+1}\right) = \boxed{0.36 \cdot u[n]\left(0.8^n - (-1)^n\right)}$

$\frac{z}{z-0.8} \downarrow a^n u[n]$, $|z|>0.8$ $\qquad \frac{z}{z+1} \downarrow a^n u[n]$, $|z|>1$

10.7.2011. 9

A) $x[n] = u[n+1] - u[n-3]$

B) $x[n] = \delta[n-1] - 0{,}5^n u[n-3] =$

$\delta[n-1] = 1 \cdot z^{-1}$

$0{,}5^n \cdot u[n] = \frac{z}{z-0{,}5}$

$0{,}5^{n-3} \cdot u[n-3] = z^{-3} \cdot \frac{z}{z-0{,}5} = z^{-2} \cdot \frac{1}{z-0{,}5}$

$0{,}5^3 \cdot 0{,}5^{n-3} \cdot u[n-3] = 0{,}125 \cdot \frac{z^{-2}}{z-0{,}5}$

$= \boxed{z^{-1} \cdot 0{,}125 \frac{z^{-2}}{z-0{,}5}}$

C) $X(z) = \frac{z^{-2}}{1-z^{-1}} = \frac{1}{z^2(1-z^{-1})} = \frac{1}{z^2 \cdot \frac{z-1}{z}} = \frac{1}{z(z-1)}$

$|z|<1$

$X(z) = z^{-2} X_1(z)$

$X_1(z) = \frac{z}{z-1} = -u[-n-1] \quad |z|<1$

$x[n] = x_1[n-2] = -u[-(n-2)-1] = -u[-n+2-1]$

$\boxed{x[n] = -u[1-n]}$

D) $X(z) = \frac{2}{(z-0{,}5)(z+2)} \cdot \frac{z}{z} \quad |z|<0{,}5$

(IMA GA NA 11 STR)

17.6.2012. 5

A) $\mathcal{Z}\{u[u+2]\} = z^2 \cdot \frac{z}{z-1} = \frac{z^3}{z-1} \quad |z|>1 \rightarrow ROC$

B) $\mathcal{Z}\{(-1)^u \cdot u[u]\} = \frac{z}{z+1} \quad |z|>1 \rightarrow ROC$

C) $\mathcal{Z}^{-1}\left\{\frac{1}{z^2-1}\right\} = (ROC: |z|>1) \frac{z}{z} \Rightarrow X(z) = z^{-1} \cdot X_1(z)$

$$X_1(z) = \frac{z}{z^2-1} = \frac{z}{(z-1)(z+1)} = \frac{A}{z-1} + \frac{B}{z+1} =$$

$$= \frac{A(z+1)+B(z-1)}{(z-1)(z+1)} = \frac{Az+A+Bz-B}{(z+1)(z-1)}$$

$A+B=1 \quad A-B=0$

$A=B$

$2A=1$

$\boxed{A=\frac{1}{2}} \quad \boxed{B=\frac{1}{2}}$

$$X_1(z) = \frac{1}{2}\frac{z}{z-1} + \frac{1}{2}\frac{z}{z+1} =$$

$$X_1(z) = \frac{1}{2}u[u] + \frac{1}{2}a^u u[u]$$

$$x[u] = z^{-1} \cdot X_1(z) =$$

$$= \frac{1}{2}u[u-1] + \frac{1}{2}(-1)^{u-1} \cdot u[u-1]$$

$$x[u-1] = \boxed{\frac{1}{2}u[u-1]\left(1+(-1)^{u-1}\right)}$$

D) $\mathcal{Z}^{-1}\left\{\frac{1}{(z+0{,}8)(z-0{,}4)(z+1)}\right\} = (ROC: |z| \in (0{,}4;\ 0{,}8)) = X(z)$

$X(z) = X_1(z) \cdot z^{-1} \qquad X_1(z) = \frac{z}{(z+0{,}8)(z-0{,}4)(z+1)}$

$X_1(z) = \frac{A}{z+0{,}8} + \frac{B}{z-0{,}4} + \frac{C}{z+1} \quad \dots \quad$ $A=-4{,}2$, $B=0{,}6$, $C=3{,}6$

$$X_1(z) = -4{,}2\frac{z}{z+0{,}8} + 0{,}6 \cdot \frac{z}{z-0{,}4} + 3{,}6 \cdot \frac{z}{z+1}$$

$$X_1(z) = -4{,}2 \cdot (-0{,}8)^u \cdot u[-u-1]) + 0{,}6 \cdot a^u \cdot u[u] - 3{,}6 \cdot a^u \cdot u[-u-1]$$

$$x[u-1] = \boxed{-4{,}2 \cdot (-0{,}8)^{u-1} \cdot u[-u]) + 0{,}6 \cdot (0{,}4)^{u-1} u[u-1] - 3{,}6(-1)^{u-1} \cdot u[-u])}$$

19.6.2011. (8)

A) $h[n] = 0.8^n u[n]$

$H(z) = \frac{z}{z-0,8}$ $\quad |z| > 0,8 \rightarrow ROC$

B) $x[n] = (-1)^n u[n]$

$X(z) = \frac{z}{z+1}$ $\quad |z| > 1 \rightarrow ROC$

$Y(z) = H(z) \cdot X(z)$

$Y(z) = \frac{z}{z-0,8} \cdot \frac{z}{z+1} = \frac{z^2}{(z-0,8)(z+1)}$

$\frac{Y(z)}{z} = \frac{z}{(z-0,8)(z+1)} = \frac{A}{z-0,8} + \frac{B}{z+1}$

$\frac{Y(z)}{z} = \frac{A(z+1)+B(z-0,8)}{(z-0,8)(z+1)} = \frac{Az+A+Bz-0,8B}{(z-0,8)(z+1)}$

$A+B=1 \qquad A-0,8B=0$

$1,8B=1 \qquad A=0,8B$

$B = \frac{1}{\frac{18}{10}} \qquad A = \frac{8}{10} \cdot \frac{5}{9} = \frac{40}{90} = \frac{4}{9}$

$B = \frac{10}{18} = \frac{5}{9}$

$\boxed{A = \frac{4}{9} \quad B = \frac{5}{9}}$

$\frac{Y(z)}{z} = \frac{4}{9} \cdot \frac{1}{z-0,8} + \frac{5}{9} \cdot \frac{1}{z+1}$

$Y(z) = \frac{4}{9} \cdot \frac{z}{z-0,8} + \frac{5}{9} \cdot \frac{z}{z+1}$

$y[n] = \frac{4}{9} \cdot (0,8)^n u[n] + \frac{5}{9} \cdot (-1)^n u[n]$

$\boxed{y[n] = \left(\frac{4}{9} \cdot 0,8^n + \frac{5}{9}(-1)^n\right) \cdot u[n]}$

.6.2011. 9

A) $x[n] = u[n] - u[n-4] = \frac{z}{z-1} - z^{-4}\cdot\frac{z}{z-1} =$

$= \frac{z}{z-1} - \frac{1}{z^3(z-1)} = \frac{z^4-1}{z^3(z-1)} = \boxed{\frac{z^4-1}{z^4-z^3}}$ ROC:

B) $x[n] = \delta[n-1] - 0{,}5^n u[n-3] =$

$\delta[n-1] = z^{-1}\cdot 1 = z^{-1}$

$0{,}5^n u[n-3] \Rightarrow 0{,}5^n u[n] = \frac{z}{z-0{,}5}$

$0{,}5^{n-3} u[n-3] = z^{-3}\cdot\frac{z}{z-0{,}5} = \frac{1}{z^2(z-0{,}5)}$

$0{,}5^3\cdot 0{,}5^{n-3} u[n-3] = \frac{0{,}5^3}{z^2(z-0{,}5)} = \frac{0{,}125}{z^2(z-0{,}5)}$

$= \boxed{z^{-1} - \frac{0{,}125}{z^2(z-0{,}5)}}$ $|z| > 0{,}5 \rightarrow$ ROC

C) $X(z) = z^{-2}\cdot\frac{1}{1-z^{-1}}$; $|z|>1$

$X(z) = z^{-2}\cdot\frac{z}{z-1} \Rightarrow \boxed{x[n] = u[n-2]}$

D) $X(z) = \frac{2}{(z-0{,}5)(z+2)}$, $0{,}5<|z|<2$

$X(z) = X_1(z)\cdot z^{-1}$

$X_1(z) = \frac{2z}{(z-0{,}5)(z+2)} = \frac{A}{z-0{,}5} + \frac{B}{z+2} =$

$= \frac{Az+2A+Bz-0{,}5B}{(z-0{,}5)(z+2)}$

$A+B=2$ $\quad 2A-0{,}5B=0$ $\quad B = \frac{4}{\frac{25}{10}} = \frac{40}{25} = \frac{8}{5}$

$A=2-B$ $\quad 4-2B-0{,}5B=0$ $\quad A = 2-\frac{8}{5} = \frac{2}{5}$

$2{,}5B=4$

$$X_1(z) = \frac{2}{5} \cdot \frac{z}{z-0{,}5} + \frac{8}{5} \cdot \frac{z}{z+2} = \frac{2}{5} \cdot (0{,}5)^n \cdot u[n] - \frac{8}{5} \cdot (-2)^n \cdot u[-n-1]$$

$$X(z) = X_1(z) \cdot z^{-1}$$

$$x[n-1] = \boxed{\frac{2}{5} \cdot 0{,}5^{n-1} \cdot u[n-1] - \frac{8}{5} \cdot (-2)^{n-1} \cdot u[-n]}$$

19.6. 2011. 10

A) $\mathcal{Z}\{x[n-2]\} = z^{-2} \cdot \frac{z}{z-1} = \boxed{z^{-2} \cdot X(z)}$

B) $\mathcal{Z}\{x[n+1] + 0{,}2^n x[n]\}$

1.2008 6

A) $h[n] = [1+(-1)^n]u[n] = u[n] + (-1)^n u[n]$

$$H(z) = \frac{z}{z-1} + \frac{z}{z+1} = \frac{z(z+1)+z(z-1)}{(z-1)(z+1)} \quad |z|>1$$

$$H(z) = \frac{z^2+\cancel{z}+z^2-\cancel{z}}{(z^2-1)} = \boxed{\frac{2z^2}{z^2-1}}$$

c) $x[n] = 0{,}8^n u[n]$

$$X(z) = \frac{z}{z-0{,}8} \quad |z|>0{,}8$$

$$Y(z) = X(z)\,H(z) = \frac{2z^2}{(z-1)(z+1)} \cdot \frac{z}{(z-0{,}8)}$$

$$\frac{Y(z)}{z} = \frac{2z^2}{(z-1)(z+1)(z-0{,}8)} = \frac{A}{z-1} + \frac{B}{z+1} + \frac{C}{z-0{,}8} =$$

$$= \frac{A(z+1)(z-0{,}8) + B(z-1)(z-0{,}8) + C(z-1)(z-0{,}8)}{(z-1)(z+1)(z-0{,}8)}$$

$$= \frac{(Az+A)(z-0{,}8) + (Bz-B)(z-0{,}8) + (Cz-C)(z-0{,}8)}{(z-1)(z+1)(z-0{,}8)} =$$

$$= \frac{Az^2-0{,}8Az+Az-0{,}8A+Bz^2-0{,}8Bz-Bz+0{,}8B+Cz^2-0{,}8Cz-Cz+0{,}8C}{(z-1)(z+1)(z-0{,}8)}$$

$A+B+C=2$

$0{,}2A-1{,}8B-1{,}8C=0$

$-1{,}8(B+C)=0$

$-0{,}8A+0{,}8B+0{,}8C=0$

$-A+B+C=0$

$A=B+C$

$A=0$

$B+C+B+C=2$

$2B+2C=2$

$B+C=0$

LAPLASOVE TRANSFORMACIJE

8.7.2012. (4)

A) $x(t) = u(t) + e^{-2t}u(t) - e^{3t}u(-t) =$

$X(s) = \frac{1}{s} + \frac{1}{s+2} + \frac{1}{s-3}$ ROC: $Re\{s\} \in (0,3)$

$Re\{s\}>0$ $Re\{s\}>-2$ $Re\{s\}<3$

B) $X(s) = \frac{2s+1}{s^2+6s+25} = \frac{2s+1}{(s+3)^2+16} = \frac{2s+1}{(s+3)^2+4^2} =$

$= \frac{2(s+3)-6+1}{(s+3)^2+4^2} = 2\,\frac{(s+3)}{(s+3)^2+4^2} - 5 \cdot \frac{1}{(s+3)^2+4^2} =$

$= 2 \cdot e^{-3t}\cos(4t) \cdot u(t) - \frac{5}{4} \cdot \frac{4}{(s+3)^2+4^2} =$

$= 2e^{-3t}\cos(4t)u(t) - \frac{5}{4} \cdot e^{-3t} \cdot \sin(4t)u(t) =$

$= e^{-3t}\left[2\cos(4t) - \frac{5}{4}\sin(4t)\right] \cdot u(t)$

$\boxed{8.7.2012. \ ③}$

$x(t) \to X(s) \quad Re\{s\} > -2$

$x_1(t) = 2x(2t) - 3x(t/2) + e^{-3t}x(t), \quad X_1(s) = ?$

$X_1(s) = 2 \cdot \frac{1}{2} \cdot X\left(\frac{s}{2}\right) - 3 \cdot \frac{1}{\frac{1}{2}} \cdot X\left(\frac{s}{\frac{1}{2}}\right) + X(s+3) =$

$= X\left(\frac{s}{2}\right) - 6X(2s) + X(s+3)$

$Re\{\frac{s}{2}\} > -2 \quad s > -4$

$Re\{2s\} > -2 \quad s > -1 \qquad Re\{s\} \in (-1, +\infty)$

$Re\{s+3\} > -2 \quad s > -5$

$\boxed{17.6.2012. \ ④}$

A) $\mathcal{L}\{2\delta(t) + 3\delta(t-2)\} = 2 + 3 \cdot e^{-2t} \cdot 1 = 2 + 3e^{-4t}$

B) $\mathcal{L}\{e^{-2t}u(t) + \sin(3t) \cdot u(t)\} = \frac{1}{s+2} + \frac{3}{s^2+9} \quad Re\{s\} > 0$

$Re\{s\} > -2 \quad Re\{s\} > 0$

C) $\mathcal{L}^{-1}\left\{\frac{4s}{s^2+6s+13}\right\} = (Re\{s\} > -3) =$

$= \frac{4s}{(s+3)^2 - 9 + 13} = \frac{4s}{(s+3)^2+4} = \frac{4(s+3) - 12}{(s+3)^2+4} =$

$= 4\frac{s+3}{(s+3)^2+2^2} - \frac{12}{2} \cdot \frac{1 \cdot 2}{(s+3)^2+2^2} =$

$= 4 \cdot e^{-3t} \cdot \cos(2t)u(t) - 6 \cdot e^{-3t} \sin(2t)u(t) =$

$= e^{-3t}\left[4\cos(2t) - 6\sin(2t)\right] \cdot u(t)$

b) $L^{-1}\left\{\frac{5}{s(s^2-4)}\right\} = \left(Re\{s\} \in (-2,0)\right) =$

$= \frac{5}{s(s-2)(s+2)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} =$

$= \frac{A(s-2)(s+2) + Bs(s+2) + Cs(s-2)}{s(s-2)(s+2)} =$

$= \frac{(As-2A)(s+2) + Bs^2 + 2Bs + Cs^2 - 2Cs}{s(s-2)(s+2)} =$

$= \frac{As^2 + \cancel{2As} - \cancel{2As} - 4A + Bs^2 + 2Bs + Cs^2 - 2Cs}{s(s-2)(s+2)} =$

$= \frac{As^2 - 4A + Bs^2 + 2Bs + Cs^2 - 2Cs}{s(s-2)(s+2)}$

$A + B + C = 0 \qquad 2B - 2C = 0 \qquad -4A = 5$

$\boxed{A = -\frac{5}{4}}$

$B = C$

$\boxed{B = \frac{5}{8}}$

$-\frac{5}{4} + 2C = 0$

$2C = \frac{5}{4}$

$C = \frac{\frac{5}{4}}{2} = \boxed{\frac{5}{8}}$

$a = -2$

$= -\frac{5}{4} \cdot \frac{1}{s} + \frac{5}{8} \cdot \frac{1}{s-2} + \frac{5}{8} \cdot \frac{1}{s+2} =$

$= -\frac{5}{4} \cdot (-u(-t)) + \frac{5}{8} \cdot \left(-e^{-(-2)t} u(t)\right) + \frac{5}{8} \cdot e^{-2t} \cdot u(t)$

$= \frac{5}{4} u(-t) - \frac{5}{8} e^{2t} u(t) + \frac{5}{8} e^{-2t} u(t)$

10.7.2011. ⑥

A) $\mathcal{L}\{e^{-3t}\cos(2t)u(t)\} = \frac{s+3}{(s+3)^2+4}$ $Re\{s\} > -3$

B) $\mathcal{L}\{e^{-3t}\delta(t-2)\} =$ [KADA IMAŠ e i δ UVEK RADIŠ PREKO INTEGRALA]

$$= \int_{-\infty}^{+\infty} e^{-3t}\delta(t-2)\cdot e^{-st}dt = \int_{-\infty}^{+\infty} e^{-t(3+s)}\delta(t-2)dt =$$

$$= e^{-t(3+s)}\Big|_{t=2} = e^{-2(3+s)} = e^{-6-2s} \quad Re\{s\} \in \mathbb{C}$$

c) $\mathcal{L}^{-1}\left\{\frac{2}{(s^2-0{,}25)(s+1)}\right\} = \left(Re\{s\} \in (-1, -0{,}5)\right)$

$$= \frac{2}{(s-0{,}5)(s+0{,}5)(s+1)} = \frac{A}{(s-0{,}5)} + \frac{B}{(s+0{,}5)} + \frac{C}{(s+1)} =$$

$$= \frac{A(s+0{,}5)(s+1) + B(s-0{,}5)(s+1) + C(s^2-0{,}25)}{(s-0{,}5)(s+0{,}5)(s+1)} =$$

$$= \frac{(As+0{,}5A)(s+1) + (Bs-0{,}5B)(s+1) + Cs^2 - 0{,}25C}{(s-0{,}5)(s+0{,}5)(s+1)}$$

$$= \frac{As^2 + As + 0{,}5As + 0{,}5A + Bs^2 + Bs - 0{,}5Bs - 0{,}5B + Cs^2 - 0{,}25C}{(s-0{,}5)(s+0{,}5)(s+1)}$$

$$= \frac{As^2 + 1{,}5As + 0{,}5A + Bs^2 + 0{,}5Bs - 0{,}5B + Cs^2 - 0{,}25C}{(s-0{,}5)(s+0{,}5)(s+1)}$$

$A + B + C = 0$ $\quad$ $1{,}5A + 0{,}5B = 0$ $\quad$ $0{,}5A - 0{,}5B - 0{,}25C = 2$

$A - 3A + 8A = 8$

$6A = 8$

$A = \frac{8}{6}$

$\boxed{A = \frac{4}{3}}$

$0{,}5B = -1{,}5A$

$\boxed{B = -3A}$

$\boxed{B = -4}$

$2A - 0{,}25C = 2$

$0{,}25C = 2A - 2$

$\boxed{C = 8A - 8}$

$C = 8 \cdot \frac{4}{3} - 8$

$C = \frac{32}{3} - \frac{24}{3}$

$\boxed{C = \frac{8}{3}}$

$$= \frac{4}{3} \cdot \frac{1}{s-0{,}5} - 4 \cdot \frac{1}{s+0{,}5} + \frac{8}{3} \cdot \frac{1}{s+1}$$

$$x(t) = \frac{4}{3} \cdot \left(-e^{-(-0{,}5)t} \cdot u(-t)\right) - 4 \cdot \left(-e^{-0{,}5t} \cdot u(-t)\right) + \frac{8}{3} \cdot e^{-t} \cdot u(t)$$

$$x(t) = 4e^{-0{,}5t} u(-t) - \frac{4}{3} e^{0{,}5t} u(-t) + \frac{8}{3} e^{-t} u(t)$$

7.6.2009. 4

A) $\mathcal{L}\{e^{-t}\sin(4t)\cdot u(t)\} = \frac{4}{(s+1)^2+16}$ $\quad Re\{s\} > -1$

B) $\mathcal{L}\{e^{-3t}\delta(t-2)\} =$

$= \int_{-\infty}^{+\infty} e^{-3t}\delta(t-2)\cdot e^{-st}dt = \int_{-\infty}^{+\infty} e^{-t(s+3)}\delta(t-2)dt =$

$= e^{-t(s+3)}\Big|_{t=2} = e^{-6-2s} \quad \forall s \in \mathbb{C}$

c) $\mathcal{L}^{-1}\left\{\frac{2}{(s^2-0{,}25)(s+1)}\right\} = \left(Re\{s\} \in (-1, -0{,}5)\right)$

URAĐEN NA STRANI 4

LAPLASOVA TRANSFORMACIJA

	SIGNAL	L.T.	ROC
1	$\delta(t)$	1	$s \in \mathbb{C}$
2	$u(t)$	$\frac{1}{s}$	$Re\{s\}>0$
3	$-u(-t)$	$\frac{1}{s}$	$Re\{s\}<0$
4	$e^{-at} \cdot u(t)$	$\frac{1}{s+a}$	$Re\{s\}>-a$
5	$-e^{-at} u(-t)$	$\frac{1}{s+a}$	$Re\{s\}<-a$
6	$t \cdot u(t)$	$\frac{1}{s^2}$	$Re\{s\}>0$
7	$t \cdot e^{-at} \cdot u(t)$	$\frac{1}{(s+a)^2}$	$Re\{s\}>-a$
8	$\sin(at) \cdot u(t)$	$\frac{a}{s^2+a^2}$	$Re\{s\}>0$
9	$\cos(at) \cdot u(t)$	$\frac{s}{s^2+a^2}$	$Re\{s\}>0$
· 10	$e^{-bt} \cdot \sin(at) u(t)$	$\frac{a}{(s+b)^2+a^2}$	$Re\{s\}>-b$
· 11	$e^{-bt} \cos(at) u(t)$	$\frac{s+b}{(s+b)^2+a^2}$	$Re\{s\}>-b$

LAPLASOVA

$$\mathcal{L}\{x(t)\} = X(s) = \boxed{\int_{-\infty}^{+\infty} x(t)\cdot e^{-st}\,dt}$$

GRANIČNE TEOREME:

① $x(0) = \lim_{s\to+\infty} s\cdot X(s)$

② $x(\infty) = \lim_{s\to 0} s\cdot X(s)$

8.7.2012

2) $x(t) = e^{2t}\cdot u(-t)$ $\left(\frac{1}{a+s}\right)$

$= \frac{1}{2-s}$

$$\boxed{x(t) = \frac{1}{j2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)\cdot e^{st}\,ds}$$ – SINTETIKA

1) $\mathcal{L}\{\delta(t-2)\} = e^{-2s}\cdot \cancel{X(s)}^{1} = \boxed{e^{-2s}}$

$\int_{-\infty}^{t-1} x(\tau)\,d\tau = \frac{1}{s}\cdot e^{-1s}X(s) = \boxed{\frac{e^{-s}}{s}X(s)}$

$Re\{s\} > -2$

$x_1(t) = 2x(2t) - 3x(t/2) + e^{-3t}x(t)$

$X_1(s) = \cancel{2} \cdot \frac{1}{\cancel{2}} \cdot X\left(\frac{s}{2}\right) - 3 \cdot \frac{1}{|\frac{1}{2}|} \cdot X\left(\frac{s}{\frac{1}{2}}\right) +$

$+$

$= X(s/2) - 6X(2s) + X(s+3)$

$x(t) \cdot e^{-2t} =$
$= X(s+2)$
$x(t) \cdot e^{2t} = X(s-2)$

$Re(s/2) > -2$

$Re(2s) > -2$

$Re(s+3) > -2$

$s > -4$
$s > -1$
$s > -5$

$\boxed{Re\{s\} \in (-1, +\infty)}$

17.6.2012.

FLZ

1) a) $x(t) = e^{-2t}u(t)$

$\frac{1}{2+s}$ $\quad Re\{s\} > -2$

$H(s) = \mathcal{L}\{h(t)\}$
$H(j\omega) = F\{h(t)\}$
$H(z) = Z\{h(t)\}$
F-JA PRENOSA SISTEMA

j) $H(s) = \frac{Y(s)}{X(s)}$

4) a) $\mathcal{L}\{2\delta(t) + 3\delta(t-4)\} = 2 + 3\cdot 1 \cdot e^{-s4} = \boxed{2+3e^{-4s}}$

$\forall s \in \mathbb{C}$

b) ~~$\mathcal{L}\{e^{3t}u(-t) + e^{-2t}u(t)\} = \frac{1}{s+2} + \frac{1}{s-3}$~~

$\mathcal{L}\{e^{-2t}u(t) + \sin(3t)u(t)\} = \frac{1}{s+2} + \frac{3}{s^2+3^2}$

---

c) $\mathcal{L}^{-1}\left\{\frac{4s}{s^2+6s+13}\right\} = \frac{4s}{(s+3)^2+2^2}$ ~~$= \frac{2^2}{(s+3)^2+2^2} \cdot s$~~

$\frac{4s}{(s+3)^2+2^2} = \frac{4(s+3) - 12}{(s+3)^2+2^2} = 4\underbrace{\frac{(s+3)}{(s+3)^2+2^2}}_{\cos} - \frac{12}{2}\cdot\frac{2}{(s+3)^2+2^2} =$

$= 4\frac{s+3}{(s+3)^2+2^2} - 6\cdot\frac{2}{(s+3)^2+2^2}$

$Re\{s\} \in (-2,0)$

$$L^{-1}\left\{\frac{5}{s(s^2-4)}\right\} = \frac{5}{s(s-2)(s+2)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} =$$

$$= \frac{As-\cancel{2A}+\cancel{2}sA+\cancel{2A}+Bs+Bs+2B+sC+sC-2C}{s(s-2)(s+2)}$$

$$= \frac{\cancel{2As+2Bs+2B+2sC-2C}}{As^2-4A+Bs^2+2Bs+Cs^2-2sC}$$

$A+B+C=0 \qquad -4A=5 \qquad 2B-2C=0$

$B-C=0$

$-\frac{5}{4}+2C=0 \qquad A=-\frac{5}{4} \qquad B=C$

$2C=\frac{5}{4}$

$\boxed{B=\frac{5}{8}}$

$C=\frac{\frac{5}{4}}{\frac{2}{1}}=\boxed{\frac{5}{8}}$

$$X(s) = -\frac{5}{4}\cdot\frac{1}{s} + \frac{5}{8}\cdot\frac{1}{s-2} + \frac{5}{8}\frac{1}{s+2}$$

$$X(s) - \left(-\frac{5}{4}u(-t)\right) + \frac{5}{8}\left(-e^{+2t}u(-t)\right) +$$

$$\frac{5}{8}e^{-2t}u(t) \qquad \frac{1}{s-(-2)}$$

19.6.2011

6) A) $L\{e^{-2t}\sin(4t)u(t)\} = \frac{4}{(s+2)^2+4^2} \quad Re\{s\}>-2$

B) $\cancel{L\{e^{-3t}\sin(4t)u(t)\}} =$

$L\{e^{-3t}\delta(t+2)\}$ (e · δ UVEK PREKO INTEGRALA)

$= \int_{-\infty}^{+\infty} e^{-3t}\delta(t+2)\cdot e^{-st}dt =$

$= e^{-t(3+s)}\Big|_{t=-2} = e^{+2(3+s)} = e^{6+2s}$

$\boxed{s \in \mathbb{C}}$

KOD e

.7.2011.

(6) A) $\mathcal{L}\{e^{-3t}\cos(2t)u(t)\} = \frac{s+3}{(s+3)^2+2^2}$

B) $\mathcal{L}\{e^{-3t}\delta(t-2)\} =$

$$= \int_{-\infty}^{+\infty} e^{-3t}\delta(t-2)\, e^{-st}\, dt$$

$$= e^{-t(3+s)}\Big|_{t=2} = e^{-6-2s} \quad \forall s \in \mathbb{C}$$

FOURIEROVA TRANSFORMACIJA

$\boxed{17.6.2012.}$ 3

A) $F\{2u(t)-3u(t-2)\} = 2\cdot\left(\frac{1}{j\omega}+\pi\delta(\omega)\right) -$

$- 3\left[\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)\cdot e^{-j\omega 2}\cdot X(j\omega)\right] =$

$= \boxed{\left(\frac{1}{j\omega}+\pi\delta(\omega)\right)\left(2-3e^{-j\omega 2}\cdot X(j\omega)\right)}$

b) $F\{e^{2t}u(-t)+e^{-2t}u(t)\} = \boxed{\frac{1}{2-j\omega}+\frac{1}{2+j\omega}}$

c) $F\{4\cos(7t)\} = 4\left(\frac{1}{2}\left(e^{j7t}+e^{-j7t}\right)\right) =$

$= 2\left(e^{j7t}+e^{-j7t}\right) = 2\left(2\pi\delta(\omega-7)+2\pi\delta(\omega+7)\right) =$

$= \boxed{4\pi\left(\delta(\omega-7)+\delta(\omega+7)\right)}$

D) $F^{-1}\left\{\frac{2}{j\omega}\right\} = 2\left(\frac{1}{j\omega}+\pi\delta(\omega)-\pi\delta(\omega)\right) =$

$= 2u(t) - 2\pi\delta(\omega) = 2u(t) - 2\pi\cdot\frac{1}{2\pi} = \boxed{2u(t)-1}$

17.6.2012. ⑥

$$y(t) = \int_{-\infty}^{t-1} x(\tau-2)\,d\tau$$

$$y(j\omega) = \int_{-\infty}^{t-1} x(\tau-2)\,d\tau = \begin{Bmatrix} \tau-2=\lambda \\ d\tau = d\lambda \end{Bmatrix} =$$

$$= \int_{-\infty}^{t-3} x(\lambda)\,d\lambda = e^{-j\omega 3} \cdot x(j\omega) =$$

$$= \boxed{e^{-j3\omega} \cdot \left(\frac{1}{j\omega} \cdot x(j\omega) + \pi x(j0) \cdot \delta(\omega) \right)}$$

17.6.2012. ⑦

$$x(t) = e^{-at} \cdot u(t)$$

$$x(j\omega) = \boxed{\frac{1}{a+j\omega}}$$

$$E_x = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |x(j\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\frac{1}{\sqrt{a^2+\omega^2}} \right)^2 d\omega =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{a^2+\omega^2} d\omega = \frac{1}{2\pi} \cdot \frac{1}{a} \cdot \operatorname{arctg} \frac{\omega}{a} \Big|_{-\infty}^{+\infty} =$$

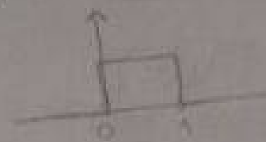
$$= \frac{1}{2\pi a} \left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right) = \frac{1}{2\pi a} \cdot \pi = \boxed{\frac{1}{2a}}$$

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} e^{-2at} dt = -\frac{1}{2a} e^{-2at} \Big|_{-\infty}^{+\infty} =$$

$$= -\frac{1}{2a} (0-1) = \boxed{\frac{1}{2a}}$$

D) $x(t) = \cos(2t)[u(t) - u(t-1)]$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt$$



$$X(j\omega) = \int_0^1 \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right) \cdot e^{-j\omega t} dt =$$

$$= \frac{1}{2}\int_0^1 e^{-jt(\omega-2)} dt + \frac{1}{2}\int_0^1 e^{-jt(\omega+2)} dt =$$

$$= -\frac{1}{2} \cdot \frac{1}{(\omega-2)j} \cdot e^{-jt(\omega-2)}\Big|_0^1 - \frac{1}{2} \cdot \frac{1}{(\omega+2)j} \cdot e^{-jt(\omega+2)}\Big|_0^1 =$$

$$= \boxed{-\frac{1}{2(\omega-2)j}\left(e^{-(\omega-2)} - 1\right) - \frac{1}{2j(\omega+2)}\left(e^{-j(\omega+2)} - 1\right)}$$

17.6.2010 ①D

$x(t) = e^{-at} \cdot u(t)$

$X(j\omega) = \frac{1}{a+j\omega}$

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} e^{-2at} dt = -\frac{1}{2a} e^{-2at}\Big|_{-\infty}^{+\infty} =$$

$$= -\frac{1}{2a}(0-1) = \boxed{\frac{1}{2a}} \qquad a>0$$

8.7.2012. ③

$h(t) = e^{-2t}u(t)$

$x(t) = 5\sin(3t)$

$H(j\omega) = \frac{1}{2+j\omega}$ $\qquad \arg\{H(j\omega)\} = -\mathrm{arctg}\left(\frac{\omega}{2}\right)$

$|H(j\omega)| = \frac{1}{\sqrt{2^2+\omega^2}}$ $\qquad \varphi = -\mathrm{arctg}\,\frac{3}{2}$

$A = 5\cdot|H(j3)| = 5\cdot\frac{1}{\sqrt{4+9}} = \frac{5}{\sqrt{13}}$

$$\boxed{y(t) = 1{,}4\sin(3t-0{,}98)}$$

17.6.2010 ①

b) $x(t) = u(t+1) - u(t-1)$

$X(j\omega) = \int_{-1}^{1} 1\cdot e^{-j\omega t}\,dt =$

$= -\frac{1}{j\omega}\cdot e^{-j\omega t}\Big|_{-1}^{1} = -\frac{1}{j\omega}\left(e^{-j\omega} - e^{+j\omega}\right) =$

$= \boxed{\frac{1}{j\omega}\left(e^{j\omega} - e^{-j\omega}\right)}$

c) $x(t) = e^{-2t}u(t^{0}) = e^{-2t}u(t)$

$$\boxed{X(j\omega) = \frac{1}{2+j\omega}}$$

9.6.2011. 2

A) $x(t) = u(t+1) - u(t-1)$

$X(j\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt$

$X(j\omega) = \int_{-1}^{1} e^{-j\omega t} dt = -\frac{1}{j\omega} e^{-j\omega t} \Big|_{-1}^{1} =$

$= -\frac{1}{j\omega} \cdot \left(e^{-j\omega} - e^{j\omega} \right) = \boxed{\frac{1}{j\omega} \left(e^{j\omega} - e^{-j\omega} \right)}$

B) $x(t) = e^{-2|t|} \cdot u(t^2)$

$X(j\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt =$

$= \int_{-\infty}^{0} e^{-(-2t)} \cdot e^{-j\omega t} dt + \int_{0}^{+\infty} e^{-2t} \cdot e^{-j\omega t} =$

$= \int_{-\infty}^{0} e^{(2t - j\omega t)} + \int_{0}^{+\infty} e^{-(2t + j\omega t)} =$

$= \int_{-\infty}^{0} e^{t(2-j\omega)} + \int_{0}^{+\infty} e^{-t(2+j\omega)} =$

$= \frac{1}{2-j\omega} \cdot e^{t(2-j\omega)} \Big|_{-\infty}^{0} - \frac{1}{2+j\omega} \cdot e^{-t(2+j\omega)} \Big|_{0}^{+\infty} =$

$= \frac{1}{2-j\omega} (1 - 0) - \frac{1}{2+j\omega} (0 - 1) =$

$= \frac{1}{2-j\omega} + \frac{1}{2+j\omega} = \frac{2+j\omega+2-j\omega}{(2-j\omega)(2+j\omega)} = \boxed{\frac{4}{4+\omega^2}}$

c) $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$

$a_k = \frac{1}{T}\int_T \delta(t) e^{-j\omega t} dt = \frac{1}{T}$

$\omega_0 = \frac{2\pi}{T}$

$X(j\omega) = \sum_{k=-\infty}^{+\infty} \frac{1}{T} \cdot 2\pi\delta(\omega - k\omega_0) = \sum_{k=-\infty}^{+\infty} \omega_0 \delta(\omega - k\omega_0)$

$$\boxed{X(j\omega) = \omega_0 \sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_0)}$$

D) $X(j\omega) = \frac{2}{j\omega} + \delta(\omega) = 2\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) - 2\pi\delta(\omega) + \delta(\omega) =$

$x(t) = 2u(t) - \cancel{2\pi} \cdot \frac{1}{\cancel{2\pi}} + \frac{1}{2\pi}$

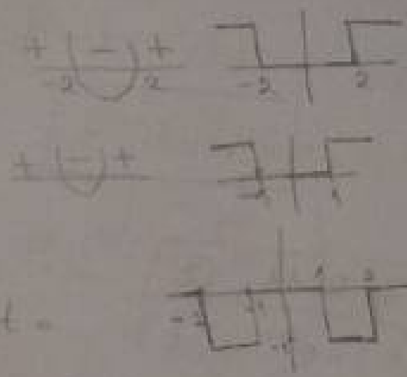
$$\boxed{x(t) = 2u(t) + \frac{1}{2\pi} - 1}$$

10.7.2011. ②

A) $x(t) = u(t^2-4) - u(t^2-1)$

$u(t^2-4) = \begin{cases} 1, & t^2-4>0 \\ 0, & t^2-4<0 \end{cases}$

$u(t^2-1) = \begin{cases} 1, & t^2-1>0 \\ 0, & t^2-1<0 \end{cases}$



$$X(j\omega) = -\int_{-2}^{-1} e^{-j\omega t}\,dt - \int_{1}^{2} e^{-j\omega t}\,dt =$$

$$= +\frac{1}{j\omega} e^{-j\omega t}\Big|_{-2}^{-1} + \frac{1}{j\omega} e^{-j\omega t}\Big|_{1}^{2} =$$

$$= \boxed{\frac{1}{j\omega}\left(e^{j\omega} - e^{j2\omega}\right) + \frac{1}{j\omega}\left(e^{-j2\omega} - e^{-j\omega}\right)}$$

B) $x(t) = e^{-2|t|}u(t)$ $\quad |t| = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases}$

$$X(j\omega) = \int_{0}^{+\infty} e^{-2t} e^{-j\omega t}\,dt =$$

$$= \int_{0}^{+\infty} e^{-t(2+j\omega)}\,dt = -\frac{1}{2+j\omega}\cdot e^{-t(2+j\omega)}\Big|_{0}^{+\infty} =$$

$$= -\frac{1}{2+j\omega}(0-1) = \boxed{\frac{1}{2+j\omega}}$$

D) $X(j\omega) = \frac{2}{j\omega} + \pi\delta(\omega) = 2\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) - 2\pi\delta(\omega) + \pi\delta(\omega)$

$x(t) = 2u(t) - 2\pi \cdot \frac{1}{2\pi} + \pi \cdot \frac{1}{2\pi}$

$x(t) = 2u(t) - 1 + \frac{1}{2}$

$x(t) = \boxed{2u(t) - \frac{1}{2}}$

10.7.2011. 3

$x(t) = e^{3t} \cdot u(-t)$

$X(j\omega) = \int_{-\infty}^{+\infty} \left(e^{3t} \cdot u(-t)\right) \cdot e^{-j\omega t}\, dt =$

$= \int_{-\infty}^{0} e^{3t} \cdot e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{t(3-j\omega)}\, dt =$

$= \frac{1}{3-j\omega} \cdot e^{t(3-j\omega)}\Big|_{-\infty}^{0} = \frac{1}{3-j\omega} \cdot (1-0) = \boxed{\frac{1}{3-j\omega}}$

$E_x = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2\, d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\frac{1}{\sqrt{3^2+\omega^2}}\right)^2 d\omega =$

$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{9+\omega^2}\, d\omega = \frac{1}{2\pi} \cdot \frac{1}{3} \cdot \mathrm{arctg}\frac{\omega}{3}\Big|_{-\infty}^{+\infty} =$

$= \frac{1}{6\pi}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \frac{1}{6\pi} \cdot \pi = \boxed{\frac{1}{6}}$

.6.2008 ②

a) $F\{u(t)-u(t-2)\} =$

$= \int_0^2 1 \cdot e^{-j\omega t}\,dt = -\frac{1}{j\omega} \cdot e^{-j\omega t}\Big|_0^2 =$

$= \boxed{\frac{j}{\omega} \cdot \left(e^{-j2\omega} - 1\right)}$

b) $F\{\sin(2t)+\cos^2(t)\} = \int_{-\infty}^{+\infty} \frac{1}{2j}\left(e^{j2t}-e^{-j2t}\right) e^{-j\omega t}\,dt + \frac{1}{2}$

$+ \frac{1}{2}\int_{-\infty}^{+\infty} \frac{1}{2}\left(e^{j2t}+e^{-j2t}\right) e^{-j\omega t}\,dt = \frac{1}{2j}\left(\int_{-\infty}^{+\infty} e^{t(j2-j\omega)}\,dt - \int_{-\infty}^{+\infty} e^{t(j2-j\omega)}\,dt\right) +$

$+ \frac{1}{2} + \frac{1}{4}\left(\int_{-\infty}^{+\infty} e^{t(2j-j\omega)}\,dt + \int_{-\infty}^{+\infty} e^{t(j2-j\omega)}\right) = \frac{1}{2j}\left(\frac{1}{j2-j\omega}\right($

c) $F\{e^{|-t|}\} = \int_{-\infty}^{0} e^{t} \cdot e^{-j\omega t}\,dt + \int_{0}^{+\infty} e^{-t} \cdot e^{-j\omega t}\,dt =$

$= \int_{-\infty}^{0} e^{t(1-j\omega)}\,dt + \int_{0}^{+\infty} e^{-t(1+j\omega)}\,dt =$

$= \frac{1}{1-j\omega} e^{t(1-j\omega)}\Big|_{-\infty}^{0} - \frac{1}{1+\omega t} \cdot e^{-t(1+j\omega)}\Big|_0^{+\infty} =$

$= \frac{1}{1-j\omega} \cdot (1-0) - \frac{1}{1+j\omega}(0-1) =$

$= \frac{1}{1-j\omega} + \frac{1}{1+j\omega} = \frac{1+\cancel{j\omega}+1-\cancel{j\omega}}{(1-j\omega)(1+j\omega)} = \boxed{\frac{2}{1+\omega^2}}$

D) $F\{\delta(t-1)-\delta(t+1)\} = \int_{-1}^{1} 1 \cdot e^{-j\omega t} =$

$= -\frac{1}{j\omega} \cdot e^{-j\omega t}\Big|_{-1}^{1} = \frac{1}{\omega}\left(e^{-j\omega} - e^{j\omega}\right) = \cos(\omega)$

$= -\frac{1}{\omega}\left(e^{j\omega} - e^{-j\omega}\right) = -\frac{1}{\omega} \cdot \frac{1}{2} \cdot 2 \cdot \left(e^{j\omega} - e^{-j\omega}\right) =$

$= \boxed{-\frac{1}{\omega} \cdot 2\sin(\omega)}$

E) $F^{-1}\left\{\frac{6}{j\omega} - 6\pi\delta(\omega)\right\} = 6\left(\frac{1}{j\omega} - \pi\delta(\omega)\right) =$

$= 6\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) - 7\pi\delta(\omega) = 6u(t) - 7\pi \cdot \frac{1}{2\pi} =$

$= 6u(t) - \frac{7}{2} = \boxed{6u(t) - 3{,}5}$

PARSEVALOVA TEOREMA

$$\boxed{E_x = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega}$$ → PARSEVALOVA TEOREMA (FREKVENCIJSKI DOMEN)

$$\boxed{E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt}$$ → DEFINICIJA ZA ENERGIJU U VREMENSKOM DOMENU

17.6.2012.

A) $X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$

DJ) $X(j\omega) = 2\delta(\omega-3)$, $x(t) = 2\pi\,\delta(\omega-3)\cdot\frac{1}{\pi} = e^{j3t}\cdot\frac{1}{\pi}$

E) $F\{x(t-2)\} = e^{-j\omega(+2)} \cdot X(j\omega)$

$F\{e^{-3t}x(t)\} = \boxed{X(3+j\omega)} = X(j(\omega+3))$ DA IMA j

$F\{2^t x(t)\} \overset{?}{=} X(j\omega - \ln 2)$

Ž) $H(j\omega) - \underline{F\{h(t)\}} = Y(j\omega)/X(j\omega)$

FURIJE

① SINTETIČKA $x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}\, d\omega$

$$\boxed{\text{(II)} \quad X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \rightarrow \text{(II) ANALITIČKA}}$$

KONVERGENCIJA

$$E = \int_{-\infty}^{+\infty} |e(t)|^2 dt = 0 \quad , \quad e(t) = x(t) - \tilde{x}(t)$$

$$\tilde{x}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}\, dt$$

MERA e

PERIODIČAN

$$\sin(3t) = \frac{1}{2j}\left(e^{j3t} - e^{-j3t}\right)$$

$$\boxed{e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi\delta(\omega - \omega_0)}$$

$$X(j\omega) = \frac{1}{2j}\left[2\pi\delta(\omega-3) - 2\pi\delta(\omega+3)\right]$$

$$\cos(3t) = \frac{1}{2}\left(e^{j3t} + e^{-j3t}\right)$$

OSOBINE

$\mathcal{F}$

① LINEARNOST

$$F\{a\,x(t)+b\,y(t)\} = \boxed{a\,F\{x(t)\} + b\,F\{y(t)\}}$$ (I)

② POMERANJE U VREMENU

$x(t-t_0) \xleftrightarrow{F} ?$

$$F\{x(t-t_0)\} = \int_{-\infty}^{+\infty} x(t-t_0)\cdot e^{-j\omega t}\,dt = \int_{-\infty}^{+\infty} x(\lambda)\cdot e^{-j\omega(\lambda+t_0)}\,d\lambda$$

$\left(\lambda = t-t_0 \Rightarrow t = \lambda + t_0\right)$

(II)

$$= \underbrace{\int_{-\infty}^{+\infty} x(\lambda)\cdot e^{-j\omega\lambda}}_{X(j\omega)}\cdot e^{-j\omega t_0}\,d\lambda = \boxed{e^{-j\omega t_0}\cdot X(j\omega)}$$

③ MODULACIJA

$x(t)\cdot e^{j\omega_0 t} \xleftrightarrow{F} ?$

$$F\{x(t)\,e^{j\omega_0 t}\} = \int_{-\infty}^{+\infty} x(t)\,e^{j\omega_0 t}\cdot e^{-j\omega t}\,dt =$$

$$= \int_{-\infty}^{+\infty} x(t)\,e^{jt(\omega_0-\omega)}\,dt = \int_{-\infty}^{+\infty} x(t)\,e^{-jt\overbrace{(\omega-\omega_0)}^{\omega}}\,dt =$$

$$= \boxed{X(j(\omega-\omega_0))}$$ (III)

SKALIRANJE

$x(at) \xleftrightarrow{F} ?$

$at = \tau \rightarrow t = \frac{\tau}{a}$

$F\{x(at)\} = \int_{-\infty}^{+\infty} x(at) \cdot e^{-j\omega t} dt = \frac{1}{|a|}\int_{-\infty}^{+\infty} x(\tau)\, e^{-j\omega \frac{\tau}{a}} d\tau =$

$= \frac{1}{|a|} \int_{-\infty}^{+\infty} x(\tau) \cdot e^{-j\left(\frac{\omega}{a}\right)\tau} d\tau = \boxed{\frac{1}{|a|} X\left(j\frac{\omega}{a}\right)}$ (IV)

⑤ OSOBINA DIFERENCIRANJA I INTEGRIRANJA

$\frac{dx}{dt} \xleftrightarrow{F} \boxed{j\omega \cdot X(j\omega)}$ (V)

(ŠAMAR NINI)! :)

$\int_{-\infty}^{t} x(t)dt \xleftrightarrow{F} \boxed{\frac{1}{j\omega} X(j\omega) + \pi \cdot X(j0)\delta(\omega)}$ (V)

$* \; F\{u(t)\} = \boxed{\frac{1}{j\omega} + \pi\delta(\omega)}$

⑥ KONVOLUCIJA (DOLAZI IZVOĐENJE)!

$x(t) * y(t) \xleftrightarrow{F} ?$

$F\{x(t) * y(t)\} = \int_{-\infty}^{+\infty} \underbrace{x(t) * y(t)}_{x(t)} \cdot e^{-j\omega t} dt =$

$\int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} x(\tau) y(t-\tau) d\tau \right] e^{-j\omega t} dt =$

$= \boxed{\int_{-\infty}^{+\infty} x(\tau)\, e^{-j\omega\tau}} \int_{-\infty}^{+\infty} y(t-\tau)\, e^{-j\omega t} \cdot e^{+j\omega\tau}\, \boxed{d\tau}\, dt =$

$e^{-j\omega(t-\tau)}$

$= \boxed{X(j\omega) \cdot Y(j\omega)}$ (VI)

7 MNOŽENJE SIGNALA

$$x(t)\cdot y(t) \xleftrightarrow{F} \boxed{\frac{1}{2\pi}\left[X(j\omega) * Y(j\omega)\right]} \quad \text{VII}$$

8 SIMETRIČNOST SIGNALA

$$x^*(t) \xleftrightarrow{F} ?$$

$$F\{x^*(t)\} = \int_{-\infty}^{+\infty} x^*(t)\cdot e^{-j\omega t}\, dt = \left[\int_{-\infty}^{+\infty} x(t)\, e^{+j\omega t}\, dt\right]^* =$$

$$= \boxed{X^*(-j\omega)} \quad \text{VIII}$$

$$\mathcal{F}\left\{\sin(2t) + \cos^2(t)\right\}$$

$$\cos^2(t) = (\cos(t))^2$$

$$= \left(\frac{1}{2}\left(e^{jt} + e^{-jt}\right)\right)^2$$

$$= \boxed{\frac{1}{4}\left(e^{j2t} + 2 \cdot + e^{-j2t}\right)}$$