Fuzzy logic

Fuzzy Control

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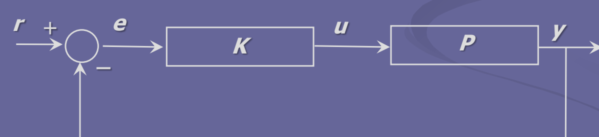
Conventional Feedback Control

- The term **control** is generally defined as a mechanism used to guide or regulate the operation of a machine, apparatus or their constellations.
- Often the notion of control is inextricably linked with **feedback**:
  a process of returning the output (regulated) variable signal to the input of a device (optionally compared with some reference value) in order to obtain appropriate control signal.
- Feedback can be (and usually is) **negative**, whereby feedback opposes the output increase by reducing the control input, or feedback can be **positive** whereby feedback reinforces the input.

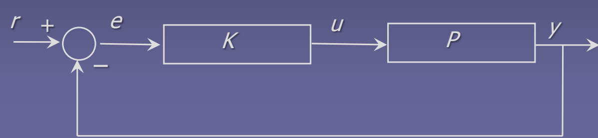


- The machinery or apparatus to be guided (regulated) is denoted by *P (Plant)*, the reference input by *r*, the controlled output by *y*, and the feedback controller by *K*. Controller's input is the so-called error signal *e* and the purpose of the controller is to generate control *u* in order to obtain desired output response *y*.

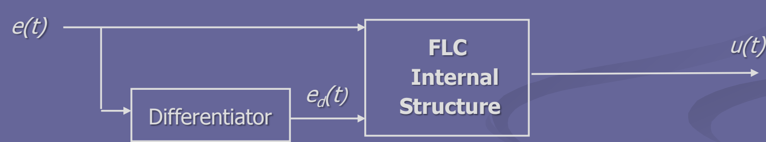
Fuzzy Logic Controller (FLC)

- By a **fuzzy logic controller** (**FLC**) we mean a control law that is described by a knowledge-based system consisting of linguistic IF...THEN rules with vague predicates and a fuzzy logic inference mechanism.
- The rule base (main FLC part) is formed as generalisation of the human control experience by a family of logical rules describing the mapping of the FLC input variables (*e, x, y*) into the FLC output (*u*).
- The main difference between a conventional control system and a fuzzy logic controlled system is not only in the type of logic (conventional or fuzzy) but in the inspiration:
  - the former attempts to increase the efficiency of control algorithms;
  - the latter is based on the implementation of human understanding and human thinking in control algorithms.
- As well as conventional controller, the FLC controller can be used with the process in two modes:
  - feedback mode when the fuzzy controller will act as a control device;
  - feedforward mode where the controller can be used as a prediction device.

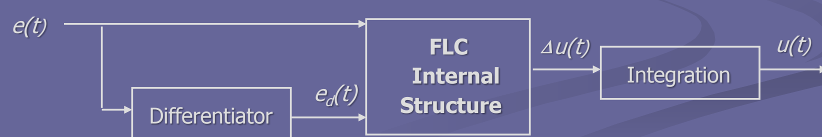
FLC within Conventional Feedback Control Structure



Structure of the Conventional Output Feedback Control System



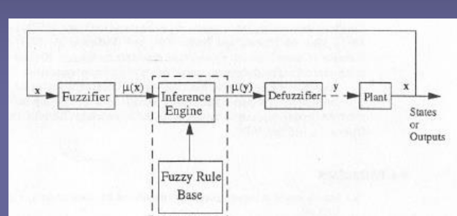
Example of the Fuzzy-logic PD based Controller Implementation



Example of the Fuzzy-logic Incremental Controller Implementation

Intuitive Approach to FLC Design

- Basic architecture of FLC:



- Example of the resulting control lookup table for the PD based FLC with output feedback:

e \ $\dot{e}$	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6
-6	·	·	·	·	·	·	-6	·	·	·	·	·	·
-5	·	·	·	·	·	-6	·	·	·	·	·	·	·
-4	·	·	·	·	·	·	·	·	·	·	·	·	·
-3	·	·	·	·	·	·	-3	·	·	·	·	·	·
-2	·	·	·	·	·	·	-5	+1	·	·	·	·	·
-1	·	·	·	+1	+1	-1	-2	+2	·	·	·	·	·
0	·	·	+2	+4	+3	·	0	+5	+5	·	·	·	·
+1	·	·	·	·	+5	·	·	+5	·	·	·	·	·
+2	·	·	·	·	·	+6	·	·	·	·	·	·	·
+3	·	·	·	·	·	+6	+6	·	·	·	·	·	·
+4	·	·	·	·	·	·	·	·	·	·	·	·	·
+5	·	·	·	·	·	·	·	·	·	·	·	·	·
+6	·	·	·	·	·	·	·	·	·	·	·	·	·

Intuitive Approach to FLC Design

- Some guidelines for the output feedback PD based FLC -

- Example: Tank control problem
- PD based FLC controlling plants with astatism (embedded integration):
  - Determination of the input and the output universes:
    - $u \in [U_{min}, U_{max}]$ – limit values of control (should be known a-priori),
    - $y \in [Y_{min}, Y_{max}] \rightarrow e = r - y \in [E_{min}, E_{max}]$ is known,
    - e_d universe can be obtained experimentally by the open-loop step response.
  - Rule base construction:

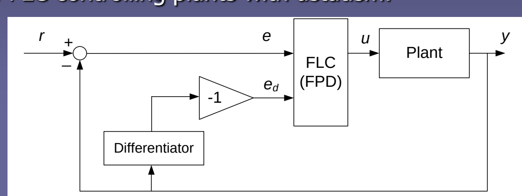
Rule 1:	IF *e* is *negative*		THEN *u* is *negative*
Rule 2:	IF *e* is *positive*		THEN *u* is *positive*
Rule 3:	IF *e* is *zero*		THEN *u* is *zero*
Rule 4:	IF *e* is *zero*	AND e_d is *positive*	THEN *u* is *positive_small*
Rule 5:	IF *e* is *zero*	AND e_d is *negative*	THEN *u* is *negative_small*

- PD based FLC for incremental control of plants without astatism:
  - Integrator added in front of the plant, so generalized plant has astatism and previous experiment to determine e_d universe can be applied.

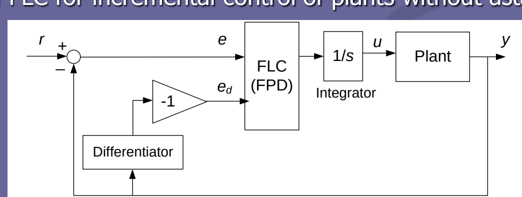
Intuitive Approach to FLC Design

- Implementation -

- PD based FLC controlling plants with astatism:



- PD based FLC for incremental control of plants without astatism:



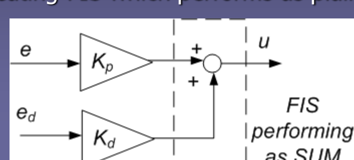
- Note: $e_d = de/dt = dr/dt - dy/dt$ approximated: $e_d = -dy/dt$ in order to avoid e_d excessive peaks due to step changes in reference.

Another Approach to FLC Design: Fuzzifying Conventional PID

- One systematic procedure for fuzzy controller design is based on transferring conventionally designed PID into the fuzzy domain, according to following steps:
  - Obtain parameters for the conventional PID.
  - Substitute PID with ***equivalent linear fuzzy controller.***
  - Transform to the nonlinear fuzzy controller by changing rules and membership functions.
  - Fine tuning.

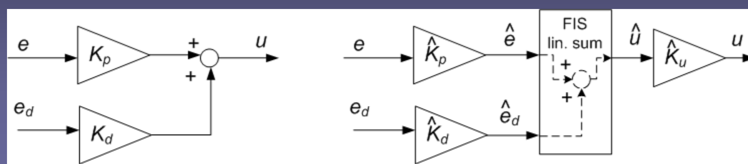
Fuzzifying Linear PD Block

- Basic idea is creating FIS which performs as plain linear sum of two inputs:



- Choices that make fuzzy inference equivalent to plain ***sum*** :
  - ***Input*** membership functions – **triangular** with **50% overlap**.
  - ***AND*** *operation* – ***Prod*** method (algebraic product).
  - **Rule base** – ***AND*** combination **of all** input memberships.
  - ***Output*** membership functions – ***singletons*** on the positions of peak sums of input membership values.
  - **Defuzzification** – ***Weighted Average*** method.
- Resulting input-output surface is flat diagonal
  i.e. output signal is the sum of inputs
- Program code: *gen_lin_fpd.m*

Fuzzy PD



- Equivalence of conventional PD (left) and linear FPD (right):

$$K_p = \hat{K}_p \hat{K}_u, \; K_d = \hat{K}_d \hat{K}_u$$

  where "capped" variables should respect the (normalized) universes of FIS linear sum:
  - inputs $\hat{e}$ and $\hat{e}_d$ should be within the range [-1, +1],
  - output $\hat{u}$ is generated within the range [-2, +2].

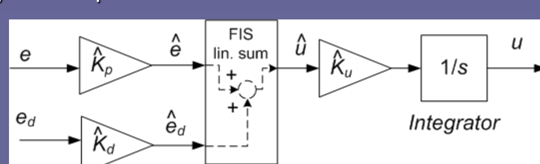
- Now, FIS linear sum can be detuned (membership functions changed, rules deleted/changed, etc) to obtain more efficient nonlinear FPD control solution.

Fuzzy Incremental PI

- Conventional PI in incremental form

$$K_{PI}(s) = K_p + \frac{K_i}{s} = \left(K_p s + K_i\right) \cdot \frac{1}{s}$$

  consists of PD block in series with the integrator.
- So, the fuzzy counterpart should the series of linear FPD and integrator:



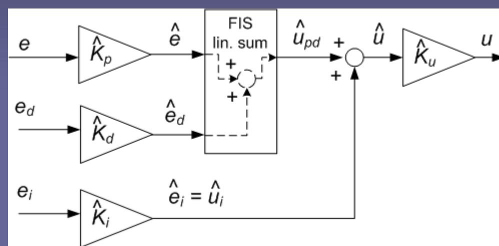
- Equivalence of conventional PI and incremental FPI:

$$K_i = \hat{K}_p \hat{K}_u, \; K_p = \hat{K}_d \hat{K}_u$$

  where normalized ("capped") FIS inputs $\hat{e}$ and $\hat{e}_d$ should respect the input universe [-1, +1] of FIS.
- Now, FIS can be detuned from linear sum to some other nonlinear transfer in order to obtain more efficient (now nonlinear) incremental FPI solution.

Fuzzy PID: Variant FPD+I

- Conventional PD part is fuzzified, leaving conventional I in parallel:



- Equivalence of conventional PID and FPD+I:

$$K_p = \hat{K}_p \hat{K}_u, \; K_d = \hat{K}_d \hat{K}_u, \; K_i = \hat{K}_i \hat{K}_u$$

  where normalized ("capped") FIS inputs $\hat{e}$ and $\hat{e}_d$ should respect the input universe [-1, +1] of FIS.
- Now, FIS linear sum can be detuned to obtain more efficient nonlinear FPD+I solution.

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