Fuzzy logic

Introduction 3

Fuzzy Inference

Aleksandar Rakić
rakic@etf.rs

Contents

- Mamdani Fuzzy Inference
  - Fuzzification of the input variables
  - Rule evaluation
  - Aggregation of rules' output
  - Defuzzification
- Sugeno Fuzzy Inference
- Mamdani or Sugeno?

Mamdani Fuzzy Inference

- The most commonly used fuzzy inference technique is the so-called **Mamdani** method.
- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.
- The Mamdani-style fuzzy inference process is performed in four steps:
  1. Fuzzification of the input variables,
  2. Rule evaluation (inference),
  3. Aggregation of the rule outputs (composition),
  4. Defuzzification.

3

Mamdani Fuzzy Inference

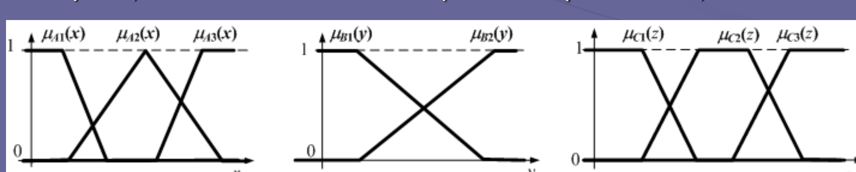
- We examine a simple two-input one-output problem that includes three rules:

Rule 1:	IF *x* is *A3*	OR	*y* is *B1*	THEN	*z* is *C1*
Rule 2:	IF *x* is *A2*	AND	*y* is *B2*	THEN	*z* is *C2*
Rule 3:	IF *x* is *A1*			THEN	*z* is *C3*

- Real-life example for these kinds of rules:

Rule 1:	IF *project_funding* is *adequate*	OR	*project_staffing* is *small*	THEN	*risk* is *low*
Rule 2:	IF *project_funding* is *marginal*	AND	*project_staffing* is *large*	THEN	*risk* is *normal*
Rule 3:	IF *project_funding* is *inadequate*			THEN	*risk* is *high*

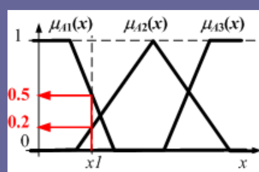
- Fuzzy sets, defined over universes of input and output variables, are:

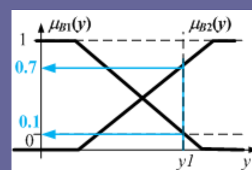


4

Step 1: Fuzzification

- The first step is to take the crisp inputs, *x1* and *y1* (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.





$\mu_{A1}(x1) = 0.5$
$\mu_{A2}(x1) = 0.2$

$\mu_{B1}(y1) = 0.1$
$\mu_{B2}(y1) = 0.7$

Step 2: Rule Evaluation

- The second step is to take the fuzzified inputs, $\mu_{A1}(x) = 0.5$, $\mu_{A2}(x) = 0.2$, $\mu_{B1}(y) = 0.1$ and $\mu_{B2}(y) = 0.7$, and apply them to the antecedents of the fuzzy rules.
- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND / OR) is used to obtain a single number that represents the result of the antecedent evaluation.

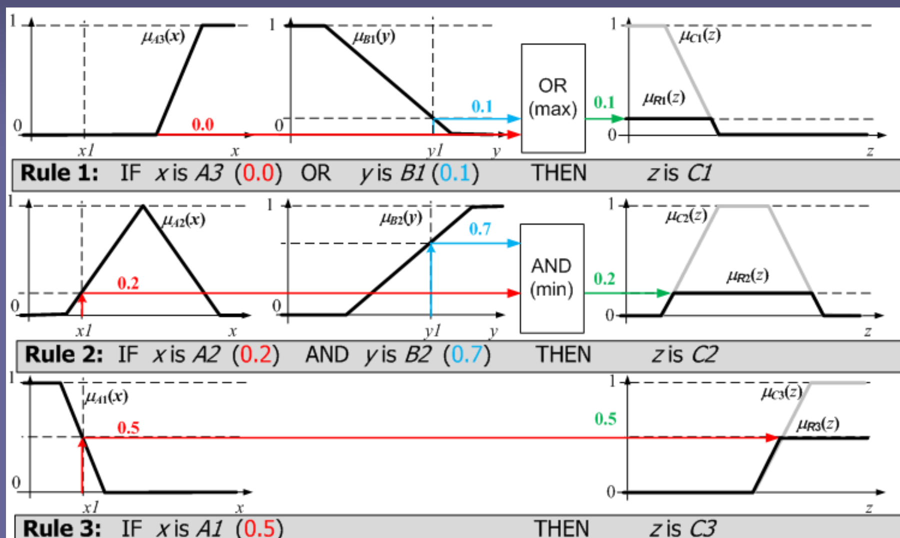
RECALL: To evaluate the disjunction of the rule antecedents, we use the **OR** fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation union:

$$\mu_{A \cup B}(x) = \max\,[\mu_A(x),\ \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND** fuzzy operation intersection:

$$\mu_{A \cap B}(x) = \min\,[\mu_A(x),\ \mu_B(x)]$$

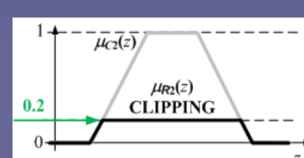
Step 2: Rule Evaluation



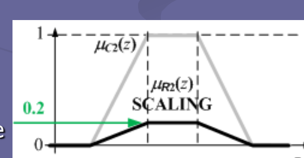
Step 2: Rule Evaluation

- Now the result of the antecedent evaluation can be applied to the membership function of the consequent (fuzzy implication).

- The most common method is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping** (alpha-cut).
  - Since the top of the membership function is sliced, the clipped fuzzy set loses some information.
  - However, clipping is still often preferred for less complex and faster mathematics, and it generates an aggregated output that is easier to defuzzify.



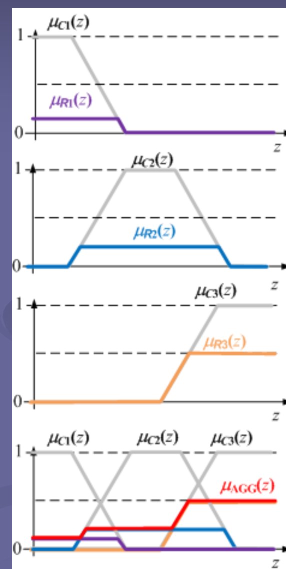
- The other frequently used method is **scaling**, where the membership function of the rule's consequent is multiplied by the truth value of the rule antecedent.
  - This method, which generally loses less information, offers a better approach for preserving the original shape of the output fuzzy set.



Step 3: Aggregation of Rules

- Aggregation is the process of unification of the outputs of all rules.
- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.
- The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.
- The most common aggregation method is maximum of rules' output membership functions:

$\mu_{AGG}(z)$ = max [$\mu_{R1}(z)$, $\mu_{R12}(z)$, $\mu_{R3}(z)$]



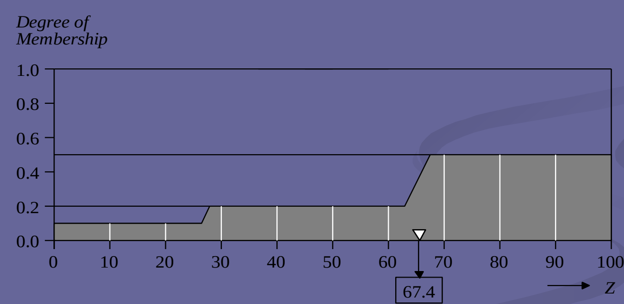
Step 4: Defuzzification

- The last step in the fuzzy inference process is defuzzification.
- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.
- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity** (**COG**) can be expressed as:

$$COG = \frac{\int_a^b \mu_{AGG}(z)\, z\, dz}{\int_a^b \mu_{AGG}(z)\, dz}$$

Step 4: Defuzzification

- Centroid defuzzification method finds a point representing the centre of gravity of the aggregated fuzzy set *AGG*, on the interval [*a, b*].
- A reasonable estimate can be obtained by calculating it over a sample of points.



$$COG = \frac{(0+10+20)\times 0.1+(30+40+50+60)\times 0.2+(70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

11

Sugeno Fuzzy Inference

- Mamdani-style inference, as we have just seen, requires us to find the centroid of aggregated membership function by integrating this function over the (output variable) universe.
- In general, this process is not exact, since defuzzification integrals are approximated in calculation by sampling and defuzzified value is effectivelly estimated by weighted sums.
- Michio Sugeno suggested to use a single spike, a singleton, as the membership function of the rule consequent.
- Recall: A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

12

Sugeno Fuzzy Inference

- Sugeno-style fuzzy inference is very similar to the Mamdani method.
- Sugeno changed only a rule consequent: instead of a fuzzy set, he used a mathematical function of the input variable.
- The format of the **Sugeno-style fuzzy rule** is

  IF ***x*** **is** ***A*** **AND** ***y*** **is** ***B*** **THEN** ***z*** **is** ***f*** **(*x, y*)**

  where:
  - *x*, *y* and *z* are linguistic variables;
  - A and *B* are fuzzy sets on universe of discourses *X* and *Y*, respectively;
  - *f* (*x*, *y*) is a mathematical function.

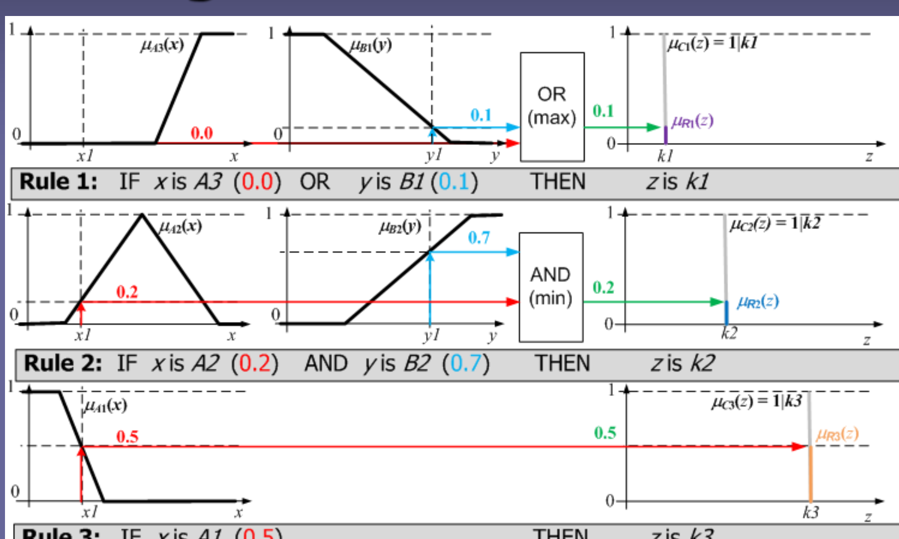
- The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

  IF ***x*** **is** ***A*** **AND** ***y*** **is** ***B*** **THEN** ***z*** **is** ***k***

- where *k* is a constant.
- In this case, the output of each fuzzy rule is constant and all consequent **membership functions** are represented by **singleton spikes.**

13

Sugeno Rule Evaluation



14

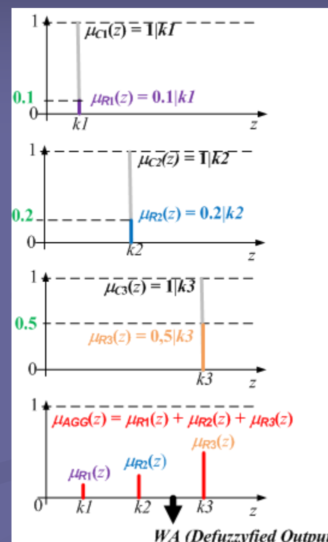
Sugeno Aggregation and Defuzzification

- MAX aggregation becomes the sum of singleton spikes

$$\mu_{AGG}(z) = \max[\mu_{R1}(z), \mu_{R12}(z), \mu_{R3}(z)] = \mu_{R1}(z) + \mu_{R2}(z) + \mu_{R3}(z) = 0.1|k1 + 0.2|k2 + 0.5|k3$$

- COG defuzzification becomes Weighted Average (WA)

$$WA = \frac{\sum_{j=1}^{n}\left(\mu_{AGG}(k_j)\times k_j\right)}{\sum_{j=1}^{n}\mu_{AGG}(k_j)} = \frac{\mu_{AGG}(k1)\times k1 + \mu_{AGG}(k2)\times k2 + \mu_{AGG}(k3)\times k3}{\mu_{AGG}(k1)+\mu_{AGG}(k2)+\mu_{AGG}(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$



15

Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.

- On the other hand, Sugeno method is computationally effective and works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.

16