Fuzzy logic

Introduction

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Contents

2

Definition

- Experts rely on **common sense** when they solve problems.
- **How can we represent expert knowledge that uses vague and ambiguous terms in a computer?**
- Fuzzy logic is not logic that is fuzzy, but logic that is used to describe fuzziness. Fuzzy logic is the theory of fuzzy sets, sets that calibrate vagueness.
- Fuzzy logic is based on the idea that all things admit of **degrees**. Temperature, height, speed, distance, beauty – all come on a sliding scale.
  - The motor is running **slightly hot**.
  - Tom is a **very tall** guy.

3

Definition

- The concept of a set and set theory are powerful concepts in mathematics. However, the principal notion underlying set theory, that an element can (exclusively) either belong to set or not belong to a set, makes it well nigh impossible to represent much of human discourse. How is one to represent notions like:
  - large profit
  - high pressure
  - tall man
  - moderate temperature
- Ordinary set-theoretic representations will require the maintenance of a crisp differentiation in a very artificial manner:
  - high
  - not quite high
  - very high … etc.

4

Definition

- Many decision-making and problem-solving tasks are too complex to be understood quantitatively, however, **people succeed by using knowledge that is imprecise rather than precise**.
- Fuzzy set theory resembles human reasoning in its **use of approximate information and uncertainty to generate decisions**.
- It was specifically **designed to mathematically represent uncertainty** and vagueness and provide formalized tools for dealing with the imprecision intrinsic to many problems.
- Since knowledge can be expressed in a more natural way by using fuzzy sets, many engineering and decision problems can be greatly simplified.
- Boolean logic uses sharp distinctions. It forces us to draw lines between members of a class and non-members.
- For instance, we may say, Tom is tall because his height is 181 cm. If we drew a line at 180 cm, we would find that David, who is 179 cm, is small. Is David really a small man or we have just drawn an arbitrary line in the sand?

5

Bit of History

- Fuzzy, or multi-valued logic, was introduced in the 1930s by Jan Lukasiewicz, a Polish philosopher. While classical logic operates with only two values 1 (true) and 0 (false), Lukasiewicz introduced logic that extended the range of truth values to all real numbers in the interval between 0 and 1.
- For example, the possibility that a man 181 cm tall is really tall might be set to a value of 0.86. It is likely that the man is tall. This work led to an inexact reasoning technique often called **possibility theory**.
- In 1965 Lotfi Zadeh, published his famous paper "Fuzzy sets". Zadeh extended the work on possibility theory into a formal system of mathematical logic, and introduced a new concept for applying natural language terms. This new logic for representing and manipulating fuzzy terms was called **fuzzy logic**.

6

The Term "Fuzzy Logic"

- **Why "fuzzy"?**
  As Zadeh said, the term is concrete, immediate and descriptive; we all know what it means. However, many people were repelled by the word fuzzy, because it is usually used in a negative sense.

- **Why "logic"?**
  Fuzziness rests on fuzzy set theory, and fuzzy logic is just a small part of that theory.

- The term fuzzy logic is **used in two senses**:
  - **Narrow sense**: Fuzzy logic is a branch of fuzzy set theory, which deals (as logical systems do) with the representation and inference from knowledge. Fuzzy logic, unlike other logical systems, deals with imprecise or uncertain knowledge. In this narrow, and perhaps correct sense, fuzzy logic is just one of the branches of fuzzy set theory.
  - **Broad Sense**: fuzzy logic synonymously with fuzzy set theory.

7

Fuzzy Applications

- Theory of fuzzy sets and fuzzy logic has been applied to problems in a variety of fields: pattern recognition, decision support, data mining & information retrieval, medicine, law, taxonomy, topology, linguistics, automata theory, game theory, etc.
- More recently fuzzy machines have been developed for: automatic train control, tunnel digging machinery, home appliances: washing machines, air conditioners, etc.

Advertisement for Extraklasse Washing Machine 1200 rpm:

- Fuzzy Logic detects the type and amount of laundry in the drum and allows only as much water to enter the machine as is really needed for the loaded amount. And less water will heat up quicker - which means less energy consumption.
- **Foam detection:** Too much foam is compensated by an additional rinse cycle: If Fuzzy Logic detects the formation of too much foam in the rinsing spin cycle, it simply activates an additional rinse cycle. Fantastic!
- **Imbalance compensation**: In the event of imbalance, Fuzzy Logic immediately calculates the maximum possible speed, sets this speed and starts spinning. This provides optimum utilization of the spinning time at full speed […]
- **Washing without wasting - with automatic water level adjustment**: Fuzzy automatic water level adjustment adapts water and energy consumption to the individual requirements of each wash programme, depending on the amount of laundry and type of fabric […]

EXTRA KLASSE from Siemens

More Definitions

- Fuzzy logic is a set of mathematical principles for knowledge representation based on **degrees of membership**.
- Unlike two-valued Boolean logic, fuzzy logic is **multi-valued**. It deals with **degrees of membership** and **degrees of truth**.
- Fuzzy logic uses the continuum of logical values between 0 (completely false) and 1 (completely true). Instead of just black and white, it employs the spectrum of colours, accepting that things can be partly true and partly false at the same time.

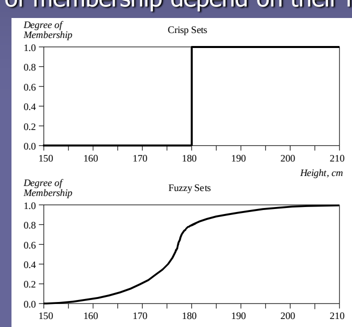
- *(a) Boolean Logic* *(b) Multi-valued Logic*

- The concept of a **set** is fundamental to mathematics.
- However, our own language is also the supreme expression of sets. For example, *car* indicates the *set of cars*. When we say a car, we mean one out of the set of cars.

Crisp vs. Fuzzy Sets

- The classical example in fuzzy sets is tall men. The elements of the fuzzy set "tall men" are all men, but their degrees of membership depend on their height.

Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00



- The x-axis represents the **universe of discourse** – the range of all possible values applicable to a chosen variable. In our case, the variable is the man height. According to this representation, the universe of men's heights consists of all tall men.
- The y-axis represents the **membership value of the fuzzy set**. In our case, the fuzzy set of "tall men" maps height values into corresponding membership values.

A Fuzzy Set has Boundaries

- Let X be the universe of discourse and its elements be denoted as x. In the classical set theory, **crisp set A of X is defined as function $f_A(x)$ called the characteristic function of A**

  $f_A(x) : X \rightarrow \{0, 1\}$, where $$f_A(x) = \begin{cases} 1, \text{if } & x \in A \\ 0, \text{if } & x \notin A \end{cases}$$

  This set maps universe X to a set of two elements. For any element x of universe X, characteristic function $f_A(x)$ is equal to 1 if x is an element of set A, and is equal to 0 if x is not an element of A.
- In the fuzzy theory, **fuzzy set A of universe X is defined by function $\mu_A(x)$ called the membership function of set A**

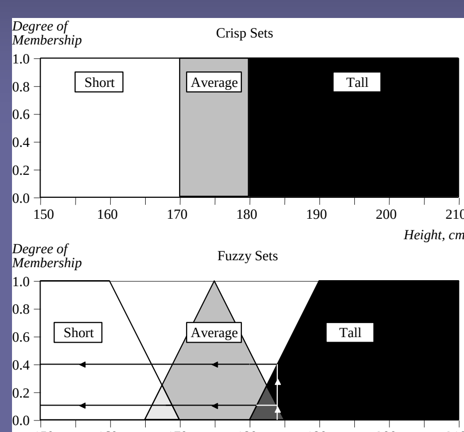
  $\mu_A(x) : X \rightarrow [0, 1]$, where

  $\mu_A(x) = 1$ if x is totally in A;
  $\mu_A(x) = 0$ if x is not in A;
  $0 < \mu_A(x) < 1$ if x is partly in A.

- This definition of set allows a continuum of possible choices. For any element x of universe X, membership function $\mu_A(x)$ equals the degree to which x is an element of set A. This degree, a value between 0 and 1, represents the **degree of membership**, also called **membership value**, of element x in set A.

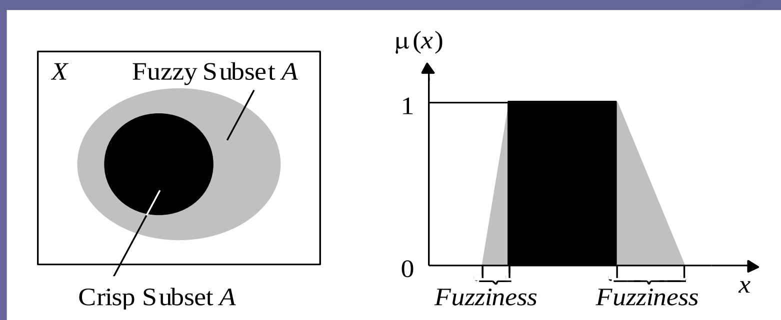
Fuzzy Set Representation

- The universe of discourse for defined fuzzy sets consist of all possible values of variable (the men's heights).
- We determine the membership functions. In our "tall men" example, we can define fuzzy sets of *tall*, *short* and *average* men.
- **Element belong to all fuzzy sets** (defined over the universe of discourse) **at the same time**, but with different membership degree (for example, John, 184 cm tall, is a member of: the *average* set with degree of 0.1, the *tall* set with degree of 0.4, and the *short* set with degree of 0).



Fuzzy Set Representation

- Typical membership functions that can be used to represent a fuzzy set are sigmoid, gaussian and pi.
- However, these functions increase the time of computation. Therefore, in practice, most applications use **linear fit functions**.



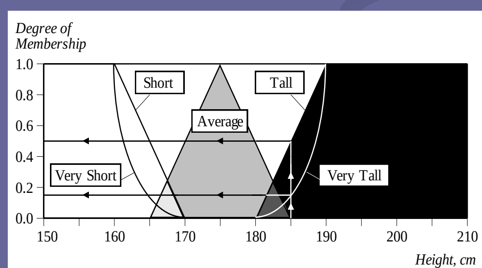
Linguistic Variables

- At the root of fuzzy set theory lies the idea of linguistic variables.
- **A linguistic variable is a fuzzy variable**. For example, the statement "John is tall" implies that the linguistic variable John takes the linguistic value tall.
- In fuzzy expert systems, **linguistic variables are used in fuzzy rules**. For example:

IF	wind	is strong
THEN	sailing	is good
IF	project_duration	is long
THEN	completion_risk	is high
IF	speed	is slow
THEN	stopping_distance	is short

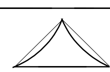
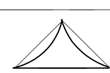
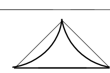
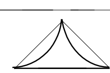
Linguistic Variables and Hedges

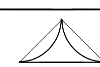
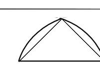
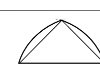
- The range of possible values of a linguistic variable represents the universe of discourse of that variable. For example, the universe of discourse of the linguistic variable *speed* might have the range between 0 and 220 km/h and may include such fuzzy subsets as *very slow*, *slow*, *medium*, *fast*, and *very fast*.
- A linguistic variable carries with it the concept of fuzzy set qualifiers, called **hedges**.
- Hedges are terms that modify the shape of fuzzy sets. They include adverbs such as *very*, *somewhat*, *quite*, *more or less* and *slightly*.



Linguistic Variables and Hedges

- Typical hedges:

Hedge	*Mathematical Expression*	*Graphical Representation*
A little	$[\mu_A(x)]^{1.3}$	
Slightly	$[\mu_A(x)]^{1.7}$	
Very	$[\mu_A(x)]^{2}$	
Extremely	$[\mu_A(x)]^{3}$	

Hedge	*Mathematical Expression*	*Graphical Representation*
Very very	$[\mu_A(x)]^{4}$	
More or less	$\sqrt{\mu_A(x)}$	
Somewhat	$\sqrt{\mu_A(x)}$	
Indeed	$2[\mu_A(x)]^2$ if $0 \le \mu_A \le 0.5$ $1 - 2[1 - \mu_A(x)]^2$ if $0.5 < \mu_A \le 1$