Elektrostatika

Rešeni zadaci iz zbirke

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ZBIRKA ELEKTROSTATIKA

(4.)

$Q_1 = Q_2 = 20\,pC$

$Q_3 = -50\,pC$

$A(-a,0,0)$

$B(a,0,0)$

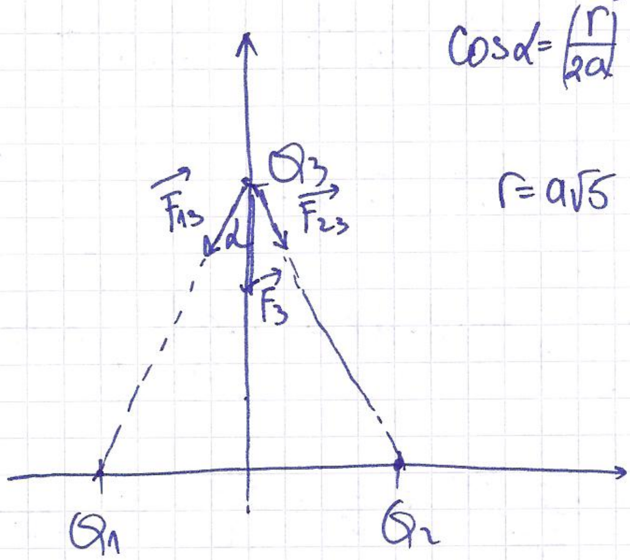
$C(0,2a,0)$

$a = 0{,}2\,m$

$\vec{F}_{e3} = ?$

$\cos\alpha = \left(\frac{r}{2a}\right)^{-1} = \frac{2a}{r} = \frac{2}{\sqrt{5}}$

$r = a\sqrt{5}$



$\vec{F}_{e3} = 2 \cdot |\vec{F}_{13}| \cos\alpha \cdot (-\vec{i}_y)$

$F_{e3} = 2 \cdot \frac{1}{4\pi\varepsilon_0} \frac{Q_1 \cdot |Q_3|}{r^2} \cdot \frac{2}{\sqrt{5}} = 8{,}04 \cdot 10^{-11}\,N$

$\vec{F}_{e3} = -8{,}04 \cdot 10^{-11}\,N \cdot \vec{i}_y$

(14.)

$x \in \left[-\frac{a}{2}, \frac{a}{2}\right]$

$Q'(x) = 8Q_0' \cdot \left|\frac{x^3}{a^3}\right|$

Q_0', a const >0

$Q = ?$

$Q = \int dQ = \int Q' dx = \int_{-\frac{a}{2}}^{\frac{a}{2}} 8Q_0' \left|\frac{x^3}{a^3}\right| dx$

$Q = \int_{-\frac{a}{2}}^{0} 8Q_0' \frac{(-x^3)}{a^3} dx + \int_{0}^{\frac{a}{2}} 8Q_0' \frac{x^3}{a^3} dx = \boxed{\frac{Q_0' a}{4}}$

15. PRAVOUGAONA POVRŠ $a \times b$

Q, $\rho_s(x) = \rho_{s0} \cdot \frac{x}{a}$ $\quad \rho_s(x) = ?$

$Q = \int dQ = \int \rho_s \cdot dS$ $\quad dS = b \cdot dx$

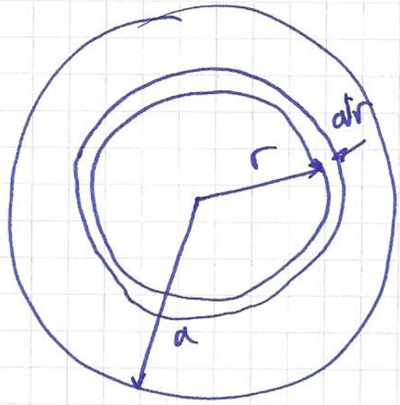
$$Q = \int_0^a \rho_{s0} \frac{x}{a} \cdot b \cdot dx = \frac{\rho_{s0} b}{a} \cdot \int_0^a x\,dx = \frac{\rho_{s0} b}{a} \cdot \frac{x^2}{2}\Big|_0^a = \frac{\rho_{s0} b a^{\cancel{2}}}{2\cancel{a}}$$

$$Q = \frac{\rho_{s0} b a}{2} \Rightarrow \rho_{s0} = \frac{2Q}{ab} \Rightarrow \cancel{\rho_{s0}}\ \rho_s(x) = \rho_{s0} \cdot \frac{x}{a}$$

$$\underline{\rho_s(x) = \frac{2Q}{a^2 b} x}$$

16. KRUŽNA POVRŠ

$\rho_s(r) = \frac{a-r}{a} \cdot \rho_{s0}$, a, ρ_{s0} const > 0



$dS = 2\pi r\,dr$

$Q = \int dQ$

$Q = \int \rho_s dS$

$$Q = \int_0^a \rho_{s0} \frac{a-r}{a} \cdot 2\pi r\,dr = \frac{\rho_{s0} 2\pi}{a} \left[\int_0^a a r\,dr - \int_0^a r^2 dr \right]$$

$$= \frac{\rho_{s0} \cancel{2}\pi}{\cancel{a}} \frac{a^{\cancel{3}\,2}}{\cancel{6}\,3} = \boxed{\frac{1}{3} \rho_{s0} \pi a}$$

(17.) LOPTA a, $\rho(r) = \rho_0 \frac{a-r}{a}$ $r \in [0, a]$

$Q = ?$

$dV = 4r^2\pi dr$

$Q = \int \rho dV$

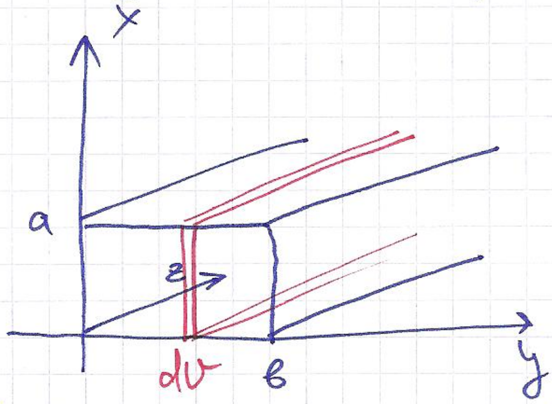
$Q = \int \rho_0 \frac{a-r}{a} 4r^2\pi dr$

$$Q = \int_{r=0}^{a} \rho_0 \frac{a-r}{a} 4r^2\pi dr = \frac{\rho_0 4\pi}{a}\left[\int_0^a ar^2 dr - \int_0^a r^3 dr\right]$$

$$Q = \frac{\rho_0 4\pi}{a}\left[\frac{a^4}{3} - \frac{a^4}{4}\right] = \frac{\rho_0 4\pi}{\cancel{a}} \frac{a^{\cancel{4}3}}{\cancel{12}3} = \frac{1}{3}\rho_0 \pi a^3$$

(18.) VEOMA DUGAČKI CILINDAR, a, b

$\rho(y) = \rho_0 \frac{y}{b}$ $x \in [0, a]$ $y \in [0, b]$ $z \in (-\infty, +\infty)$



$Q' = \frac{Q}{z}$

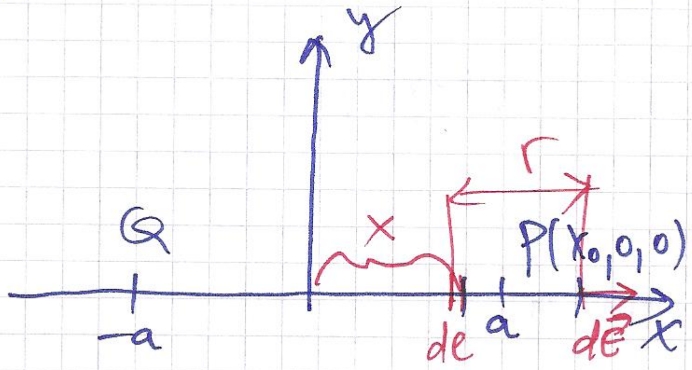
$Q = \int \rho dV$

$dV = a \cdot z \cdot dy$

$$Q = \int_0^b \rho_0 \frac{y}{b} a \cdot z \cdot dy = \frac{\rho_0 a \cdot z}{b}\int_0^b y dy = \frac{\rho_0 a z}{\cancel{b}} \cdot \frac{b^{\cancel{2}}}{2}$$

$$Q' = \frac{Q}{z} = \frac{1}{2}\rho_0 a b$$

19. $2a$, P, $|x_0| > a$ Q $E(\text{u TAČKI } P) = ?$



$$E = \int dE$$

$$E = \frac{1}{4\pi\varepsilon_0} Q \int \frac{dr}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q' dl}{r^2}$$

$$r = x_0 - x$$

$$r = |x_0 - x|$$

$x_0 > a$ $\quad \vec{r_0} = \vec{i_x}$

$x_0 < -a$ $\quad \vec{r_0} = -\vec{i_x}$

$$d\vec{E} = dE_x \vec{i_x}$$

$$\vec{r_0} = \vec{i_x} \, \mathrm{sgn}(x_0 - x)$$

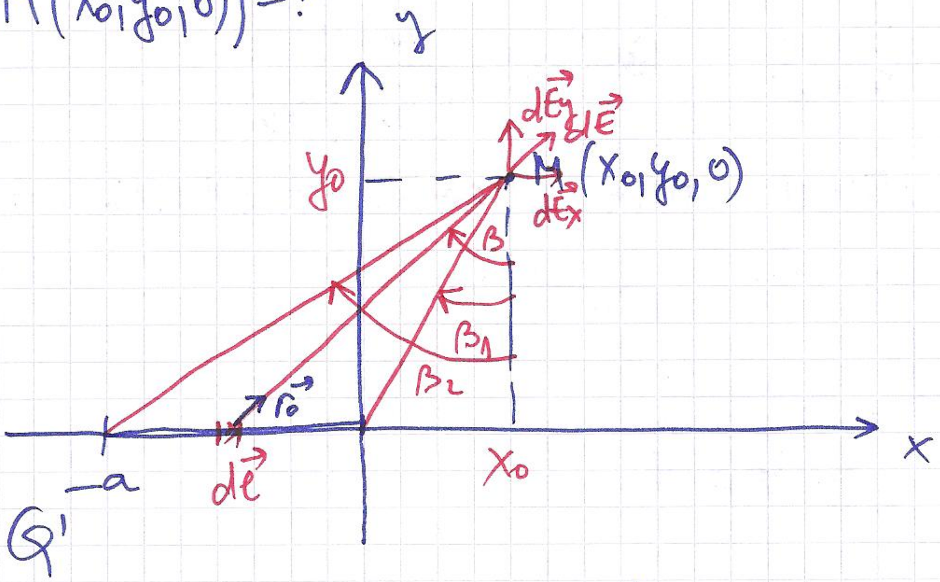
$$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{Q' dx}{(x_0 - x)^2} \mathrm{sgn}(x_0 - x)$$

$$E_x = \int dE_x = \frac{Q' \overset{\mathrm{sgn}\, x_0}{\cancel{\mathrm{sgn}(x_0 - x)}}}{4\pi\varepsilon_0} \left. \frac{1}{x_0 - x} \right|_{x=-a}^{a} \qquad Q' = \frac{Q}{2a}$$

$$\vec{E} = \frac{Q \, \mathrm{sgn}\, x_0}{4\pi\varepsilon_0 (x_0^2 - a^2)} \vec{i_x}, \quad |x_0| > a$$

20.* Q', a, ε_0

$E(M(x_0, y_0, 0)) = ?$



$E = \int dE$ SUPERPOZICIJA

$dE_x = dE \sin\beta$

$dE_y = dE \cos\beta$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{Q' dl}{r^2}$$

$$tg\beta = \frac{x_0 + l}{y_0} \qquad \cos\beta = \frac{y_0}{r}$$

$$r = \frac{y_0}{\cos\beta} \qquad dl = \frac{y_0 d\beta}{\cos^2\beta}$$

$$dE_x = \frac{Q'}{4\pi\varepsilon_0 y_0}\sin\beta d\beta$$

$$dE_y = \frac{Q'}{4\pi\varepsilon_0 y_0}\cos\beta d\beta$$

$$E_x = \int_{\beta_1}^{\beta_2} dE_x = \frac{Q'}{4\pi\varepsilon_0 y_0}(\cos\beta_1 - \cos\beta_2)$$

$$E_y = \int_{\beta_1}^{\beta_2} dE_y = \frac{Q'}{4\pi\varepsilon_0 y_0}(\sin\beta_2 - \sin\beta_1)$$

$$\vec{E} = E_x \vec{i_x} + E_y \vec{i_y} = \frac{Q'}{4\pi\varepsilon_0 y_0}\left((\cos\beta_1 - \cos\beta_2)\vec{i_x} + (\sin\beta_2 - \sin\beta_1)\vec{i_y}\right)$$

$$\cos\beta_1 = \frac{y_0}{\sqrt{x_0^2+y_0^2}} \qquad \cos\beta_2 = \frac{y_0}{\sqrt{(x_0+a)^2+y_0^2}}$$

$$\sin\beta_1 = \frac{x_0}{\sqrt{x_0^2+y_0^2}} \qquad \sin\beta_2 = \frac{x_0+a}{\sqrt{(x_0+a)^2+y_0^2}}$$

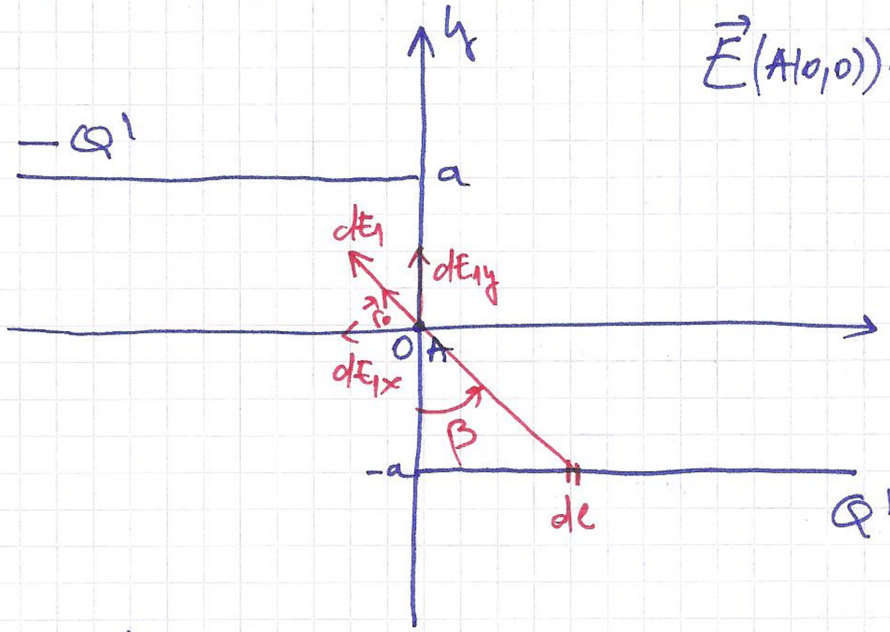
b) U SLUČAJU VEOMA DUGAČKE PRAVOLINIJSKE NITI

$$\beta_1 \rightarrow -\frac{\pi}{2} \qquad \beta_2 \rightarrow \pi/2$$

$$\underline{\underline{\vec{E} = \frac{Q'}{2\pi\varepsilon_0 y_0}\vec{i}_y}}$$

21. DVE RAVNOMERNO NAELEKTRISANE POLUPRAVE Q'

$\vec{E}(A(0,0)) = ?$



Q' i $-Q'$ U OVOM SLUČAJU DAJU ISTI DOPRINOS $d\vec{E}_2 = dE_1 \vec{r}_0$

$\vec{E} = 2\vec{E}_1 \qquad \vec{E}_1 = \int d\vec{E}_1$

$\beta_1 = -\pi/2$

$\beta_2 = 0$

$dl = dx$

$dE_{1x} = dE_1 \sin\beta$

$dE_{1y} = dE_1 \cos\beta$

$dQ = Q'dx$

$dx = dl \qquad a\,tg\beta = x \; / d$

$$dE_1 = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{Q'dx}{4\pi\varepsilon_0 r^2}$$

$$dx = \frac{a}{\cos^2\beta} d\beta$$

$$r = \frac{a}{\cos\beta}$$

$$\vec{E}_1 = \int d\vec{E} = \int \left(dE_{1x}(-\vec{i}_x) + dE_{1y}\vec{i}_y \right)$$

$$\vec{E}_1 = \frac{Q'}{4\pi\varepsilon_0}\left[\int_{\beta=-\pi/2}^{0} \frac{\frac{\cancel{a}}{\cancel{\cos^2\beta}} \sin\beta\, d\beta}{\frac{a^{\cancel{2}}}{\cancel{\cos^2\beta}}} (-\vec{i}_x) + \int_{\beta=-\pi/2}^{0} \frac{\frac{\cancel{a}}{\cancel{\cos^2\beta}} \cos\beta\, d\beta}{\frac{a^{\cancel{2}}}{\cancel{\cos^2\beta}}} \vec{i}_y \right]$$

$$\vec{E}_1 = \frac{Q'}{4\pi\varepsilon_0 a}\left[\int_{-\pi/2}^{0} \sin\beta\, d\beta\, (-\vec{i}_x) + \int_{-\pi/2}^{0} \cos\beta\, d\beta\, \vec{i}_y \right]$$

$$\vec{E_1} = \frac{Q'}{4\pi\varepsilon_0 a}\left(-\cos\beta \Big|_0^{-\pi/2}\vec{i_x} + \sin\beta \Big|_{-\pi/2}^{0}\vec{i_y}\right)$$

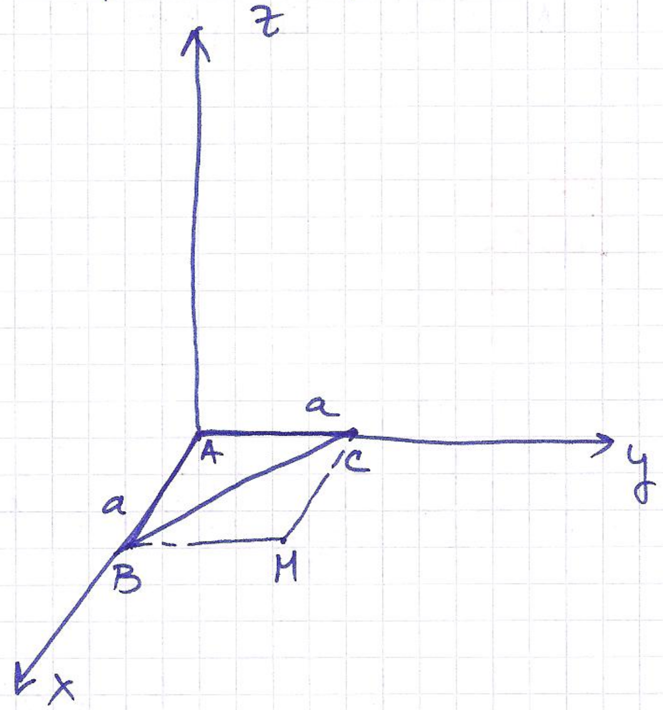
$$\vec{E_1} = \frac{Q'}{4\pi\varepsilon_0 a}\left(-\vec{i_x} + \vec{i_y}\right)$$

$$\vec{E} = 2\vec{E_1}$$ SUPERPOZICIJA

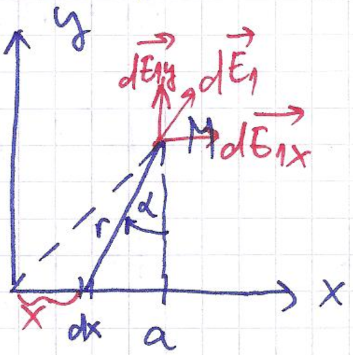
$$\underline{\vec{E} = \frac{Q'}{2\pi\varepsilon_0 a}\left(-\vec{i_x} + \vec{i_y}\right)}$$

22. TEMENA TROUGLA $A(0,0,0)$ $B(a,0,0)$ $C(0,a,0)$

Q', $\vec{E}(M(a,a,0)) = ?$



I SEGMENT



$dE_{1y} = dE_1 \cos\alpha$

$dE_{1x} = dE_1 \sin\alpha$

$x = a - a\,tg\,\alpha$

$dx = -\frac{a}{\cos^2\alpha} d\alpha$

$r = \frac{a}{\cos\alpha}$

$$E_{1y} = \int dE_{1y} = \int_0^{\pi/4} \frac{Q' \frac{a}{\cos^2\alpha}}{4\pi\varepsilon_0 \frac{a^2}{\cos^2\alpha}} \cos\alpha \, d\alpha$$

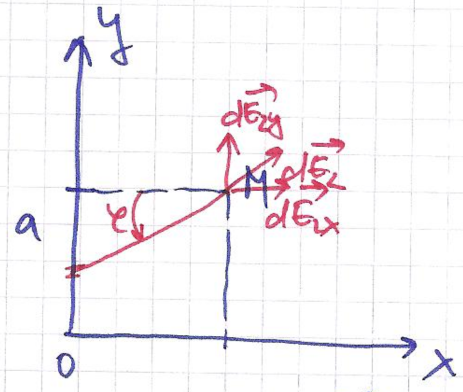
$$E_{1y} = \frac{Q'}{4\pi\varepsilon_0 a} \sin\alpha \Big|_0^{\pi/4} = \frac{Q'}{4\pi\varepsilon_0 a} \frac{\sqrt{2}}{2}$$

$$E_{1x} = -\int_{\pi/4}^{0} \frac{Q' \frac{a}{\cos^2\alpha}}{4\pi\varepsilon_0 \frac{a^2}{\cos^2\alpha}} \sin\alpha \, d\alpha = \frac{Q'}{4\pi\varepsilon_0 a} \cos\alpha \Big|_{\pi/4}^{0} = \frac{Q'}{4\pi\varepsilon_0 a}\left(1 - \frac{\sqrt{2}}{2}\right)$$

$$\vec{E}_1 = E_{1x} \vec{i_x} + E_{1y} \cdot \vec{i_y} = \frac{Q}{4\pi\varepsilon_0 a}\left(\left(1 - \frac{\sqrt{2}}{2}\right)\vec{i_x} + \frac{\sqrt{2}}{2}\vec{i_y}\right)$$

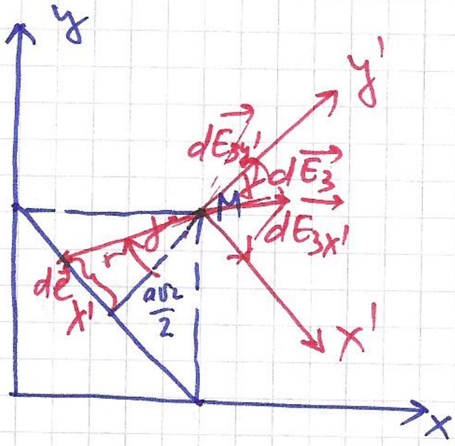
II SEGMENT

UOČAVAMO ISTI SLUČAJ KAO ZA PRVI SEGMENT
(OSE X i Y SU ZAMENILE MESTA, PA ĆE I KOMPONENTE)



PA JE IZRAZ: $\vec{E_2} = \frac{Q'}{4\pi\varepsilon_0 a}\left(\frac{\sqrt{2}}{2}\vec{i_x} + \left(1-\frac{\sqrt{2}}{2}\right)\vec{i_y}\right)$

III SEGMENT



$dE_{3y'} = dE_3 \cos\gamma$

$dE_{3x'} = dE_3 \sin\gamma$

$tg\gamma = \frac{x'}{\frac{a\sqrt{2}}{2}} = \frac{2x'}{a\sqrt{2}}$

$dx' = \frac{a\sqrt{2}}{2\cos^2\gamma}d\gamma$

$dE_{3x'} = \frac{Q'\frac{\cancel{a}\sqrt{2}}{2\cancel{\cos^2\gamma}}d\gamma}{\cancel{4}\pi\varepsilon_0\frac{2a^{\cancel{2}}}{\cancel{4\cos^2\gamma}}} = \frac{Q'\sqrt{2}\sin\gamma}{4\pi\varepsilon_0 a} \qquad r = \frac{a\sqrt{2}}{2\cos\gamma}$

$E_{3x'} = \int dE_{3x'} = \frac{Q'\sqrt{2}}{4\pi\varepsilon_0 a}\int_{-\pi/4}^{\pi/4}\sin\gamma\, d\gamma = 0$

MOŽE SE UOČITI SIMETRIJA PA POSTOJI SAMO y' KOMPONENTA

$E_{3y'} = \int dE_{3y'}$

$dE_{3y'} = \frac{Q'\sqrt{2}}{4\pi\varepsilon_0 a}\cos\gamma\, d\gamma$

$E_{3y'} = \frac{Q'\sqrt{2}}{4\pi\varepsilon_0 a}\int_{-\pi/4}^{\pi/4}\cos\gamma\, d\gamma = \frac{Q'}{2\pi\varepsilon_0 a}$

$$\vec{E_3} = E_{3y'}\vec{i_{y'}} \qquad \vec{i_x}+\vec{i_y} = \sqrt{2}\,\vec{i_{y'}}$$

$$\vec{i_{y'}} = \frac{\sqrt{2}}{2}\left(\vec{i_x}+\vec{i_y}\right)$$

$$\frac{Q'}{2\pi\varepsilon_0 a}\cdot\frac{\sqrt{2}}{2}\left(\vec{i_x}+\vec{i_y}\right) = \vec{E_3}$$

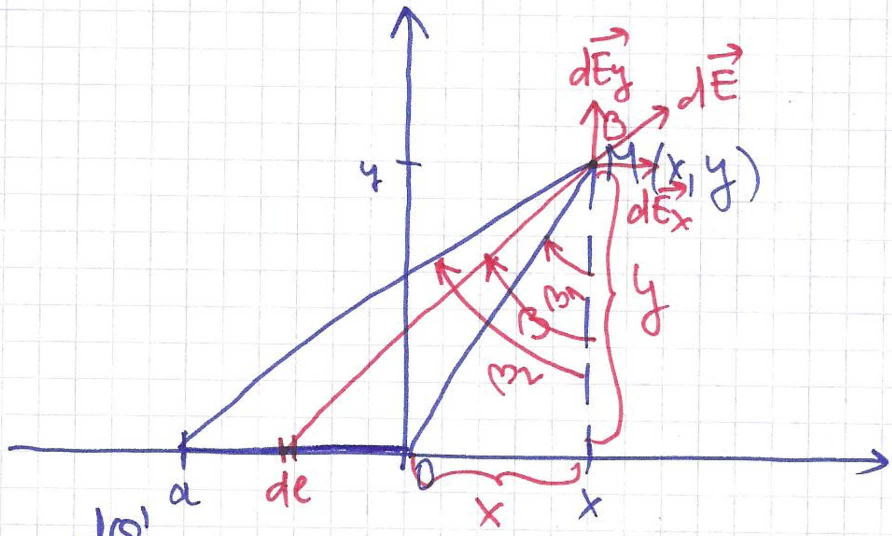
$$\underline{\vec{E_3} = \frac{Q'\sqrt{2}}{4\pi\varepsilon_0 a}\left(\vec{i_x}+\vec{i_y}\right)}$$

$$\vec{E} = \vec{E_1}+\vec{E_2}+\vec{E_3}$$

$$\underline{\underline{\vec{E} = \frac{Q'(1+\sqrt{2})}{4\pi\varepsilon_0 a}\left(\vec{i_x}+\vec{i_y}\right)}}$$

23. VRLO DUGAČKA TANKA TRAKA , a , ρ_s

a) $\vec{E} = ?$ U PROIZVOLJNOJ TAČKI



$$dE = \frac{dQ'}{2\pi\varepsilon_0 r}$$

$$dQ = \rho_s dS = \rho_s \cdot z \cdot dl$$

$$dQ = dQ' \cdot z \Rightarrow dQ' = \rho_s \cdot de$$

$$r = \frac{y}{\cos\beta} \qquad l = x + y\,tg\beta$$

$$dE_x = dE\sin\beta$$

$$dE_y = dE\cos\beta$$

$$dl = \frac{y\,d\beta}{\cos^2\beta}$$

$$dE_x = \frac{dQ'}{2\pi\varepsilon_0 r}\sin\beta \qquad dE_y = \frac{dQ'}{2\pi\varepsilon_0 r}\cos\beta$$

$$d\vec{E} = dE_x\vec{i_x} + dE_y\vec{i_y}$$

$$E_x = \int dE_x = \int_{\beta_1}^{\beta_2} \frac{\rho_s \cdot \frac{\cancel{y}\,d\beta}{\cancel{\cos^2\beta}}}{2\pi\varepsilon_0 \frac{\cancel{y}}{\cancel{\cos\beta}}} \cdot \sin\beta = \frac{\rho_s}{2\pi\varepsilon_0}\int_{\beta_1}^{\beta_2} \frac{\sin\beta}{\cos\beta}d\beta$$

$$t = \cos\beta$$

$$dt = -\sin\beta\, d\beta$$

$$E_x = \frac{\rho_s}{2\pi\varepsilon_0}\int_{\cos\beta_2}^{\cos\beta_1} \frac{dt}{t}$$ (GRANICE SE ZBOG MINUSA OBRĆU)

$$E_x = \frac{\rho_s}{2\pi\varepsilon_0} \ln|t| \Big|_{\cos\beta_2}^{\cos\beta_1} = \frac{\rho_s}{2\pi\varepsilon_0}\ln\frac{\cos\beta_1}{\cos\beta_2}$$ (POZITIVAN ~~KOSINUS~~ KOSINUS UGLOVI IZ ~~ISTOG~~ I KVADRANTA)

$$E_y=\int dE_y=\int_{\beta_1}^{\beta_2}\frac{\rho_s\frac{y\,d\beta}{\cos^2\beta}}{2\pi\varepsilon_0\frac{y}{\cos\beta}}\cdot\cos\beta=\frac{\rho_s}{2\pi\varepsilon_0}(\beta_2-\beta_1)$$

$$\vec{E}=E_x\vec{i_x}+E_y\cdot\vec{i_y}=\frac{\rho_s}{2\pi\varepsilon_0}\left[\ln\frac{\cos\beta_1}{\cos\beta_2}\cdot\vec{i_x}+(\beta_2-\beta_1)\vec{i_y}\right]$$

б) U SLUČAJU NAELEKTRISANE RAVNI

$\beta_1\to-\pi/2$ $\quad\beta_2\to\pi/2$

$\beta_1=-\beta_2$

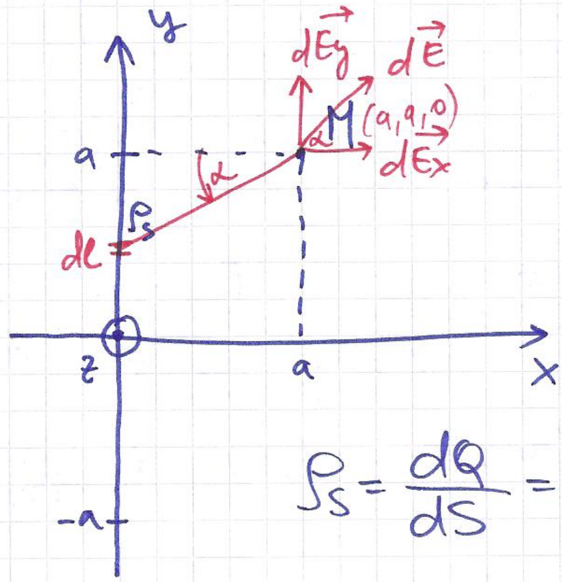
ZBOG SIMETRIJE $E_x=0$

$$E_y=\frac{\rho_s}{2\varepsilon_0}$$

$$\vec{E}=\frac{\rho_s}{2\varepsilon_0}\operatorname{sgn}y\,\vec{i_y}$$

*24 : $\rho_s(y)=\rho_{s0}\frac{y}{a}$ $\quad\rho_{s0}$ – const , $a>0$

$\vec{E}$(U TAČKI $M(a,a,0)$) = ?



$dE_x=dE\cos\alpha$

$dE_y=dE\sin\alpha$

$$\rho_s=\frac{dQ}{dS}=\frac{dQ}{l\,dy}=\frac{dQ'}{dy}$$

$$dQ'=\rho_s\,dy$$

$$dE = \frac{\rho_s dy}{2\pi\varepsilon_0 r}$$

$$\cos\alpha = \frac{a}{r}$$

$$r = \frac{a}{\cos\alpha}$$

$$E_x = \int dE_x$$

$$\sin\alpha = \frac{a-y}{r}$$

$$r = \sqrt{a^2+(a-y)^2}$$

$$E_x = \int_{-a}^{a} \frac{\rho_{s0}}{2\pi\varepsilon_0} \frac{\frac{y}{a}\cdot\frac{a}{r}dy}{\sqrt{a^2+(a-y)^2}}$$

$$E_x = \frac{\rho_{s0}}{2\pi\varepsilon_0}\int_{-a}^{a}\frac{y\,dy}{\sqrt{a^2+(a-y)^2}} = \left|\begin{array}{l} a-y=t \\ dt=-dy,\ y=a-t \end{array}\right.$$

$$E_x = \frac{\rho_{s0}}{2\pi\varepsilon_0}\int_{2a}^{0}\frac{(t-a)}{a^2+t^2}dt$$

$$E_x = \frac{\rho_{s0}}{2\pi\varepsilon_0}\int_{2a}^{0}\left(\frac{1}{2}\frac{2t\,dt}{a^2+t^2} - \frac{a\,dt}{a^2+t^2}\right)$$

$$E_x = \frac{\rho_{s0}}{2\pi\varepsilon_0}\frac{1}{2}\ln(a^2+t^2)\Big|_{2a}^{0} - a\cdot\frac{1}{a}\operatorname{arctg}\frac{t}{a}\Big|_{2a}^{0}$$

$$\boxed{E_x = \frac{\rho_{s0}}{2\pi\varepsilon_0}\left(-\ln\sqrt{5}+\operatorname{arctg}2\right)\vec{i_x}}$$

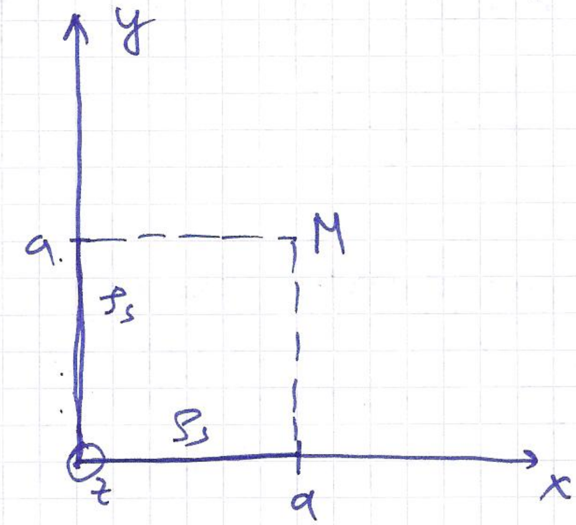
$$E_y = \int dE_y = \int_{-a}^{a}\frac{\rho_{s0}\frac{y}{a}\frac{a-y}{r}}{2\pi\varepsilon_0 r}dy = \frac{\rho_{s0}}{2\pi\varepsilon_0}\int_{-a}^{a}\frac{y(a-y)dy}{a^2+(a-y)^2} = \left|\begin{array}{l} t=a-y \\ dt=-dy \end{array}\right.\ y=a-t$$

$$E_y = \frac{\rho_{s0}}{2\pi\varepsilon_0 a}\int_{2a}^{0}\frac{(t-a)t\,dt}{a^2+t^2} = \frac{\rho_{s0}}{2\pi\varepsilon_0 a}\int_{2a}^{0}\left[\frac{t^2dt}{a^2+t^2} - \frac{at\,dt}{a^2+t^2}\right]$$

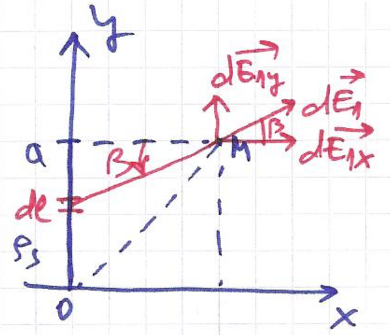
$$E_y = \frac{\rho_{s0}}{2\pi\varepsilon_0 a}\left(0-2a-a(\operatorname{arctg}0-\operatorname{arctg}2)-a\ln\sqrt{\frac{a^2}{a^2+4a^2}}\right)$$

$$\boxed{\vec{E_y} = \frac{\rho_{s0}}{2\pi\varepsilon_0 a}\left(\operatorname{arctg}2+\ln\sqrt{5}-2\right)\vec{i_y}}$$

25. VEOMA DUGAČAK DIEDAR, ρ_s



I SEGMENT



$$dQ = \rho_s dS = \rho_s l\, dl$$

$$dQ' = \rho_s dl$$

$$r = \frac{a}{\cos\beta}$$

$$y = a - a\,tg\beta$$

$$dy = -\frac{a}{\cos^2\beta} d\beta = dl$$

$$dE_{1x} = dE_1 \cos\beta$$

$$dE_{1y} = dE_1 \sin\beta$$

$$E_{1x} = \int dE_{1x} = -\int_{\pi/4}^{0} \frac{\rho_s \frac{a}{\cos^2\beta} d\beta}{2\pi\varepsilon_0 \frac{a}{\cos\beta}} \cos\beta = \frac{\rho_s}{2\pi\varepsilon_0} \pi/4$$

$$E_{1y} = -\int_{\pi/4}^{0} \frac{\rho_s \frac{a}{\cos^2\beta} d\beta}{2\pi\varepsilon_0 \frac{a}{\cos\beta}} \sin\beta = \frac{\rho_s}{2\pi\varepsilon_0} \int_0^{\pi/4} tg\beta\, d\beta$$

$$E_{1y} = \frac{\rho_s}{2\pi\varepsilon_0} \frac{1}{2} \ln 2\, \vec{i_y}$$

$$\vec{E_1} = E_{1x} \cdot \vec{i_x} + E_{1y} \cdot \vec{i_y} = \frac{\rho_s}{2\pi\varepsilon_0} \left(\pi/4 \cdot \vec{i_x} + \frac{1}{2} \ln 2\, \vec{i_y} \right)$$

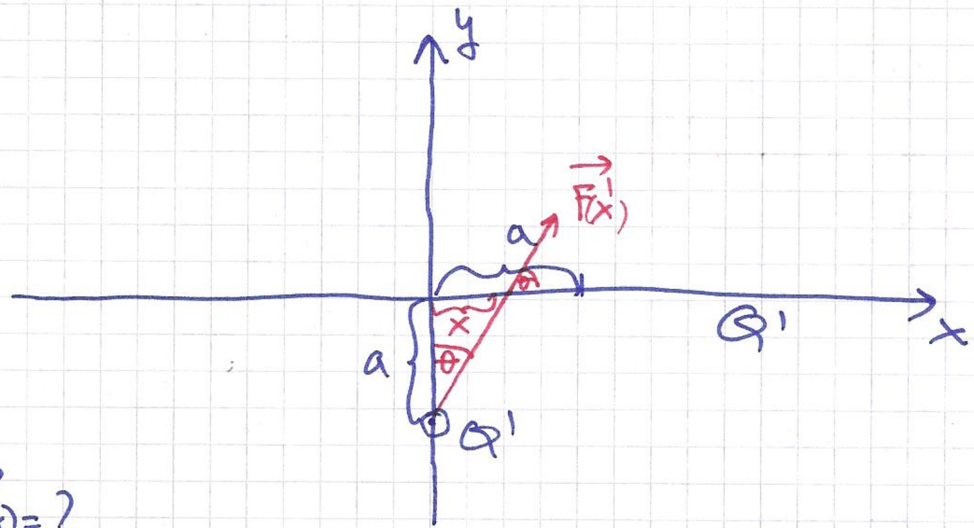
ZA DRUGI SEGMENT SE UOČAVA DA SU KOMPONENTE (OSE) X I Y ZAMENILE MESTA PA JE IZRAZ:

$$\vec{E_2} = \frac{\rho_{s0}}{2\pi\varepsilon_0}\left(\frac{1}{2}\ln 2\,\vec{i_x} + \pi/4\cdot\vec{i_y}\right)$$

PA JE POLJE REZULTANTNO (VEKTOR):

$$\boxed{\vec{E} = \vec{E_1} + \vec{E_2} = \frac{\rho_{s0}}{2\pi\varepsilon_0}\left(\frac{1}{2}\ln 2 + \pi/4\right)(\vec{i_x} + \vec{i_y})}$$

26. DVA PRAVOLINIJSKA NAELEKTRISANJA Q'



a) $\vec{F'}(x) = ?$

$$\vec{F'}(x) = Q'\cdot\vec{E(x)}$$

$$\vec{E}(x) = \vec{E_x}(x) + \vec{E_y}(x)$$

$$\vec{E}(x) = E_x(x)\cdot\vec{i_x} + E_y(x)\cdot\vec{i_y}$$

$$E_y = E\cos\theta$$
$$E_x = E\sin\theta$$

$$E = \frac{Q'}{2\pi\varepsilon_0 r}\ ;\ r = \sqrt{a^2+x^2}$$

$$\cos\theta = \frac{a}{r}\quad \sin\theta = \frac{x}{r}$$

$$E_y = \frac{Q'}{2\pi\varepsilon_0}\,\frac{a}{x^2+a^2}\qquad E_x = \frac{Q'}{2\pi\varepsilon_0}\,\frac{x}{x^2+a^2}$$

$$\boxed{\vec{F'}(x) = \frac{Q'^2}{2\pi\varepsilon_0}\,\frac{x\cdot\vec{i_x} + a\cdot\vec{i_y}}{x^2+a^2}}$$

b) $F'_{MAX} = ?$

$$F'_{MAX} = F'(x=0) = \frac{Q'^2}{2\pi\varepsilon_0 a}$$

v) $\vec{F}$ NA SEGMENTU $[0, a]$

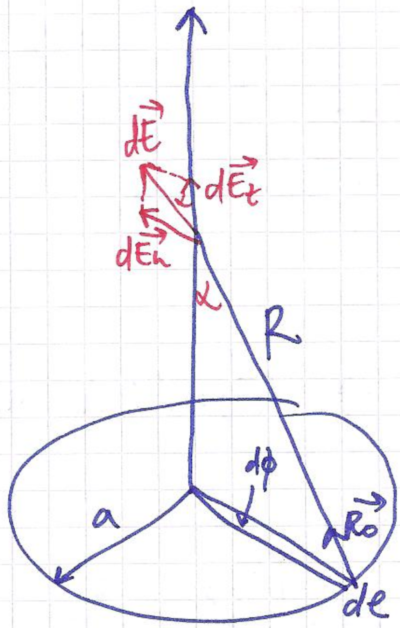
$$\vec{F} = \int_0^a \vec{F}' dx = \frac{Q'^2}{2\pi\varepsilon_0}\left(\frac{1}{2}\cdot\vec{i}_x \ln 2 + \vec{i}_y \frac{\pi}{4}\right)$$

27. KRUŽNA KONTURA, a, $Q'>0$

a) $\vec{E}(z) = ?$

b) $|E_z|_{MAX} = ?$

a)



$dl = a\,d\phi$

$\cos\alpha = \frac{z}{R} = \frac{z}{\sqrt{a^2+z^2}}$

$R = \sqrt{a^2+z^2}$

UOČAVAMO SIMETRIJU

PA JE $\oint_C dE_n = 0$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q'dl}{R^2}\cdot\vec{R}_0$$

VEKTOR $\vec{E}$ IMA SAMO z-KOMPONENTU

$$\vec{E} = \int d\vec{E}_z = E_z\cdot\vec{i}_z$$

$$dE_z = dE\cos\alpha = dE\,\frac{z}{R}$$

$$E_z = \frac{Q'a\cdot z}{4\pi\varepsilon_0 R^3}\int_0^{2\pi} d\phi = \frac{Q'az}{2\varepsilon_0 R^3}$$

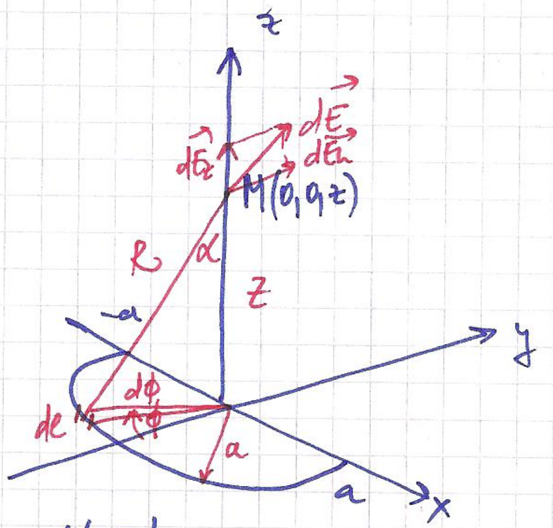
$$\vec{E}(z)=\frac{Q' a z}{2\varepsilon_0 (a^2+z^2)^{\frac{3}{2}}}\cdot\vec{i_z}$$

b)

$$\frac{dE_z}{dz}=0 \Rightarrow z=\pm\frac{a\sqrt{2}}{2}$$

$$\boxed{E_{MAX}=\frac{Q'\sqrt{3}}{9\varepsilon_0 a}}$$

28.



$dl=a\,d\phi$

$R=\sqrt{a^2+z^2}$

$\vec{E}=\int d\vec{E}$

$dE_z=dE\cos\alpha$

$dE_h=dE\sin\alpha$

$\sin\alpha=\frac{a}{R}=\frac{a}{\sqrt{a^2+z^2}}$

$\cos\alpha=\frac{z}{R}=\frac{z}{\sqrt{a^2+z^2}}$

$$dE=\frac{dQ}{4\pi\varepsilon_0 R^2}=\frac{a\,d\phi\,Q'}{4\pi\varepsilon_0 R^2}$$

$dE_y=dE_h\cos\phi \qquad \Rightarrow dE_y=dE\sin\alpha\cos\phi$

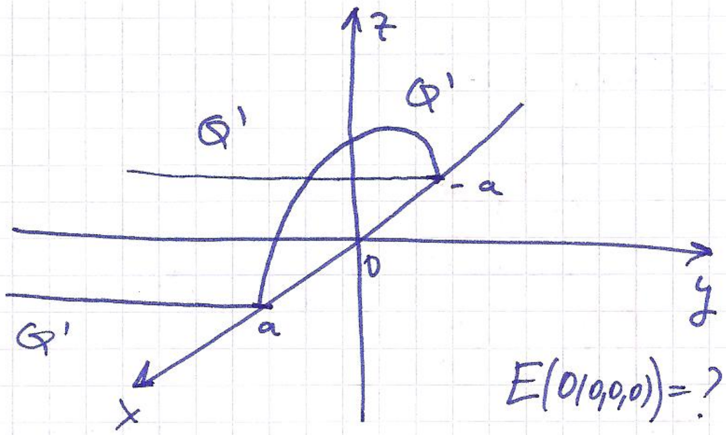
$dE_x=dE_h\sin\phi$

ZBOG SIMETRIJE $E_x=0$

$$\vec{E}=\frac{Q' a}{4\pi\varepsilon_0(a^2+z^2)}\left[\frac{a}{\sqrt{a^2+z^2}}\sin\phi\Big|_{-\pi/2}^{\pi/2}\vec{i_y}+\frac{z}{\sqrt{a^2+z^2}}\,\phi\Big|_{-\pi/2}^{\pi/2}\vec{i_z}\right]$$

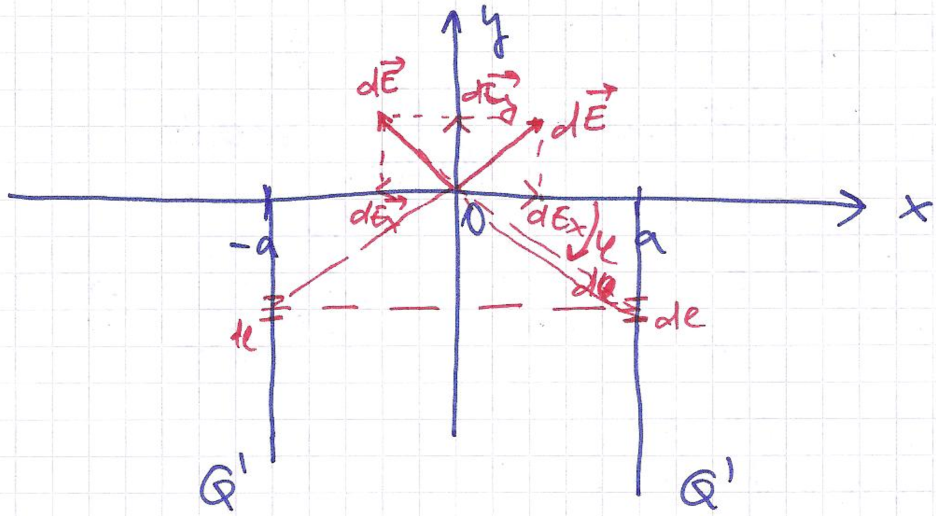
$$\vec{E}=\frac{Q' a}{4\pi\varepsilon_0(a^2+z^2)}\left[\frac{2a}{\sqrt{a^2+z^2}}\vec{i_y}+\frac{z}{\sqrt{a^2+z^2}}\cdot\pi\cdot\vec{i_z}\right]$$

29.



$E(0,0,0) = ?$

I SEGMENT (PRAVOLINIJSKI)



X-KOMPONENTE SE ZBOG SIMETRIJE PONIŠTAVAJU

$r = \frac{a}{\cos\varphi}$

$dQ = Q'dl = Q'dy$

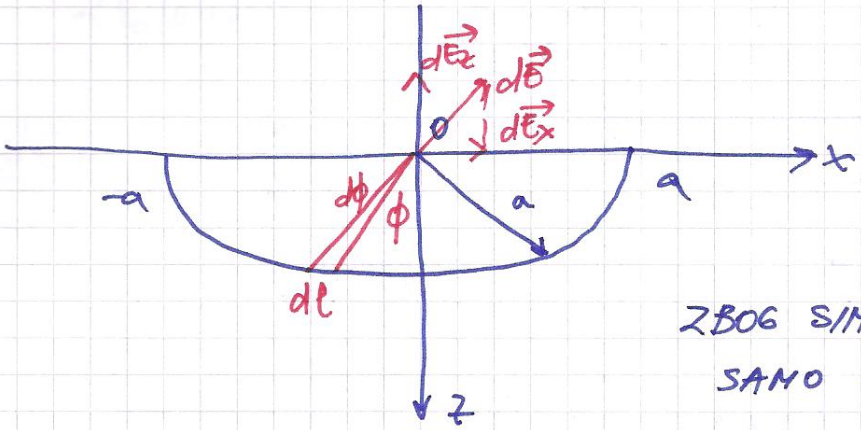
$y = a\,tg\varphi \Rightarrow dy = \frac{a}{\cos^2\varphi}d\varphi$

POSTOJI SAMO Y-KOMPONENTA

$$E_{1y} = 2\int dE_y = 2\frac{Q'}{4\pi\varepsilon_0 a}\int_{-\pi/2}^{0}\frac{\frac{d\varphi}{\cancel{\cos^2\varphi}}}{\frac{1}{\cancel{\cos^2\varphi}}}\sin\varphi = \frac{Q'}{2\pi\varepsilon_0 a}$$

$$\vec{E_{1y}} = \frac{Q'}{2\pi\varepsilon_0 a}\vec{i_y}$$

II SEGMENT (KRIVOLINIJSKI)



ZBOG SIMETRIJE
SAMO z-KOMPONENTA

$$dE_z = dE\cos\phi \qquad dl = a\,d\phi$$

$$E_z = \int dE_z = \int dE\cos\phi = \frac{Q'\,a}{4\pi\varepsilon_0 a^2}\int_{-\pi/2}^{\pi/2}\cos\phi\,d\phi$$

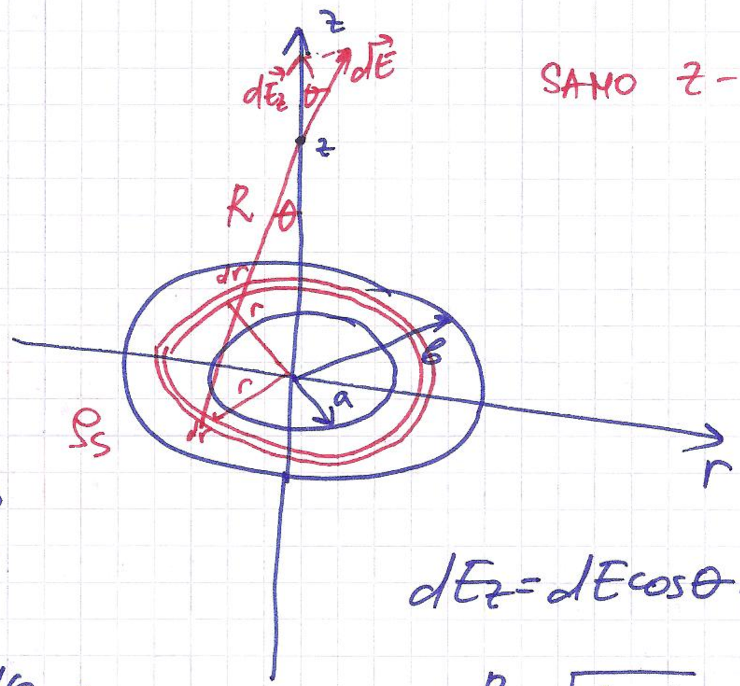
$$E_z = \frac{Q'}{4\pi\varepsilon_0 a}\sin\phi\Big|_{-\pi/2}^{\pi/2} = \frac{Q'}{2\pi\varepsilon_0 a} \qquad \vec{E}_z = \frac{Q'}{2\pi\varepsilon_0 a}(-\vec{i}_z)$$

$$\vec{E} = \frac{Q'}{2\pi\varepsilon_0 a}\left(\vec{i}_y - \vec{i}_z\right)$$

30. KRUŽNI PRSTEN a, b $(b>a)$, ρ_s

$\vec{E}(z) = ?$

SAMO z - KOMPONENTA



$$\vec{E} = \int d\vec{E_z}$$

$$dE_z = dE\cos\theta = dE\frac{z}{R}$$

$$R = \sqrt{r^2+z^2}$$

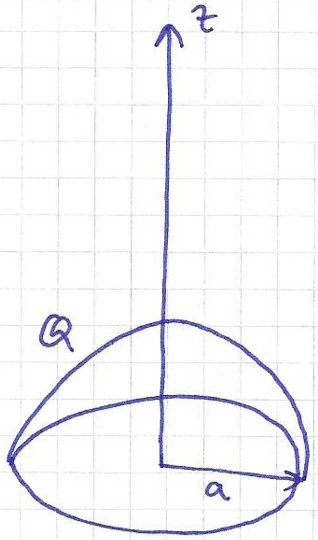
$$dE_z = \frac{dQ\cdot z}{4\pi\varepsilon_0 R^3}$$

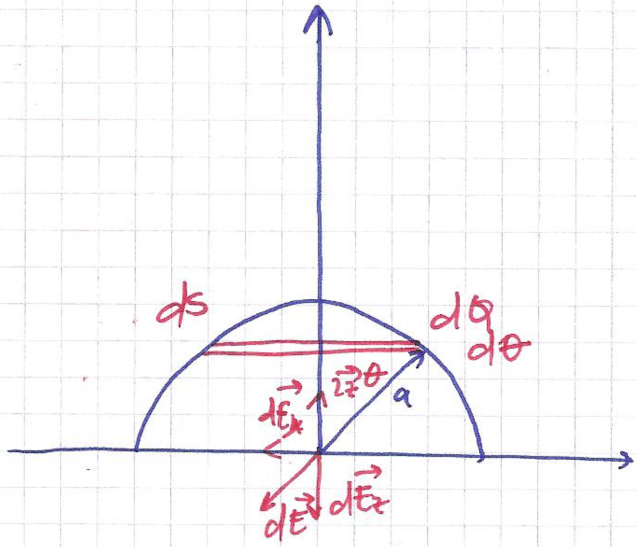
$$dE_z = \frac{\rho_s dS\cdot z}{4\pi\varepsilon_0 R^3} = \frac{\rho_s \cancel{2\pi} r dr z}{2\cancel{4\pi}\varepsilon_0 R^3} = \frac{\rho_s r z dr}{2\varepsilon_0 R^3} = \frac{\rho_s r z dr}{2\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$\vec{E} = \frac{\rho_s z}{2\varepsilon_0}\vec{a}_z \int_{r=a}^{b} \frac{r dr}{(r^2+z^2)^{3/2}} = \left\{ \begin{aligned} z^2+r^2 &= t \\ 2rdr &= dt \end{aligned} \right.$$

$$\Rightarrow \vec{E} = \frac{\rho_s z}{2\varepsilon_0}\left(\frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}}\right)\vec{a}_z$$

(32) POLUSFERNA POVRŠ





ZBOG SIMETRIJE – SAMO Z KOMPONENTA

$$dE_z = dE\cos\theta \qquad \cos\theta = \frac{z}{R}$$

$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2} \qquad dQ = \rho_S dS$$

$$dS = 2\pi a^2 \sin\theta d\theta \qquad R = a$$

$$\Downarrow$$

$$dQ = Q\sin\theta d\theta$$

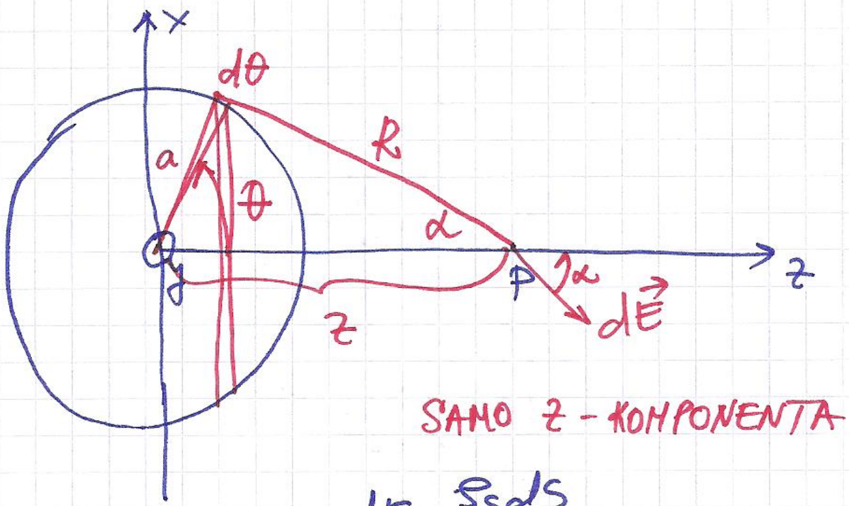
$$E_z = \int_0^{\pi/2} \frac{Q\sin\theta\cos\theta d\theta}{4\pi\varepsilon_0 a^2}$$

$$E_z = \frac{Q}{4\pi\varepsilon_0 a^2} \int_0^{\pi/2} \sin\theta\cos\theta d\theta$$

$$E_z = \frac{Q}{4\pi\varepsilon_0 a^2} \left. \frac{\sin^2\theta}{2} \right|_0^{\pi/2} = \frac{Q}{8\pi\varepsilon_0 a^2}$$

$$\underline{\vec{E}_z = E_z(-\vec{i}_z) = -\frac{Q}{8\pi\varepsilon_0 a^2}\vec{i}_z}$$

33. USAMLJENA SFERA, a



SAMO z - KOMPONENTA

$$dE=\frac{\rho_s dS}{4\pi\varepsilon_0 R^2}$$

$$dE_z=dE\cos\alpha$$

$$dS=ad\theta 2\pi a\sin\theta \quad , \cos\alpha=\frac{z-a\cos\theta}{R}$$

$R^2=z^2+a^2-2az\cos\theta$ - KOSINUSNA TEOREMA

$$dE_z=\frac{\rho_s a}{2\varepsilon_0}\frac{a\sin\theta d\theta}{R^2}\cdot\frac{z-a\cos\theta}{R}$$

$$\cos\theta=\frac{z^2+a^2-R^2}{2az} \qquad \frac{dR}{d\theta}=\frac{az\sin\theta}{R}\Rightarrow\sin\theta d\theta=\frac{RdR}{az}$$

$$dE_z=\frac{\rho_s a}{4\varepsilon_0 z^2}\left(1+\frac{z^2-a^2}{R^2}\right)dR$$

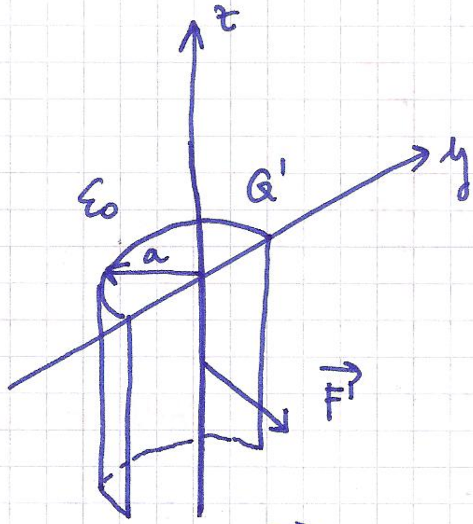
GRANICE ZA R OD $|z-a|$ DO $|z+a|$ $(\theta\in[0,\pi])$

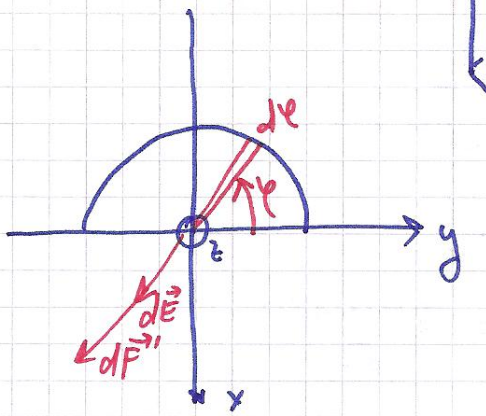
$$E_z=\frac{\rho_s a}{4\varepsilon_0 z^2}\int_{|z-a|}^{|z+a|}\left(1+\frac{z^2+a^2}{R^2}\right)dR=\frac{\rho_s a}{4\varepsilon_0 z^2}\left(|z+a|-|z-a|-(z^2-a^2)\left[\frac{1}{|z+a|}-\frac{1}{|z-a|}\right]\right.$$

$$\vec{E_z}=\vec{E}=\frac{\rho_s a}{4\varepsilon_0 z^2}\left[|z+a|-|z-a|-(z^2-a^2)\left(\frac{1}{|z+a|}-\frac{1}{|z-a|}\right)\right]\cdot\vec{i_z}$$

(34) DUGAČKA TRAKA I PRAVOLINIJSKI PROVODNIK

Q', $\vec{F'} = ?$





$$\vec{F'} = \int d\vec{F'}$$

$$\vec{F'} = \int dQ' d\vec{E}$$

$$F' = \int Q' dE \cos\varphi$$

$$\frac{dQ'}{Q'} = \frac{d\varphi}{\pi}$$

$$\Downarrow$$

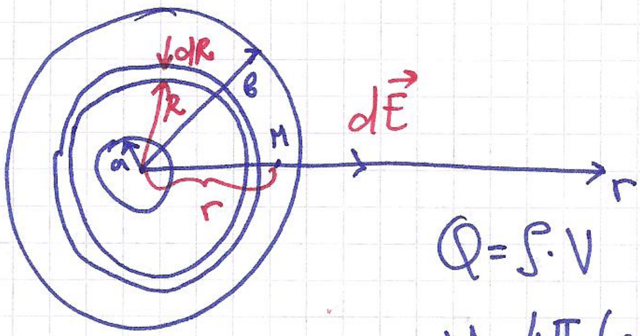
$$dQ' = \frac{Q'}{\pi} d\varphi$$

$$\vec{F'} = \int Q' \frac{dQ'}{2\pi a \varepsilon_0} \cos\varphi =$$

$$= \frac{Q'^2}{2\pi^2 a \varepsilon_0} \int_{-\pi/2}^{\pi/2} \cos\varphi \, d\varphi = \frac{Q'^2}{\pi^2 a \varepsilon_0} \Rightarrow \vec{F'} = \frac{Q'^2}{\cancel{2}\pi^2 \varepsilon_0 a} \vec{i_x}$$

$$\vec{F'} = \frac{Q'^2}{\pi^2 \varepsilon_0 a} \cdot \vec{i_x}$$

36. ρ, $\vec{E}(r) = ?$



$Q = \rho \cdot V$

$$\boxed{V = \frac{4\pi}{3}(b^3 - a^3)}$$

$dQ = 4\pi R^2 dR \cdot \rho$

$$dE(r) = \begin{cases} 0, & r < R \\ \dfrac{dQ}{4\pi\varepsilon_0 r^2}, & r > R \end{cases}$$

I)

$r \le a \quad E = 0$

II) $a < r \le b$

$$\vec{E}(r) = \int_a^b \frac{4\pi R^2 \rho\, dR}{4\pi\varepsilon_0 r^2}\vec{r_0} = \frac{\rho}{3\varepsilon_0}\left(r - \frac{a^3}{r^3}\right)\vec{r_0}$$

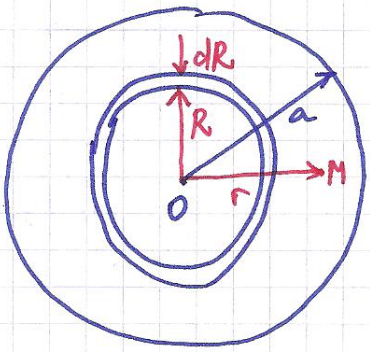
III) $r > b$

$$\boxed{\vec{E}(r) = \int_a^b \frac{4\pi R^2 \rho\, dR}{4\pi\varepsilon_0 r^2}\vec{r_0} = \frac{\rho}{3\varepsilon_0 r^2}(b^3 - a^3)\vec{r_0}}$$

37. LOPTA, a, $\rho(r) = \rho(a)\frac{r}{a}$ $\quad 0 \le r \le a$

$E(r) = ?$

$0 \le r \le a$



$dQ = \rho(R)\,dV$

$dQ = \rho(a)\frac{R}{a} 4\pi R^2 dR$

$dQ = \frac{\rho(a)}{a} 4\pi R^3 dR$

$r < R \quad dE = 0$

$r > R, \ dE = \frac{dQ}{4\pi\varepsilon_0 r^2}$

$$E = \int dE = \int_{R=0}^{r} \frac{\rho(a) R^3 \cancel{4\pi} dR}{\cancel{4\pi}\varepsilon_0 r^2 a} = \frac{\rho(a)}{\varepsilon_0 r^2 a}\int_0^r R^3 dR$$

$$E(r) = \frac{\rho(a)}{\varepsilon_0 r^2 a} \frac{R^4}{4}\Big|_0^r = \frac{\rho(a)}{4\varepsilon_0 \cancel{r^2} a}\frac{r^{\cancel{4}\,2}}{4} = \frac{\rho(a)}{4\varepsilon_0 a} r^2$$

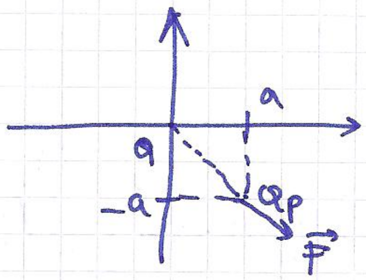
$$\vec{E(r)} = \frac{\rho(a) r^2}{4\varepsilon_0 a}\vec{i_r}$$

ZA SLUČAJ $r > a$, INTEGRAL IDE U GRANICAMA OD 0 DO a PA JE IZRAZ ZA E:

$$\boxed{\vec{E}(r>a) = \frac{\rho(a) a^3}{4\varepsilon_0 r^2}}$$

(38.) $Q, Q_p, M(a,0,-a) \quad a>0 \quad A=?$

$A=\int \vec{F} d\vec{l}$



$r=a\sqrt{2}$

$V=\dfrac{Q}{4\pi\varepsilon_0 r}$

$A=\int Q_p \vec{E} d\vec{l}$

$A=Q_p\int \vec{E} d\vec{l} = Q_p \cdot V$

$$A=\frac{Q_p\cdot Q}{4\pi\varepsilon_0 r}=\frac{Q_p\cdot Q}{4\pi\varepsilon_0 a\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\boxed{\frac{Q_p Q\sqrt{2}}{8\pi\varepsilon_0 a}}$$

(39.)

$Q=1\mu C \quad A=? \quad m=1\mu g \quad Q_p=1pC$

a) $A=-Q_p V_1=-\dfrac{Q\cdot Q_p}{4\pi\varepsilon_0 r_1}$

b) $A=\dfrac{Q\cdot Q_p}{4\pi\varepsilon_0}\left(\dfrac{1}{r_1}-\dfrac{1}{r_2}\right)=Q_p(V_1-V_2)=Q_p\cdot U_{12}$

(40.)

U PRVOJ TAČKI $\quad W_{p1}=\dfrac{Q\cdot Q_p}{4\pi\varepsilon_0 r_1}$

U DRUGOJ TAČKI $\quad W_{p2}=\dfrac{Q\,Q_p}{4\pi\varepsilon_0 r_2} \qquad W_{k2}=\dfrac{m v_2^2}{2}$

$W_{p1}=W_{p2}+W_{k2}$ ZAKON ODRŽANJA ENERGIJE

44. $\vec{A} = A_m \sin\left(\frac{\pi x}{d}\right)\vec{i_y} \quad x \in [0,d] \quad A_m, d = const$

$$\oint \vec{E}\, d\vec{\ell} = 0$$

$$\oint_{\ell} \vec{A}\, d\vec{\ell} = \int_1^2 \vec{A}\cdot d\vec{\ell} + \int_2^3 \vec{A}\cdot d\vec{\ell} + \int_3^4 \vec{A}\cdot d\vec{\ell} + \int_4^1 \vec{A}\cdot d\vec{\ell} = 0$$

$$A_m \sin\left(\frac{\pi x^2}{d}\right)\ell_{23} - A_m \sin\left(\frac{\pi x_1}{d}\right)\ell_{14} \neq 0$$

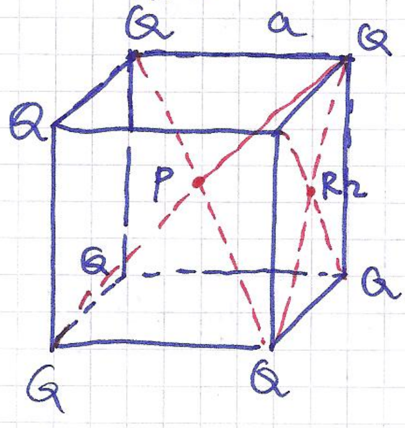
46. $V(r) = P\left(r^2 + \frac{Q}{r} + S\right), \; r \in [r_1, r_2] \quad \vec{E} = ?$

$$\vec{E(r)} = -\frac{dV}{dr}\vec{i_r} = -P\left(2r - \frac{Q}{r^2}\right)\vec{i_r} \quad, r \in [r_1, r_2]$$

47. $V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{Q_3}{r_3}$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{a} + \frac{Q}{a} - \frac{2Q}{a}\right) = 0$$

48.



a) $V_p = 8 \cdot \frac{Q}{4\pi\varepsilon_0 R} \quad | \; R = \frac{a\sqrt{3}}{2}$

$$V_p = \cancel{8}^{4} \cdot \frac{Q}{\cancel{4}\pi\varepsilon_0 \frac{a\sqrt{3}}{\cancel{2}}} = \frac{4}{3}\frac{Q\sqrt{3}}{\pi\varepsilon_0 a}$$

б) REFERENTNA TAČKA U R_2 (RASTOJANJE $\frac{a}{2}$)

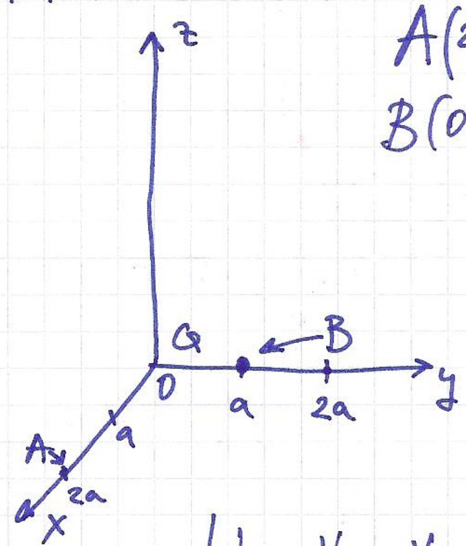
$$V_P = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_P} - \frac{1}{r_R}\right)$$

$$V_{P|R_2} = \frac{Q}{4\pi\varepsilon_0}\left(4\left(\frac{1}{\frac{a\sqrt{3}}{2}} - \frac{1}{\frac{a\sqrt{6}}{2}}\right) + 4\left(\frac{1}{\frac{a\sqrt{3}}{2}} - \frac{1}{\frac{a\sqrt{2}}{2}}\right)\right)$$

49. $Q\ (0,0,0)$ $\quad U_{AB} = ?$

$A(2a,0,0)$

$B(0,a,0)$



$U_{AB} = V_A - V_B$

$$U_{AB} = \frac{Q}{4\pi\varepsilon_0 r_A} - \frac{Q}{4\pi\varepsilon_0 r_B}$$

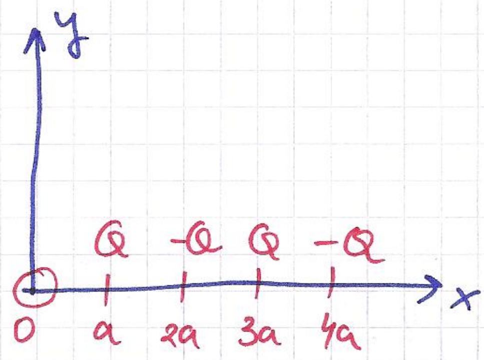
$$U_{AB} = \frac{Q}{4\pi\varepsilon_0 2a} - \frac{Q}{4\pi\varepsilon_0 a} = \frac{Q}{8\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 a}$$

$$\boxed{U_{AB} = -\frac{Q}{8\pi\varepsilon_0 a}}$$

*50

a) $V = ?$

b) $E = ?$



a) $$V = \frac{Q}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots\right) = \frac{Q}{4\pi\varepsilon_0 a}\sum_{k=1}^{+\infty}\frac{(-1)^{k+1}}{k}$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 a}\ln 2}$$

b) SAMO X KOMPONENTA

$$E = -\frac{dV}{dx}, \quad E = -\frac{Q}{4\pi\varepsilon_0 a^2}\left(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\right)$$

$$E = -\frac{Q}{4\pi\varepsilon_0 a^2}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2} = \frac{-\pi Q}{48\varepsilon_0 a^2}$$

51. $Q'(-a,0,0)$ $a>0$ $(0,0,0)$

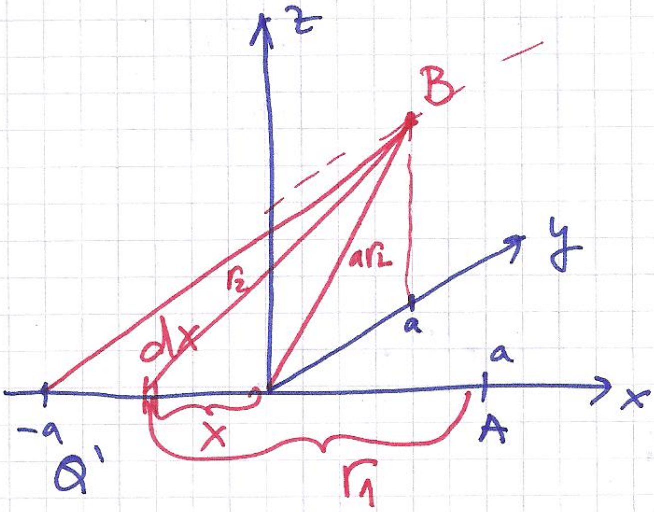
$A(a,0,0)$

$B(0,a,a)$

a) $V_A = ?$

b) $V_B = ?$

c) $U_{AB} = ?$



a) $$dV_A = \frac{1}{4\pi\varepsilon_0}\frac{Q'dx}{a+|x|} = \frac{Q'dx}{4\pi\varepsilon_0(a-x)}$$

$$V_A = \int_{x=-a}^{0} dV_A = \frac{Q'}{4\pi\varepsilon_0}\int_{-a}^{0}\frac{dx}{a-x} = \frac{Q'}{4\pi\varepsilon_0}\left(-\ln(a-x)\right)\Big|_{-a}^{0}$$

$$V_A = \frac{Q'}{4\pi\varepsilon_0}\ln\frac{2a}{a} = \boxed{\frac{Q'}{4\pi\varepsilon_0}\ln 2}$$

b) $$dV_B = \frac{Q'}{4\pi\varepsilon_0}\frac{dx}{\sqrt{2a^2+x^2}}$$

$$V_B = \frac{Q'}{4\pi\varepsilon_0}\int_{x=-a}^{0}\frac{dx}{\sqrt{a^2+x^2}} = \boxed{\frac{Q'}{4\pi\varepsilon_0}\ln\frac{\sqrt{2}}{\sqrt{3}-1}}$$

c) $$U_{AB} = V_A - V_B = \boxed{\frac{Q'}{4\pi\varepsilon_0}\ln(\sqrt{6}-\sqrt{2})}$$

(*52)

$$V=\frac{1}{4\pi\varepsilon_0}\int_{-a}^{0}\frac{Q'dx}{\sqrt{(x-x_0)^2+y_0^2+z_0^2}}=\frac{Q'}{4\pi\varepsilon_0}\ln\left(x-x_0+\sqrt{(x-x_0)^2+y_0^2+z_0^2}\right)\Big|_{-a}^{0}$$

$$V=\frac{Q'}{4\pi\varepsilon_0}\ln\frac{-x_0+\sqrt{x_0^2+y_0^2+z_0^2}}{-a-x_0\ \sqrt{(a+x_0)^2+y_0^2+z_0^2}}$$

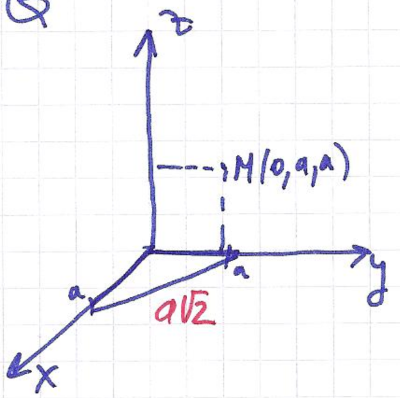
b) $E_x=-\frac{dV}{dx}=\dots$

$E_y=-\frac{dV}{dy}=\dots$

$E_z=-\frac{dV}{dz}=\dots$

$$\vec{E}=E_x\vec{i_x}+E_y\vec{i_y}+E_z\vec{i_z}$$

(53.) Q'



$V=\int dV$

$dV=\frac{Q'dl}{4\pi\varepsilon_0 r}$

$V_M=V_1+V_2+V_3$

$V(b,c)=\int\frac{dQ}{4\pi\varepsilon_0 r}=\int\frac{Q'dx}{4\pi\varepsilon_0 r}$

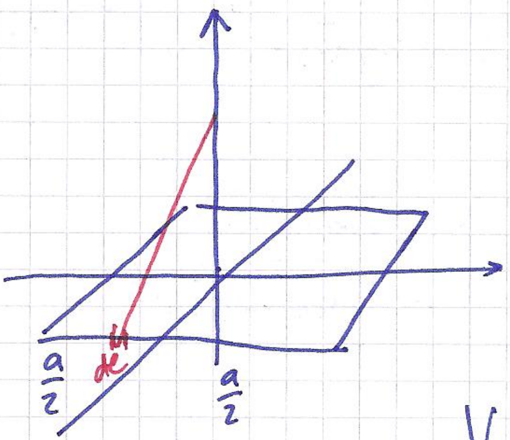
$V(b,c)=\frac{Q'}{4\pi\varepsilon_0}\int_0^b\frac{dx}{\sqrt{c^2+x^2}}$

$V_M=V(a,a)+V(a\sqrt{2},a)+V(a,a\sqrt{2})$

$=\frac{Q'}{4\pi\varepsilon_0}\left(\ln\frac{a+\sqrt{a^2+a^2}}{a}+\ln(\sqrt{2}+\sqrt{3})+\ln\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)\right.$

$$V_M=\frac{Q'}{4\pi\varepsilon_0}\ln\frac{(1+\sqrt{2})(1+\sqrt{3})(\sqrt{2}+\sqrt{3})}{\sqrt{2}}$$

54. KVADRATNA KONTURA a, Q' $V(z) = ?$



SVAKA STRANICA DAJE ISTI DOPRINOS

$$V = 4V_1$$

$$V_1 = \int \frac{dQ}{4\pi\varepsilon_0 r} = \frac{Q'}{4\pi\varepsilon_0} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{dx}{\sqrt{z^2 + \frac{a^2}{4} + x^2}}$$

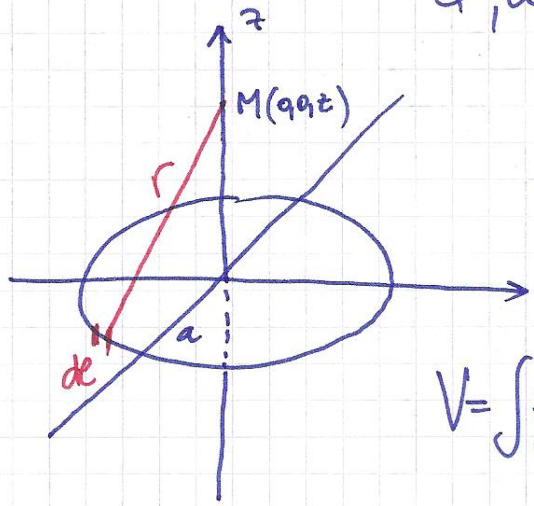
$$V_1 = \frac{Q'}{4\pi\varepsilon_0} \ln \frac{\frac{a}{2} + \sqrt{z^2 + 2\left(\frac{a}{2}\right)^2}}{-\frac{a}{2} + \sqrt{z^2 + 2\left(\frac{a}{2}\right)^2}}$$

$$V = 4V_1 = \frac{Q'}{\pi\varepsilon_0} \ln \frac{\frac{a}{2} + \sqrt{z^2 + 2\left(\frac{a}{2}\right)^2}}{-\frac{a}{2} + \sqrt{z^2 + 2\left(\frac{a}{2}\right)^2}}$$

b) $$E_z = -\frac{dV}{dz} = \frac{4Q' a z \sqrt{2}}{\pi\varepsilon_0 (4z^2 + a^2)\sqrt{2z^2 + a^2}}$$

(55.)

$Q', a \quad V(z) = ?$



$V = \int dV$

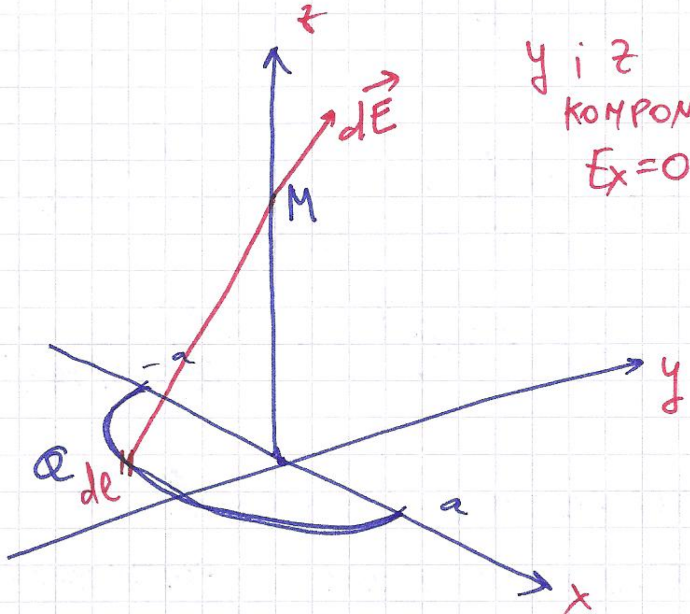
$r = \sqrt{a^2+z^2}$

$$V = \int \frac{dQ}{4\pi\varepsilon_0 r} = \int \frac{Q' dl}{4\pi\varepsilon_0 r}$$

$$V = \frac{Q'}{4\pi\varepsilon_0\sqrt{a^2+z^2}} \int dl = \frac{Q' \cancel{2\pi} a}{2 \cancel{4\pi} \varepsilon_0 \sqrt{a^2+z^2}} = \frac{Q' a}{2\varepsilon_0\sqrt{a^2+z^2}}$$

$$E_z = -\frac{dV}{dz} = \frac{Q' a z}{2\varepsilon_0 (a^2+z^2)^{3/2}}$$

(56.)



y i z
KOMPONENTE
$E_x = 0$

$V = \int dV$

$$V = \int \vec{E} \cdot d\vec{l} \qquad V = \int_{z=z_0}^{+\infty} E_z dz = \int_{z=z_0}^{\infty} \frac{Q z dz}{4\pi\varepsilon_0 (z^2+a^2)^{3/2}}$$

$\vec{i}_y \cdot \vec{i}_z = 0$

$$\vec{E} \cdot d\vec{l} = (\cancel{E_y \vec{i}_y} + E_z \vec{i}_z) \vec{i}_z dz$$

$$V = \frac{Q}{4\pi\varepsilon_0} \int_{z_0}^{\infty} \frac{z dz}{\sqrt{(z^2+a^2)^3}} = \begin{cases} z^2+a^2 = t \\ dt = 2z dz \end{cases}$$

$$V=\frac{Q}{4\pi\varepsilon_0}\,\frac{1}{2}\int_{z_0^2+a^2}^{\infty}\frac{dt}{t^{3/2}}=\frac{Q}{4\pi\varepsilon_0}\,\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\frac{1}{\sqrt{t}}\Bigg|_{z_0^2+a^2}^{\infty}$$

$$\boxed{V=\frac{Q}{4\pi\varepsilon_0}\,\frac{1}{\sqrt{z_0^2+a^2}}}$$

57.

$$V=\frac{1}{4\pi\varepsilon_0}\int_L\frac{\frac{Q}{\pi a}\,dl}{\sqrt{z^2+a^2}}=\frac{1}{4\pi\varepsilon_0}\,\frac{\frac{Q}{\pi a}}{\sqrt{z^2+a^2}}\int_L dl$$

$\frac{Q}{\pi a}\rightarrow Q'$

$$\underline{\underline{L=\pi a}}$$

$$\Downarrow$$

$$V=\frac{1}{4\pi\varepsilon_0}\int_L\frac{Q'dl}{r}\qquad\qquad V=\frac{Q}{4\pi\varepsilon_0\sqrt{z^2+a^2}}$$

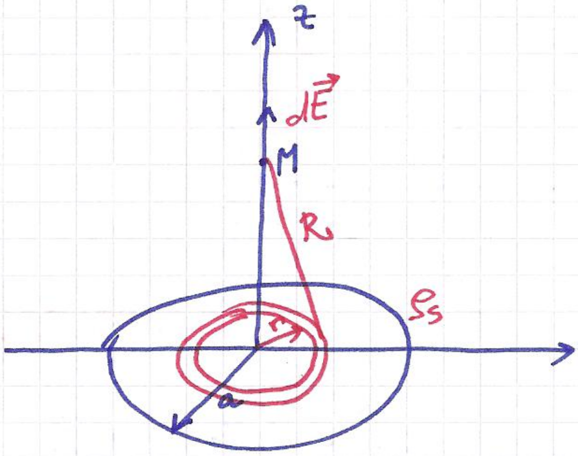
58.

$$E_z=-\frac{dV}{dz}=-\frac{Q}{4\pi\varepsilon_0}\,d\left(\frac{1}{\sqrt{z^2+a^2}}\right)=\boxed{\frac{Q\,z}{4\pi\varepsilon_0\,(z^2+a^2)^{3/2}}}$$

$$d\left((z^2+a^2)^{-1/2}\right)=-\frac{1}{\cancel{2}}\,\frac{1}{(\sqrt{z^2+a^2})^3}\cdot\cancel{2}z$$

y-KOMPONENTA VEKTORA $\vec{E}$ NE MOŽE SE ODREDITI DIREKTNO IZ IZRAZA ZA POTENCIJAL NA OSI-z.

(59.) a, KRUG, ρ_s a) $V(z)=?$ б) $E(z)=?$



$V=\int dV$

$dQ=\rho_s dS$

$dS=2\pi r dr$

$$V=\int\frac{dQ}{4\pi\varepsilon_0 R}\ , \quad R=\sqrt{r^2+z^2}$$

$$V=\frac{\rho_s \cancel{2\pi}}{{}_2\cancel{4\pi}\varepsilon_0}\int_0^a\frac{r\,dr}{\sqrt{r^2+z^2}}$$

$$V=\frac{\rho_s}{2\varepsilon_0}\int_0^a\frac{r\,dr}{\sqrt{r^2+z^2}}=\begin{cases}t=r^2+z^2\\ dt=2r\,dr\end{cases}$$

$$V=\frac{\rho_s}{2\varepsilon_0}\cdot\frac{1}{2}\int_{z^2}^{a^2+z^2}\frac{dt}{\sqrt{t}}=\frac{\rho_s}{{}_2\cancel{4}\varepsilon_0}\cancel{2}\sqrt{t}\Big|_{z^2}^{a^2+z^2}$$

$$\underbrace{V=\frac{\rho_s}{2\varepsilon_0}\left(\sqrt{a^2+z^2}-|z|\right)}$$

б) $$E_z=-\frac{dV}{dz}=\frac{\rho_s}{2\varepsilon_0}\left(\operatorname{sgn} z-\frac{z}{\sqrt{a^2+z^2}}\right)$$

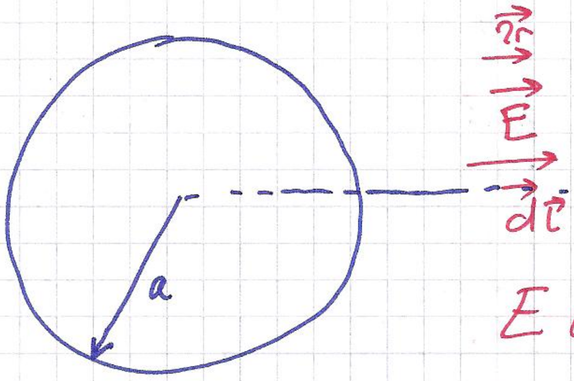
(*60.)

$$V=\frac{\rho_{s0}}{2\varepsilon_0 a}\int_0^a\frac{r^2dr}{\sqrt{r^2+z^2}}=\begin{cases}r^2+z^2=t\\ dt=2r\,dr\end{cases}$$

$$V=\frac{\rho_{s0}}{4\varepsilon_0 a}\int_{z^2}^{a^2+z^2}\frac{(t-z^2)}{\sqrt{t}}$$ DRUGAČIJA SMENA

$$V=\frac{\rho_{s0}}{4\varepsilon_0 a}\left(a\sqrt{a^2+z^2}-z^2\ln\frac{a+\sqrt{a^2+z^2}}{|z|}\right)$$

(61.) Q, a, $V = ?$



E U SFERI $\underline{\underline{=0}}$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{i}_r$$

VAN SFERE
(KAO TAČKASTO NAEL)

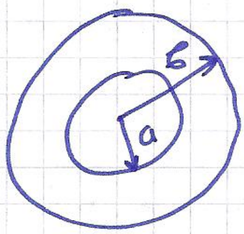
$$V = \int \vec{E}\cdot d\vec{r} = \int_{r_m}^{\infty} E_r\, dr = \frac{Q}{4\pi\varepsilon_0 r_m}, \quad r_m > a$$

$r_m \le a$

$$V = \int_M^R \vec{E}\cdot d\vec{r} = \int_a^{+\infty} \frac{Q}{4\pi\varepsilon_0 a} = \underline{\underline{const}}$$

(62.)

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 a} + \frac{Q_2}{4\pi\varepsilon_0 b}$$



$$V_2 = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 b}$$

V_1 - POTENCIJAL UNUTRAŠNJE LJUSKE

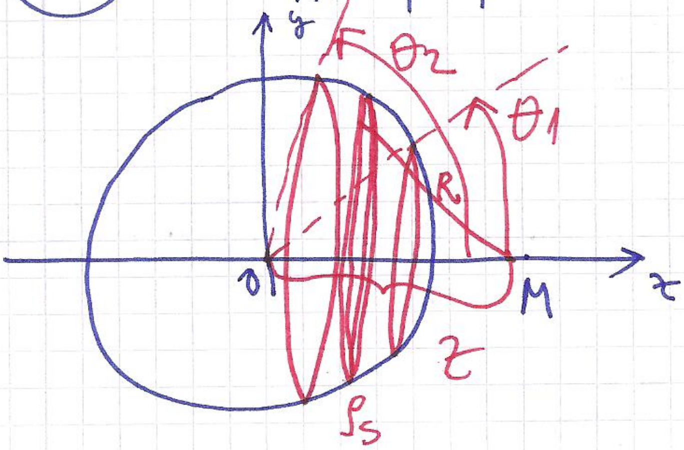
V_2 - POTENCIJAL SPOLJAŠNJE LJUSKE

$Q_1 > 0$

$Q_2 < 0$

$V_1 < 0$ ZA $\frac{Q_1}{|Q_2|} < \frac{a}{b}$

63 $a, \theta_1, \theta_2, \rho_s, \varepsilon_0 \quad V_M = ?$



$$dQ = \rho_s dS$$

$$dQ = \rho_s 2\pi a \sin\theta d\theta \cdot a$$

$$dV = \frac{dQ}{4\pi\varepsilon_0 R}$$

$$R = \sqrt{(z - a\cos\theta)^2 + (a\sin\theta)^2}$$

$$R = \sqrt{a^2 + z^2 - 2az\cos\theta}$$

$$dV = \frac{\rho_s a^2 \sin\theta d\theta}{2\varepsilon_0 R}$$

$$t = a^2 + z^2 - 2a\cos\theta$$

$$dt = 2az\sin\theta d\theta$$

$$V = \int dV = \frac{\rho_s a}{4\varepsilon_0 z}\int_{\theta_1}^{\theta_2}\frac{dt}{\sqrt{t}} \Rightarrow V(z) = \frac{\rho_s a}{2\varepsilon_0 z}\left(\sqrt{a^2+z^2-2az\cos\theta_2} - \sqrt{a^2+z^2-2az\cos\theta_1}\right)$$

$$V(z) = \frac{\rho_s a}{2\varepsilon_0 z}\left(\sqrt{a^2+z^2-2az\cos\theta_2} - \sqrt{a^2+z^2-2az\cos\theta_1}\right)$$

64. U SREDIŠTU POLUSFERE a, ε_0, Q

$\theta_1 = 0 \quad \theta_2 = \pi/2$

$$V(z) = \frac{\rho_s a}{2\varepsilon_0 z}\left(\sqrt{a^2+z^2} - \sqrt{(z-a)^2}\right) = \frac{\rho_s a}{2\varepsilon_0 z}\left(\sqrt{a^2+z^2} - |z-a|\right)$$

$$\rho_s = 2\pi a^2 \Rightarrow V(z=0) = \frac{Q}{4\pi\varepsilon_0 a}$$

E NE MOŽE PREKO $-\mathrm{grad}\, V$

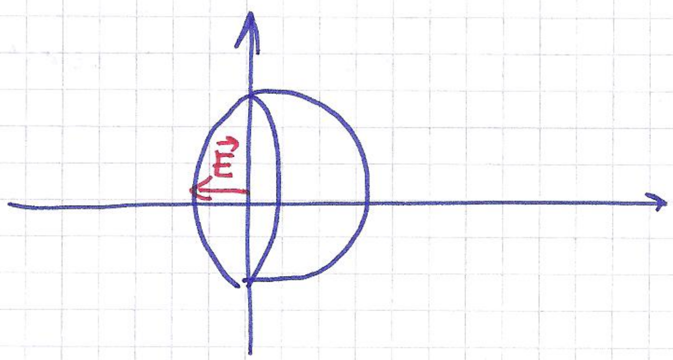
$$E(0) = \int dE(0)$$

$$dQ = \rho_s dS = \frac{Q}{\cancel{2\pi R^2}} \cdot \cancel{2\pi R} \sin\theta \cancel{R} d\theta = Q \sin\theta d\theta$$

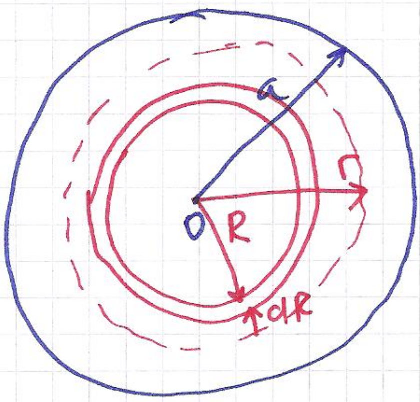
$$z = R\cos\theta$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} \int_0^{\pi/2} \sin\theta \cos\theta d\theta \Rightarrow E = \frac{Q}{8\pi\varepsilon_0 R^2}$$

$$\vec{E(0)} = -\frac{Q}{8\pi\varepsilon_0 R^2} \vec{i_z}$$



*65. $a, \rho, V=?, E=?$



$Q=\rho \cdot v$

$dQ=\rho dv$

$dv=4\pi R^2 dR$

$dQ=\rho 4\pi R^2 dR$

$$dV=\begin{cases}\dfrac{dQ}{4\pi\varepsilon_0 r}=\dfrac{\rho R^2 dR}{\varepsilon_0 r}, & r>R\\[2ex] \dfrac{dQ}{4\pi\varepsilon_0 R}=\dfrac{\rho \cancel{4\pi} R^{\cancel{2}} dR}{\cancel{4\pi}\varepsilon_0 \cancel{R}}=\dfrac{\rho R dR}{\varepsilon_0}, & r\leq R\end{cases}$$

$r\leq a$

$$V=\frac{\rho}{\varepsilon_0}\left(\frac{1}{r}\int_0^r R^2 dR+\int_r^a R dR\right)=\frac{\rho_0}{\varepsilon_0}\left(\frac{1}{\cancel{r}}\frac{r^{\cancel{3}2}}{3}+\frac{a^2}{2}-\frac{r^2}{r}\right)$$

$$V=\frac{\rho}{6\varepsilon_0}\left(2r^2-3a^2-3r^2\right)=\frac{\rho}{6\varepsilon_0}(3a^2-r^2), \quad r\leq a$$

$r>a$

$$V=\frac{\rho}{\varepsilon_0}\frac{1}{r}\int_0^a R^2 dR=\frac{\rho a^3}{3\varepsilon_0 r}=\frac{Q}{4\pi\varepsilon_0 r}, \quad r>a$$

$$E(r)=-\frac{dV}{dr}=\begin{cases}\dfrac{\rho r}{3\varepsilon_0}, & r\leq a\\[2ex] \dfrac{Q}{4\pi\varepsilon_0 r^2}, & r>a\end{cases}$$

(66.) $a, \rho \quad , \quad U_{OA} = V_O - V_A$

$$V_0 = V(r=0) = \frac{\rho a^2}{2\varepsilon_0}$$

$$V_A = V(r=a) = \frac{\rho a^2}{3\varepsilon_0}$$

$$U_{OA} = \frac{\rho a^2}{2\varepsilon_0} - \frac{\rho a^2}{3\varepsilon_0} = \frac{\rho a^2}{6\varepsilon_0}$$

GAVSOV ZAKON

(70.) $$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{us}}{\varepsilon_0} = 0 \Rightarrow Q = 0$$

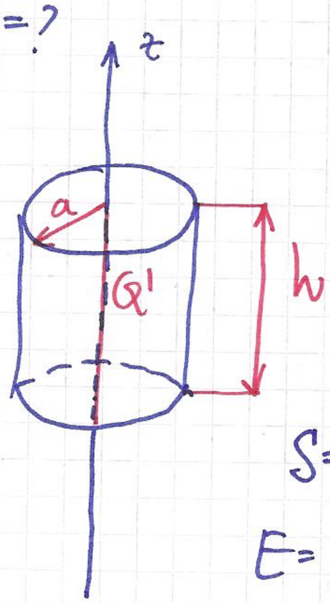
$$\vec{E} \underbrace{\int d\vec{S}}_{\neq 0} = 0$$

(73.) $r, \rho \quad$ a) $\Psi_E = ?$ $\quad$ b) $\lim\limits_{r \to 0} \frac{\Psi_E}{v} = ?$

$$\Psi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0} = \frac{\int_v \rho \, dv}{\varepsilon_0} = \frac{\rho 4\pi r^3}{\varepsilon_0}$$

b) $\lim\limits_{r \to 0} \frac{\Psi_E}{v} = \frac{\rho}{\varepsilon_0}$

(74.) $Q', h, S, \Psi_E = ?$



$$\Psi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q' h}{\varepsilon_0}$$

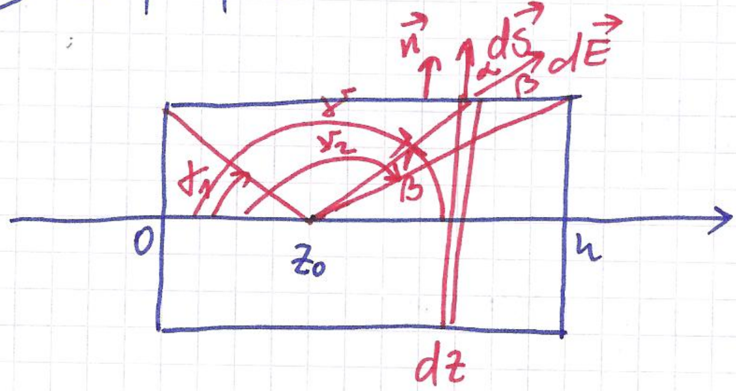
$$S = 2\pi a h$$

$$E = \frac{Q'}{2\pi \varepsilon_0 a}$$

$$\frac{Q'}{\cancel{2\pi} \varepsilon_0 \cancel{a}} \cdot \cancel{2\pi} \cancel{a} h = \frac{Q' h}{\varepsilon_0}$$

SAMO RADIJALNA KOMPONENTA

(*75) Q, a, h



$$d\vec{S} = \vec{n}\, dS$$

$$d\Psi_E = \vec{E}\, d\vec{S}$$

$$= \vec{E} \cdot \vec{n}\, dS$$

$$= E \cos(\vec{E}, d\vec{S})\, dS$$

$$\Psi_E = \int d\Psi_E$$

$$E_n = \frac{Q}{4\pi \varepsilon_0 r^2} \cos\alpha = \frac{Q}{4\pi \varepsilon_0 r^2} \sin\beta = \frac{Q a}{4\pi \varepsilon_0 r^3}$$

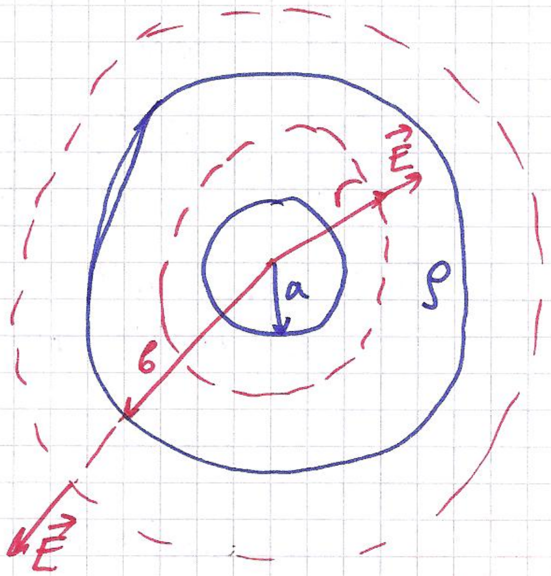
$$\vec{E} = \frac{Q \vec{r_0}}{4\pi \varepsilon_0 r^2}$$

$$E_n \cdot 2\pi a\, dz = \frac{Q a^2 d\ell}{2\varepsilon_0 r^3}$$

$$\Psi_E = \int_S \frac{Q a^2 d\ell}{2\varepsilon_0 r^3} = \int_{z=0}^{h} \frac{Q a^2 dz}{2\varepsilon_0((z-z_0)^2+a^2)^{3/2}} = \frac{Q}{2\varepsilon_0}\int_0^h \frac{a^2 dz}{((z-z_0)^2+a^2)^{3/2}}$$

$$\Psi_E = \frac{Q}{2\varepsilon_0}\left(\frac{h-z_0}{\sqrt{(h-z_0)^2+a^2}} + \frac{z_0}{\sqrt{z_0^2+a^2}}\right)$$

76.



$$\oint \vec{E}\, d\vec{s} = \frac{Q}{\varepsilon_0}$$

$Q = \rho v$

$Q = \rho \frac{4}{3} R^3 \pi \quad , R=r$

I) $r < a \quad E 4\pi r^2 = 0 \Rightarrow E = 0$

II) $a < r \leq b$

$$E 4\pi r^2 = \frac{Q_{(\text{od } a \text{ do } r)}}{\varepsilon_0}$$

$$E \cancel{4\pi} r^2 = \frac{\rho \frac{4}{3}(r^3 - a^3)\cancel{\pi}}{\varepsilon_0}$$

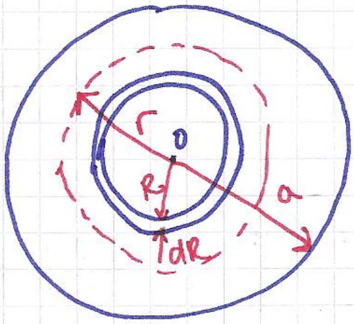
$$\underline{E = \frac{\rho}{3\varepsilon_0}\left(r - \frac{a^3}{r^2}\right)}$$

III) $r > b$

$$E 4\pi r^2 = \frac{\rho \frac{4}{3}(b^3 - a^3)\pi}{\varepsilon_0}$$

$$E(r) = \frac{\rho}{3\varepsilon_0}\left(\frac{b^3}{r^2} - \frac{a^3}{r^2}\right) \quad , \underline{\underline{r > b}}$$

77. $\rho(r) = \rho(a)\frac{r}{a}$ $\quad 0 \le r \le a$



$\vec{E(r)} = E(r)\vec{2r}$

$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{ius}}{\varepsilon_0}$

$E(r) 4\pi r^2 = \frac{1}{\varepsilon_0}\int \rho \, dv$

$\frac{1}{\varepsilon_0}\int \rho \, dv = \frac{\rho(a)}{\varepsilon_0 a}\int r \, dv$

I) $R < r \le a$

$dv = 4\pi R^2 dR$

$E \cancel{4\pi} r^2 = \frac{\rho(a)}{\varepsilon_0 a}\int_0^r R \, dR \cancel{4\pi} R^2$

$E \cdot r^2 = \frac{\rho(a)}{\varepsilon_0 a}\int_0^r R^3 dR = \frac{\rho(a)}{\varepsilon_0 a}\frac{R^4}{4}\Big|_0^r = \frac{\rho(a)}{4\varepsilon_0 a} r^4$

$\vec{E} = \frac{\rho(a)}{4\varepsilon_0 a} r^2 \vec{2r}$

II) $E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^a \rho \, dv$

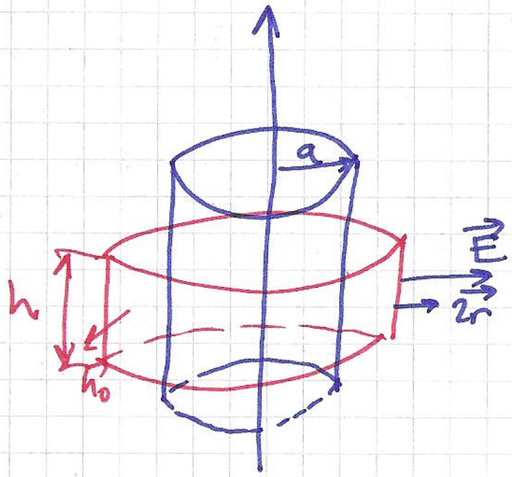
$E \cancel{4\pi} r^2 = \frac{\rho(a)}{\varepsilon_0 a}\int_0^a \cancel{4\pi} R^2 \cdot R \, dR$

$E r^2 = \frac{\rho(a)}{\varepsilon_0 a}\frac{R^4}{4}\Big|_0^a = \frac{\rho(a) a^3}{4\varepsilon_0}$

$\vec{E(r)} = \frac{\rho(a) a^3}{4\varepsilon_0 r^2}\vec{2r}$

78. VEOMA DUGAČAK CILINDAR

a, ρ_s, ε_0



$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{us}}{\varepsilon_0}$$

$$\vec{E}(r) = E(r)\vec{i}_r$$

SAMO RADIJALNA KOMPONENTA

I) $r<a$

$$E \cdot 2\pi r h = 0$$

II) $r>a$

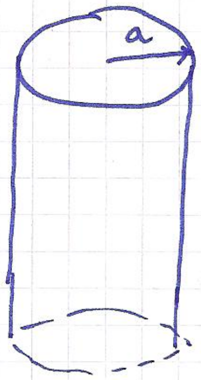
$$E \cancel{2\pi} r \cancel{h} = \frac{Q_{us}}{\varepsilon_0} = \frac{\rho_s \cancel{2\pi} a \cancel{h}}{\varepsilon_0}$$

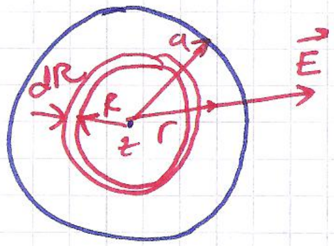
$$E(r) = \frac{\rho_s a}{\varepsilon_0 r}, \quad r>a$$

$$\boxed{\vec{E}(r) = \frac{\rho_s a}{\varepsilon_0 r}\vec{i}_r}$$

79. $\rho(R) = \rho(a)\left(\frac{R}{a}\right)^2 \quad R \in [0, a]$

$a, \rho(a)$





$$dE = \begin{cases} 0, & r < R \\ \frac{dQ'(R)}{2\pi\varepsilon_0 r}, & r > R \end{cases} \qquad dQ' = \rho \cdot 2\pi R\, dR$$

$$E(r) = \int_0^r \frac{\rho R\, dR}{\varepsilon_0 r} = \frac{\rho(a)}{\varepsilon_0 r a^2} \int_0^r R^3 dR = \frac{\rho(a)\, r^3}{4a^2\varepsilon_0} \quad r \leq a$$

$$r \geq a \quad E(r) = \frac{\rho(a)\, a^2}{4\varepsilon_0 r}$$

II NAČIN – PREKO GAUSOVOG ZAKONA

$$E \cdot 2\pi r h = \frac{Q_{us}}{\varepsilon_0}$$

$$\frac{\rho(a)}{\varepsilon_0 a^2} \int R^2\, 2\pi R h\, dR = E\, 2\pi r h$$

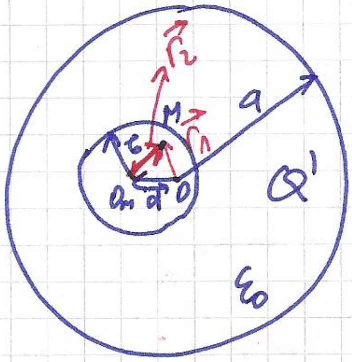
$$Er = \frac{\rho(a)}{\varepsilon_0 a^2} \int_0^r R^3 dR \Rightarrow E(r) = \frac{\rho(a)\, r^3}{4\varepsilon_0 a^2}, \; r \leq a$$

$r > a$

$$E \cdot r = \frac{\rho(a)}{\varepsilon_0 a^2} \int_0^a R^3 dR = \frac{\rho(a)\, a^{\cancel{4}\,2}}{4\varepsilon_0 \cancel{a^2}}$$

$$E(r) = \frac{\rho(a)\, a^2}{4\varepsilon_0 r}, \; \underline{r > a}$$

(80.)



$\vec{r_1} - \vec{r_2} = \vec{d}$

$\vec{E} = \vec{E_1} - \vec{E_2}$

$\vec{E}$ u TAČKI M

$$\rho_a = \frac{Q'}{\pi(a^2-b^2)}$$

$$\rho_b = -\rho_a$$

$$\oint \vec{E_1}\,d\vec{S_1} = \frac{Q_1}{\varepsilon_0}$$

$$\cancel{E_1 4\pi r_1^2 = \frac{Q'}{(a^2-b^2)} \cdot \frac{4\pi r_1^3}{3\varepsilon_0}}$$

$$\cancel{\vec{E_1} = \frac{Q'\vec{r_1}}{3\varepsilon_0(a^2-b^2)}}$$

$$\vec{E_1} = \frac{Q'\vec{r_1}}{\pi(a^2-b^2)2\varepsilon_0} \leftarrow E_1 2\cancel{\pi} \cancel{r_1} \cancel{h} = \frac{Q'}{\pi(a^2-b^2)\varepsilon_0}\cancel{\pi} r_1^{\cancel{2}} \cancel{h}$$

$$E_2 2\pi r_2 h = \frac{Q'}{\pi(a^2-b^2)\varepsilon_0}\pi r_2^2 h$$

$$\vec{E_2} = \frac{Q'\vec{r_2}}{\pi(a^2-b^2)2\varepsilon_0}$$

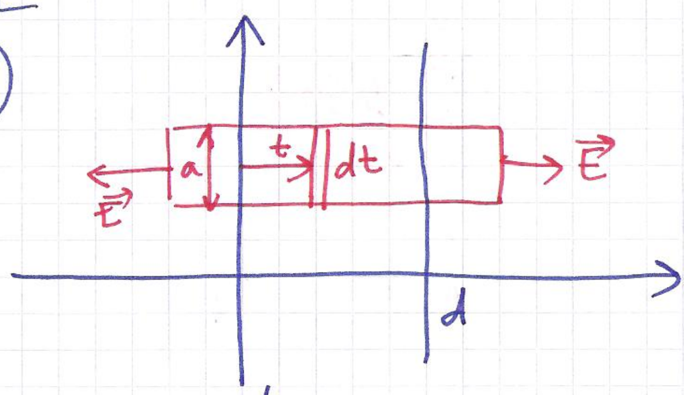
$$\vec{E} = \frac{Q'\vec{d}}{2\varepsilon_0\pi(a^2-b^2)}$$

81. $\rho(x) = \rho_0 \left(\frac{x}{d}\right)^{-\frac{2}{3}} \qquad 0 < x < d$

$\rho_0 = \text{const}$

$\vec{E} = ?$

I)



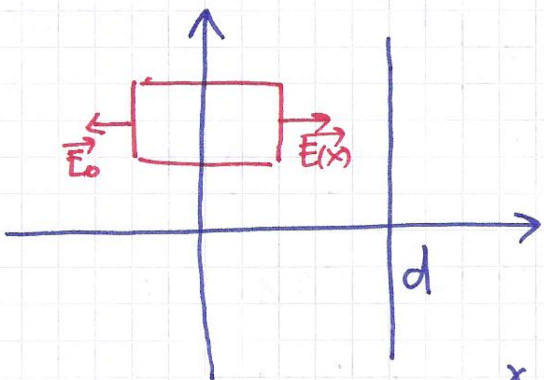
$\vec{E}$ SAMO NA BAZISIMA a^2

$$2Ea^2 = \frac{1}{\varepsilon_0} \int_0^d \rho(t) a^2 dt$$

$$2E = \frac{1}{\varepsilon_0} \int_0^d \rho_0 \left(\frac{t}{d}\right)^{-\frac{2}{3}} dt$$

$$E = \frac{3\rho_0 d}{2\varepsilon_0} \quad , x \geq d$$

II) X - UNUTAR OBLAKA



GDE JE $E_0 = \frac{3\rho_0 d}{2\varepsilon_0}$

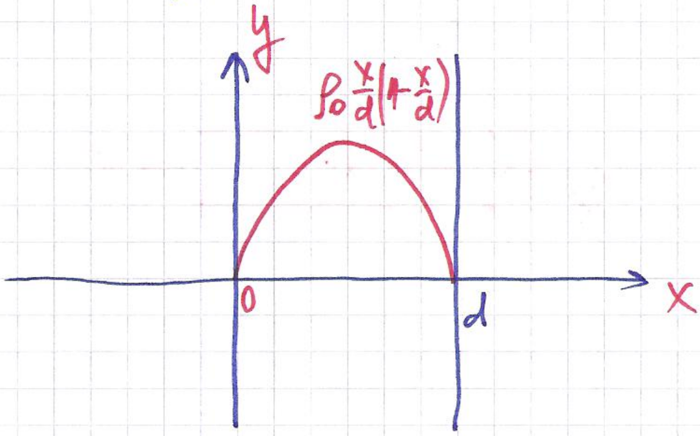
$$E_0 \cancel{a^2} + E(x)\cancel{a^2} = \frac{1}{\varepsilon_0} \int_0^x \rho(t) \cancel{a^2} dt$$

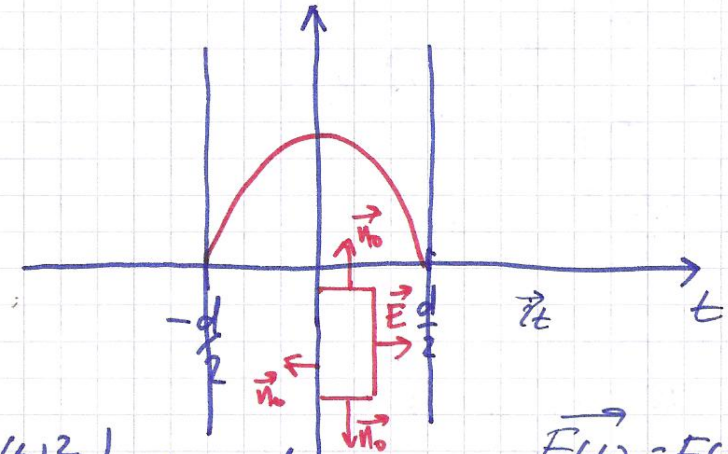
$$\frac{3\rho_0 d}{2\varepsilon_0} + E(x) = \frac{3\rho_0 d}{\varepsilon_0} \left(\frac{x}{d}\right)^{+\frac{1}{3}}$$

$$\vec{E(x)} = \frac{3\rho_0 d}{2\varepsilon_0} \left(2\left(\frac{x}{d}\right)^{\frac{1}{3}} - 1\right) \vec{i_x} \quad , 0 < x < d$$

82. $x=0$, $x=d$, $d>0$

$$\rho(x)=\begin{cases}\rho_0 \frac{x}{d}\left(1-\frac{x}{d}\right), & \frac{x}{d}\in[0,1]\\ 0, & \frac{x}{d}<0 \wedge \frac{x}{d}>1\end{cases}$$





$$\rho(t)=\begin{cases}\rho_0\left(\frac{1}{4}-\left(\frac{t}{d}\right)^2\right), & |t|\le\frac{d}{2}\\ 0, & |t|>\frac{d}{2}\end{cases}$$

$\vec{E}(t)=E(t)\vec{i}_t$

$E(t=0)=0$

I) TAČKA U OBLAKU NAELEKTRISANJA

$$Ea \overset{?}{=} \frac{1}{\varepsilon_0}\int_0^t \rho_0\left(\frac{1}{4}-\left(\frac{u}{a}\right)^2\right)a^2 du$$

$$\Rightarrow \vec{E}=\frac{\rho_0 d}{\varepsilon_0}\left(\frac{t}{d}\right)\left(\frac{1}{4}-\frac{1}{3}\left(\frac{t}{d}\right)^2\right)\vec{i}_t=E_t\vec{i}_t$$

VAŽI I ZA $-d/2\le t<0$

$\mathrm{II})\ t > \frac{d}{2}$

TAČKA SA DESNE STRANE

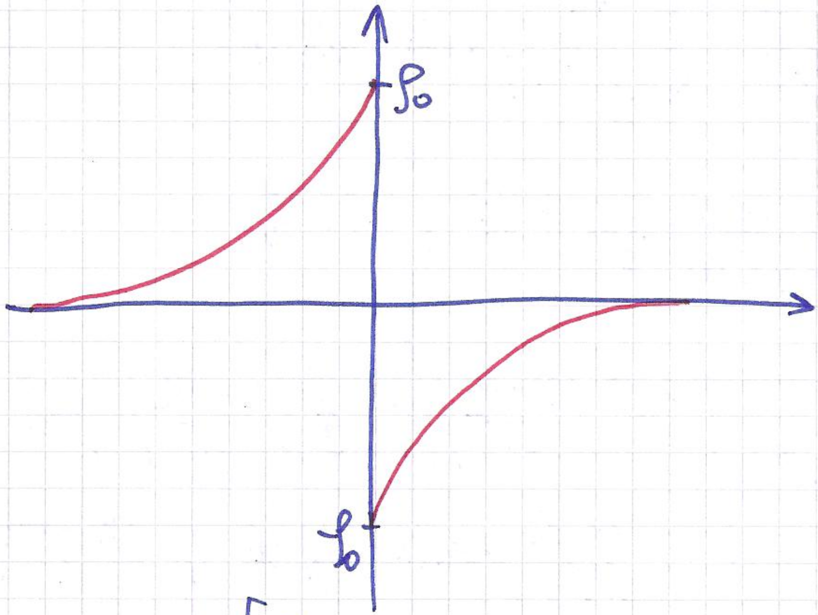
$$E\cancel{a} = \frac{1}{\varepsilon_0}\int_0^{d/2} \rho_0\left(\frac{1}{4}-\left(\frac{u}{d}\right)^2\right)\cancel{a}\,du$$

$$\vec{E} = \frac{\rho_0 d}{12\varepsilon_0}\,\mathrm{sgn}\,t\,\vec{i_t}\ ,\ |t| > \frac{d}{2}$$

$$E(x) = \begin{cases} \frac{\rho_0 d}{\varepsilon_0}\left(\frac{x-\frac{d}{2}}{d}\right)\left(\frac{1}{4}-\frac{1}{3}\left(\frac{x-d/2}{d}\right)^2\right)\vec{i_x}\ ,\ \left|\frac{x-d/2}{d}\right| \leq \frac{1}{2} \\ \frac{\rho_0 d}{12\varepsilon_0}\,\mathrm{sgn}(x-d/2)\,\vec{i_x}\ ,\ \left|\frac{x-d/2}{d}\right| > \frac{1}{2} \end{cases}$$

83.

$$\rho(x)=\begin{cases}-\rho_0 \operatorname{sgn}x\, e^{-ax\operatorname{sgn}x}, & x\neq 0\\ 0, & x=0\end{cases}$$



a)

$$\vec{E}(x)=\frac{\rho_0}{2\varepsilon_0}\left[\int_{-\infty}^{0} e^{au}du+\int_{0}^{x}(-e^{-au})du-\int_{x}^{\infty}(-e^{-au})du\right]\vec{i_x}$$

$$=\frac{\rho_0}{\varepsilon_0 a}e^{-ax}\vec{i_x},\ x>0$$

$\underline{x<0}$

$$\vec{E}(x)=\frac{\rho_0}{2\varepsilon_0}\left(\int_{-\infty}^{x} e^{au}du-\int_{x}^{0} e^{au}du-\int_{0}^{\infty}(-e^{-au})du\right)\vec{i_x}$$

$$=\frac{\rho_0}{\varepsilon_0 a}e^{ax}\vec{i_x},\ x<0$$

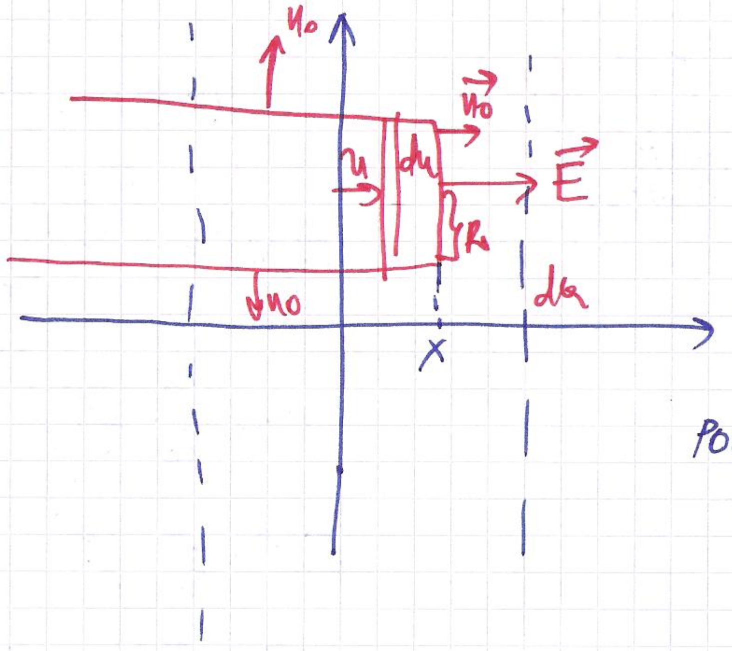
$\underline{x=0}$

$$E(0)=\frac{\rho_0}{2\varepsilon_0}\left(\int_{-\infty}^{0} e^{au}du-\int_{0}^{+\infty}(-e^{-au})du\right)\vec{i_x}=\frac{\rho_0}{\varepsilon_0 a}\vec{i_x}$$

JEDINSTVEN IZRAZ MOŽE BITI:

$$\vec{E}(x)=\frac{\rho_0}{\varepsilon_0 a}e^{-a|x|}\vec{i_x},\ \forall x$$

b) PREKO GAUSOVOG ZAKONA



POVRŠ PRAVOG VALJKA
R, JEDAN BAZIS NA $-\infty$
DRUGI NA X

$$ER^2\pi\cos 0 = \frac{1}{\varepsilon_0}\int_{-\infty}^{x}\rho(u)R^2 du\,\pi$$

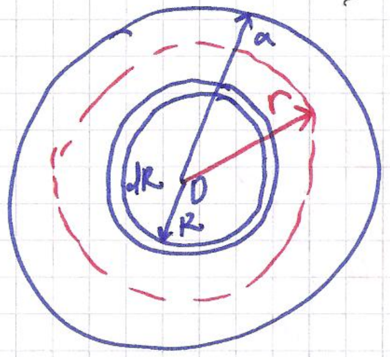
$$\cancel{E}\cancel{R^2}\cancel{\pi} = \frac{1}{\varepsilon_0}\int_{-\infty}^{x}\rho(u)\cancel{R^2}du\cancel{\pi}$$

$$\vec{E(x)} = \frac{\rho_0}{\varepsilon_0}\left(\int_{-\infty}^{0}e^{au}du - \int_{0}^{x}e^{-au}du\right)\vec{i_x}$$

$$\vec{E(x)} = \frac{\rho_0}{\varepsilon_0 a}e^{-ax}\vec{i_x}\ ,\ x>0$$

⋮

(88.) LOPTA , ρ, $\vec{E} = ?$



I) $r \leq a$

$$dQ = \rho dv = \rho 4\pi R^2 dR$$

$$E \cancel{4\pi} r^2 = \int_0^r \frac{\rho \cancel{4\pi} R^2 dR}{\varepsilon_0}$$

$$E r^2 = \frac{\rho}{\varepsilon_0} \frac{R^3}{3} \Big|_0^r$$

$$E \cancel{r^2} = \frac{\rho}{\varepsilon_0} \frac{r^{\cancel{3}}}{3}$$

$$E = \frac{\rho r}{\varepsilon_0 3}, \quad \underline{r \leq a}$$

II) $r > a$

$$\cancel{4\pi} E r^2 = \int_0^a \frac{\rho \cancel{4\pi} R^2 dR}{\varepsilon_0} = \frac{\rho a^3}{3 \varepsilon_0}$$

$$E = \frac{\rho a^3}{3 \varepsilon_0 r^2}, \quad \underline{r > a}$$

(89.) LOPTA IZ 37. ZADATKA

$$\rho(R) = \rho(a) \frac{R}{a} \quad , 0 \leq R \leq a$$

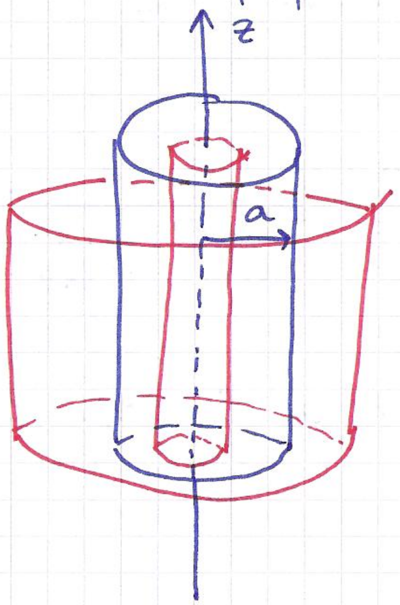
I) $r < a$

$$V = \int E dR = \int_r^a \frac{\rho(a)}{4\varepsilon_0 a} r^2 dr + \int_a^{+\infty} \frac{\rho(a)}{4\varepsilon_0 a} r^2 dr = \frac{\rho(a) r^3}{12 \varepsilon_0 a}$$

$r > a$

$$\underbrace{V = \frac{Q}{4\pi \varepsilon_0 r}} \qquad \frac{\rho(a) a^3}{4\pi \varepsilon_0 r} = \frac{Q}{4\pi \varepsilon_0 r}$$

90. CILINDAR, ρ, a) $\vec{E}=?$ b) $V=?$



a) I) $r<a$

$$E2\pi rh = \frac{\rho r^2 \pi h}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho r}{2\varepsilon_0}}$$

II) $r>a$

$$E2\pi rh = \frac{\rho a^2 \pi h}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho a^2}{2r\varepsilon_0}}$$

b)

$$V = \int \vec{E} d\vec{R}$$

$$V = \begin{cases} \frac{\rho}{4\varepsilon_0}(a^2 - r^2), & r \le a \\ -\frac{\rho a^2}{2\varepsilon_0} \ln \frac{r}{a}, & r > a \end{cases}$$

$r \le a$

$$V = \int_r^a \frac{\rho R}{2\varepsilon_0} dR = \frac{\rho}{2\varepsilon_0} \frac{R^2}{2} \Big|_r^a = \frac{\rho}{4\varepsilon_0}(a^2 - r^2)$$

$r>a$

$$V = -\frac{\rho a^2}{2\varepsilon_0} \ln \frac{r}{a}$$

92. $V(x) = V_0 \operatorname{arctg}\left(\frac{x}{a}\right) \quad V_0, a = const, \; a \neq 0$

$E_x = ?$

$$E_x = -\frac{dV(x)}{dx} = -\frac{V_0 d\left(\operatorname{arctg}\frac{x}{a}\right)}{dx} = -V_0 \frac{1}{1+\frac{x^2}{a^2}} \cdot \frac{1}{a} = \underline{-\frac{V_0 a}{a^2+x^2}}$$

94. POTPUNI SISTEM JEDNAČINA ZA EL. STAT. POLJE U VAKUMU

$$\oint \vec{E} \cdot d\vec{e} = 0$$

$$\oint \vec{E} d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho dv}{r^2} \vec{r_0}$$

95. $V(x, y, z) = V_2 \frac{x^2}{a^2} + V_1 \frac{x}{a} + V_0 \quad , a > 0$

a) $\vec{E} = ?$

$$E_x = -\frac{dV}{dx} = \underline{-\frac{2V_2 x}{a^2} - \frac{V_1}{a}}$$

б) $\rho = -\varepsilon_0 \frac{d^2V}{dx^2} = \underline{-2\varepsilon_0 \frac{V_2}{a^2}}$

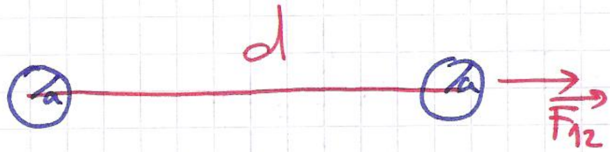
PROVODNICI U EL. STATIČKOM POLJU

98.

$a = 100\,mm$

$d = 2\,m$

$E_{MAX} \approx 3\frac{MV}{m}$

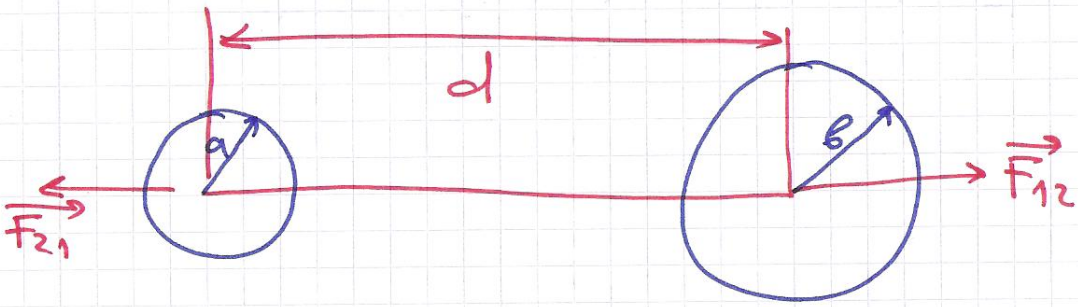


$$E = \frac{Q}{4\pi\varepsilon_0 a^2} \qquad E \leq E_{MAX}$$

$$Q_{1MAX} = Q_{2MAX} = 4\pi\varepsilon_0 a^2 E_{MAX}$$

$$\vec{F_{12}} = \frac{Q_{1MAX} \cdot Q_{2MAX}}{4\pi\varepsilon_0 d^2}\vec{r_{012}} = \frac{Q_{MAX}^2}{4\pi\varepsilon_0 d^2}\vec{r_{012}} = 25\,mN\,\vec{r_{012}}$$

101. $a, b \quad d \gg a, b \;,\; Q_1, Q_2$



I PRE POVEZIVANJA

$$V_a = \frac{Q_1}{4\pi\varepsilon_0 a} = V_1 \quad V_b = \frac{Q_2}{4\pi\varepsilon_0 b} = V_2$$

$$\vec{F_{12}} = Q_2\vec{E_1} \qquad \vec{F_{21}} = Q_1\vec{E_2}$$

$$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2}\vec{i_x} \qquad \vec{F_{21}} = -\frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2}\vec{i_x}$$

II POSLE POVEZIVANJA → POTENCIJALI KUGLI SU ISTI

$$Q_1^{(1)} + Q_2^{(1)} = Q_1 + Q_2$$

$$V_1^{(1)} = \frac{Q_1^{(1)}}{\cancel{4\pi\varepsilon_0} a} = \frac{Q_2^{(1)}}{\cancel{4\pi\varepsilon_0} b} = V_2^{(1)}$$

$$\frac{Q_1^{(1)}}{a} = \frac{Q_2^{(1)}}{b} \Rightarrow \frac{Q_1^{(1)}}{Q_2^{(1)}} = \frac{a}{b}$$

$$\Delta V_1 = V_1^{(1)} - V_1 \qquad \Delta V_2 = V_2^{(1)} - V_2$$

$$\Delta \vec{F}_{12} = \vec{F}_{12}^{(1)} - \vec{F}_{12}$$

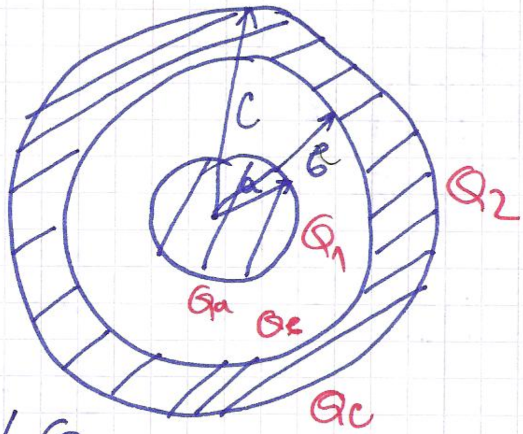
$$Q_1^{(1)} = \frac{a}{a+b}(Q_1 + Q_2) \qquad Q_2^{(1)} = \frac{b}{a+b}(Q_1 + Q_2)$$

$$\Delta Q_1 = Q_1^{(1)} - Q_1 = \frac{aQ_2 - bQ_1}{a+b}$$

$$\Delta Q_2 = Q_2^{(1)} - Q_2 = -\frac{aQ_2 - bQ_1}{a+b} = -\Delta Q_1$$

102. $a = 1cm$ $Q_1 = -5nC$ $b = 2,5cm$ $c = 4cm$

$Q_2 = 7nC$



ε_0

$Q_a = -Q_b = Q_1$

$Q_c = Q_1 + Q_2$

$$\vec{E}(r) = \begin{cases} \dfrac{Q_a}{4\pi\varepsilon_0 r^2}\vec{u_r} = \dfrac{Q_1}{4\pi\varepsilon_0 r^2}\vec{u_r}, & r \in [a, b] \\ \dfrac{Q_a + Q_b + Q_c}{4\pi\varepsilon_0 r^2}\vec{u_r} = \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 r^2}\vec{u_r} & r > c \end{cases}$$

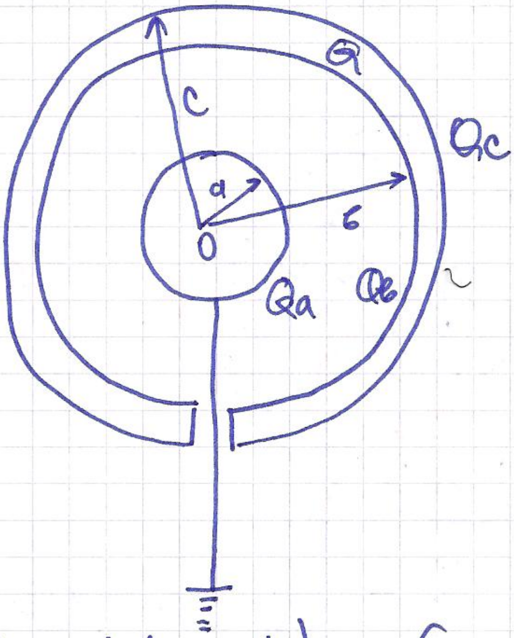
$r \in (b, c)$ $E = 0$

$$V_p = \int_P^R \vec{E} \cdot d\vec{l}$$

$$V(r) = \begin{cases} \displaystyle\int_r^\infty \frac{Q_1 + Q_2}{4\pi\varepsilon_0 t^2} dt = \frac{Q_1 + Q_2}{4\pi\varepsilon_0 r}, & r \geq c \\ \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 c} = V_c = V_b, & b \leq r \leq c \\ \displaystyle\int_r^b \frac{Q_1}{4\pi\varepsilon_0 t^2} dt = \frac{Q_1}{4\pi\varepsilon_0 r}\left(\frac{1}{r} - \frac{1}{b}\right) + V_c & a \leq r < b \\ \displaystyle + \int_c^\infty \frac{Q_1 + Q_2}{4\pi\varepsilon_0 t^2} dt \end{cases}$$

$$\frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{a} + \frac{1}{b}\right) + \frac{Q_1 + Q_2}{4\pi\varepsilon_0 c} = V_a \quad 0 < r \leq a$$

(104.) $a, b, c, Q, \quad V = ?$



$$V_z = 0 \Rightarrow V_a = 0$$

$$Q_b + Q_c = Q$$

$$Q_a + Q_b = 0$$

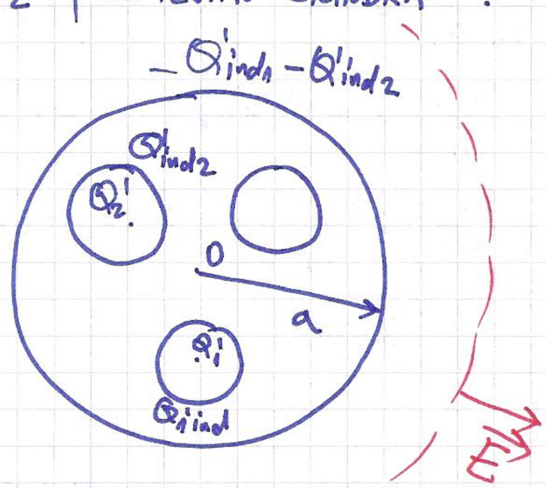
$$Q_1 = Q_a$$

$$Q_2 = Q$$

$$V_a = \frac{Q_a}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{Q_a + Q}{4\pi\varepsilon_0 c}$$

$$V_c = \frac{Q_a + Q}{4\pi\varepsilon_0 c} \qquad V_a = 0 \Rightarrow Q_a = -Q\,\frac{\frac{1}{c}}{\frac{1}{a} - \frac{1}{b} + \frac{1}{c}}$$

(109.) $a, Q_1', Q_2', \vec{E}$ IZVAN CILINDRA $= ?$



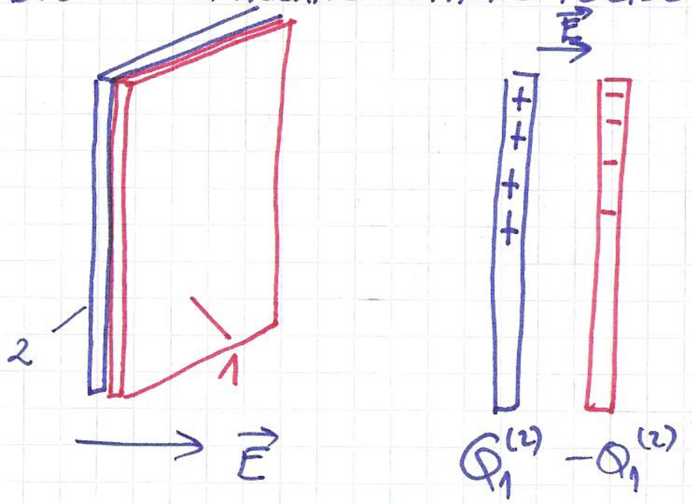
$$Q'_{ind1} = -Q_1'$$

$$Q'_{ind2} = -Q_2'$$

$$E \cdot 2\pi r \cancel{h} = \frac{(Q_1' + Q_2' + \cancel{Q'_{ind1}} + \cancel{Q'_{ind2}} - \cancel{Q'_{ind1}} - \cancel{Q'_{ind2}})\cancel{h}}{\varepsilon_0}$$

$$\vec{E} = \frac{Q'_1 + Q'_2}{2\pi r \cancel{h} \varepsilon_0}\vec{i_r} = \frac{Q_1' + Q_2'}{2\pi r \varepsilon_0}\vec{i_r}$$

110. DVE NAELEKTRISANE METALNE FOLIJE S

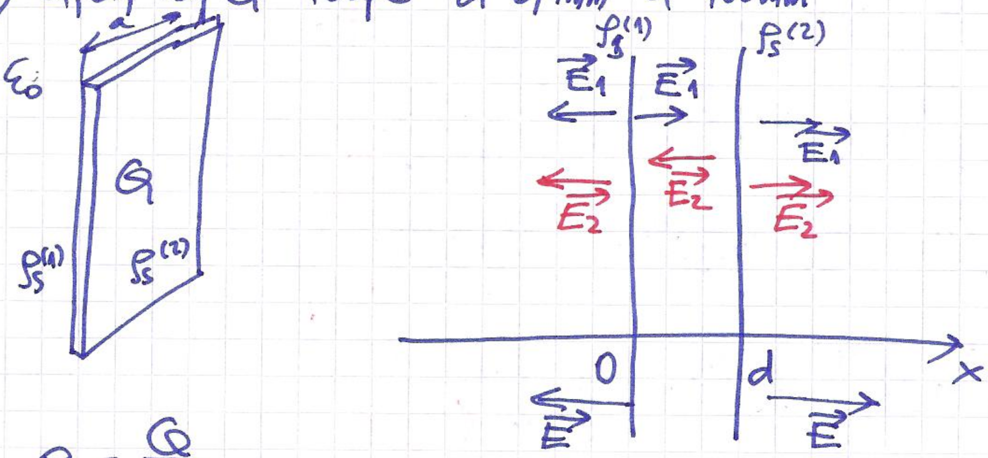


$$ES = \frac{Q_1}{\varepsilon_0}$$

$$Q_1 = ES\varepsilon_0$$

$$Q_2 = -Q_1 = -ES\varepsilon_0$$

*111. $a, d, \varepsilon_0, Q = 100pC \quad d = 0{,}1mm \quad a = 100mm$



$$\rho_s = \frac{Q}{a^2}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} \qquad \rho_s^{(1)} = \rho_s^{(2)} = \frac{\rho_s}{2} = \frac{Q}{2a^2}$$

$$E_1 \cancel{S} = \frac{\rho_s^{(1)} \cancel{S}}{2\varepsilon_0} \Rightarrow \frac{\rho_s^{(1)}}{2\varepsilon_0} = E_1$$

$$E_2 \cancel{S} = \frac{\rho_s^{(2)} \cancel{S}}{2\varepsilon_0} \Rightarrow \frac{\rho_s^{(2)}}{2\varepsilon_0} = E_2$$

$$E_{1x} = \frac{\rho_s^{(1)}}{2\varepsilon_0} \operatorname{sgn} x \qquad \vec{E} = \frac{\rho_s}{4\varepsilon_0}(\operatorname{sgn} x + \operatorname{sgn}(x-d))\vec{i_x}$$

$$E_{2x} = \frac{\rho_s^{(2)}}{2\varepsilon_0} \operatorname{sgn}(x-d)$$

*112. a, d, Q_1, Q_2, d_1

I) KADA SU FOLIJE DALEKO JEDNA OD DRUGE

$$\rho_{s1}^{(0)} = \frac{Q_1}{2a^2} \qquad \rho_{s2}^{(0)} = \frac{Q_2}{2a^2}$$

$$\rho_{s1}^{(1)} + \rho_{s1}^{(2)} = 2\rho_{s1}^{(0)}$$

$$\rho_{s2}^{(1)} + \rho_{s2}^{(2)} = 2\rho_{s2}^{(0)}$$

$$\rho_{s1}^{(1)} + \rho_{s1}^{(2)} - \rho_{s2}^{(1)} - \rho_{s2}^{(2)} = 0$$

$$\rho_{s1}^{(2)} + \rho_{s2}^{(1)} = 0$$

$$\rho_{s1}^{(1)} = \rho_{s2}^{(2)}$$

$$\rho_{s1}^{(1)} = \rho_{s1}^{(0)} + \rho_{s2}^{(0)} = \frac{Q_1+Q_2}{2a^2}$$

$$\rho_{s2}^{(1)} = -\rho_{s1}^{(0)} + \rho_{s2}^{(0)} = \frac{-Q_1+Q_2}{2a^2}$$

$$\rho_{s1}^{(2)} = \rho_{s1}^{(0)} - \rho_{s2}^{(0)} = \frac{Q_1-Q_2}{2a^2}$$

$$\rho_{s2}^{(2)} = \frac{Q_1+Q_2}{2a^2}$$

$$\vec{E} = \begin{cases} \frac{-Q_1-Q_2}{2\varepsilon_0 a^2}\vec{i_x}, & x<0 \\ 0, & 0<x<d \\ \frac{Q_1-Q_2}{2\varepsilon_0 a^2}\vec{i_x}, & d<x<d+d_1 \\ 0, & d+d_1<x<2d+d_1 \\ \frac{Q_1+Q_2}{2\varepsilon_0 a^2}\vec{i_x}, & x>2d+d_1 \end{cases}$$

(*113) TRI METALNE FOLIJE, d, ρ_{S1}, ρ_{S3}; $\rho_{S2}=0$

d_1, d_2

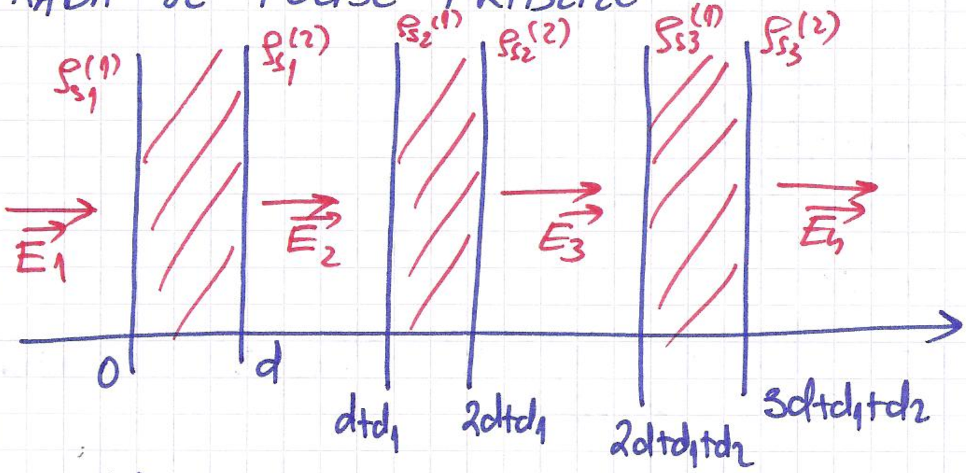
KADA SU FOLIJE DALEKO JEDNA OD DRUGE

$$\rho_{S1}^{(0)}=\frac{\rho_{S1}}{2}$$

$$\rho_{S3}^{(0)}=\frac{\rho_{S3}}{2}$$

$$\rho_{S2}^{(0)}=0$$

KADA SE FOLIJE PRIBLIŽE



$$\rho_{S1}^{(1)}+\rho_{S1}^{(2)}=\rho_{S1}$$

$$\rho_{S2}^{(1)}+\rho_{S2}^{(2)}=0$$

$$\rho_{S3}^{(1)}+\rho_{S3}^{(2)}=\rho_{S3}$$

$$\rho_{S1}^{(1)}-\rho_{S1}^{(2)}-\rho_{S2}^{(1)}-\rho_{S2}^{(2)}-\rho_{S3}^{(1)}-\rho_{S3}^{(2)}=0 \quad I$$

$$\rho_{S1}^{(1)}+\rho_{S1}^{(2)}+\rho_{S2}^{(1)}-\rho_{S2}^{(2)}-\rho_{S3}^{(1)}-\rho_{S3}^{(2)}=0 \quad II$$

$$\rho_{S1}^{(1)}+\rho_{S1}^{(2)}+\rho_{S2}^{(1)}+\rho_{S2}^{(2)}+\rho_{S3}^{(1)}-\rho_{S3}^{(2)}=0 \quad III$$

$$\rho_{S1}^{(1)}=\frac{\rho_{S1}+\rho_{S3}}{2} \qquad \rho_{S2}^{(1)}=\frac{-\rho_{S1}+\rho_{S3}}{2} \qquad \rho_{S3}^{(1)}=\frac{-\rho_{S1}+\rho_{S3}}{2}$$

$$\rho_{S1}^{(2)}=\frac{\rho_{S1}-\rho_{S3}}{2} \qquad \rho_{S2}^{(2)}=\frac{\rho_{S1}-\rho_{S3}}{2} \qquad \rho_{S3}^{(2)}=\frac{\rho_{S1}+\rho_{S3}}{2}$$

$\frac{1}{\varepsilon_0}$ 2A $\vec{E}$

*116. NA SLIČAN NAČIN

$$\rho_{s1} + \rho_{s2}^{(1)} = 0$$

$$\rho_{s2}^{(2)} + \rho_{s3}^{(1)} = 0$$

$$\rho_{s3}^{(2)} + \rho_{s4} = 0$$

$$\rho_{s3}^{(1)} + \rho_{s3}^{(2)} = \frac{Q_3}{S}$$

$$V_2 = \frac{\rho_{s2}^{(1)} d}{\varepsilon_0}$$

$$V_2 = \frac{\rho_{s2}^{(2)} d}{\varepsilon_0} + \frac{\rho_{s3}^{(2)} d}{\varepsilon_0}$$

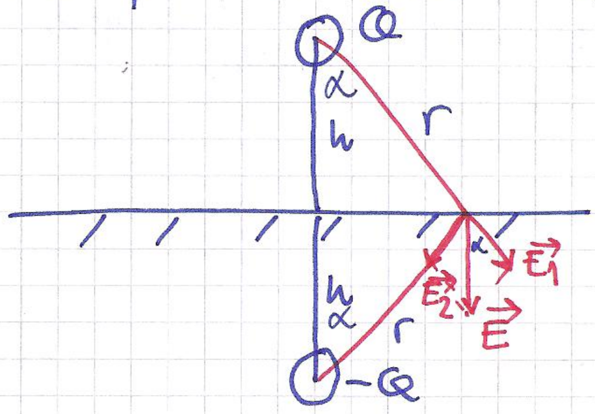
$$\rho_{s3}^{(1)} = \frac{1}{2}\left(\frac{Q_3}{S} - \frac{V_2 \varepsilon_0}{d}\right)$$

$$\rho_{s2}^{(2)} = -\rho_{s3}^{(1)} = \frac{1}{2}\left(\frac{V_2 \varepsilon_0}{d} - \frac{Q_3}{S}\right)$$

$$\rho_{s1} = -\rho_{s2}^{(1)} = -\frac{\varepsilon_0 V_2}{d}$$

TEOREMA LIKOVA

118. $a, Q > 0$, $h \gg a$ a) $E = ?$ b) $\rho_{s ind} = ?$ c) $Q_{ind} = ?$



$$\cos\alpha = \frac{h}{r}$$

$$E_1 = E_2$$

$$E = 2E_1 \cos\alpha = \cancel{2} \frac{Q}{\cancel{2} 4\pi\varepsilon_0 r^2} \cdot \frac{h}{r}$$

$$E = \frac{Qh}{2\pi\varepsilon_0 r^3}$$

$$\vec{E} = \frac{Qh}{2\pi\varepsilon_0 (h^2+x^2)^{3/2}} (-\vec{i_z})$$

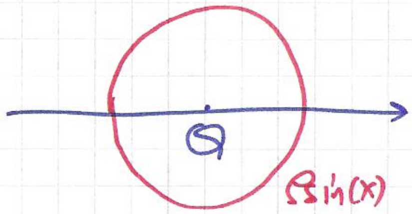
b) $\rho_{s ind} = ?$

$$\vec{n} \cdot \vec{E} = \frac{\rho_{s ind}(x)}{\varepsilon_0} \Rightarrow -E = \frac{\rho_{s ind}}{\varepsilon_0} \Rightarrow \rho_{s ind} = -\varepsilon_0 E$$

$$\rho_{s ind} = -\frac{Qh}{2\pi (h^2+x^2)^{3/2}}$$

$$\rho_{s ind}(x) = -\frac{Qh}{2\pi (h^2+x^2)^{3/2}}$$

c) $Q_{ind} = ?$



$$Q_{ind} = \int \rho_{sind}(x)\, dS \qquad dS = 2\pi x\, dx$$

$$Q_{ind} = -\frac{Qh}{\cancel{2\pi}} \int_{x=0}^{\infty} \frac{\cancel{2\pi} x\, dx}{(h^2+x^2)^{3/2}}$$

$$Q_{ind} = -\frac{Qh}{2} \int_{h^2}^{\infty} \frac{dt}{t^{3/2}} = \cancel{-}\frac{Qh}{\cancel{2}} \frac{t^{-\frac{1}{2}}}{\cancel{-\frac{1}{2}}} \Big|_{h^2}^{+\infty}$$

$$Q_{ind} = Qh \frac{1}{\sqrt{t}} \Big|_{h^2}^{\infty} = -Q\frac{\cancel{h}}{\cancel{h}} = \boxed{-Q}$$

(119) $F = ?$

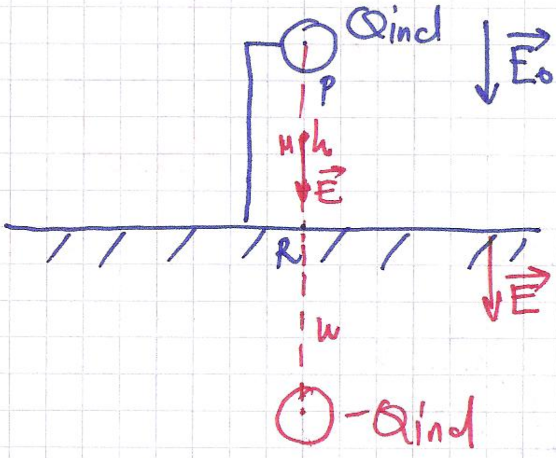
$$F = \frac{Q^2}{4\pi\varepsilon_0 R^2} = \frac{Q^2}{4\pi\varepsilon_0 4h^2} = \frac{Q^2}{16\pi\varepsilon_0 h^2}$$

(120) $a,\ h \gg a,\ V,\ F = ?$

$$V = \frac{Q}{4\pi\varepsilon_0 a} \qquad Q = V \cdot 4\pi\varepsilon_0 a$$

$$F = \frac{V^2 \cancel{16}\pi^{\cancel{2}} \varepsilon_0^{\cancel{2}} a^2}{\cancel{16}\cancel{\pi}\cancel{\varepsilon_0} h^2} = \boxed{V^2 \pi\varepsilon_0 \left(\frac{a}{h}\right)^2}$$

121. a, $h \gg a$



$Q_{ind} = ?$

$$\vec{E} = \vec{E_0} + \vec{E}^{(0)} + \vec{E}^{(1)}$$

$$E(z) = -E_z(z) = E_0 + \frac{Q_{ind}}{4\pi\varepsilon_0}\left(\frac{1}{(h-z)^2} + \frac{1}{(h+z)^2}\right), \; z \in (0, h-a)$$

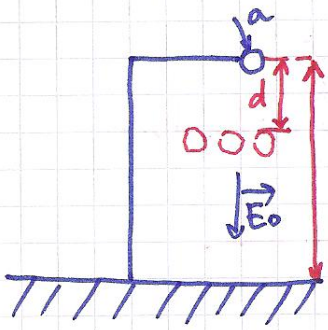
POTENCIJAL MORA BITI JEDNAK NULI JER JE UZEMLJEN

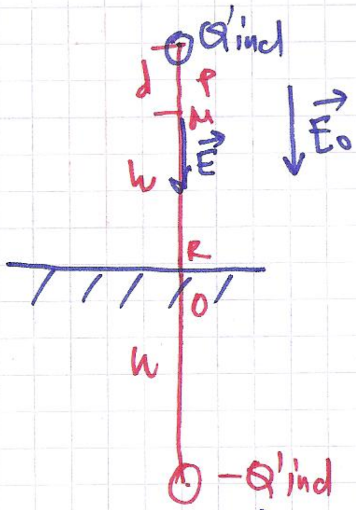
$$\int_P^R \vec{E}\cdot d\vec{\ell} = \int_{\ell=a}^{h} E\,d\ell = -\int_{z=h-a}^{0} E(z)\,dz = \int_{z=0}^{h-a} E(z)\,dz = E_0 z\Big|_0^{h-a} + \frac{Q_{ind}}{4\pi\varepsilon_0}\left(\frac{1}{h-z} - \frac{1}{h+z}\right)\Bigg|_0^{h-a} = 0$$

ZA $h \gg a$

$$\boxed{Q_{ind} = -4\pi\varepsilon_0 a h E_0}$$

*122. $E_0,\ h,\ d,\ a,\ E_0 = 2\,kV/m$





$$E(z) = -E_z(z) = E_0 + \frac{Q'_{ind}}{2\pi\varepsilon_0(h-z)} + \frac{Q'_{ind}}{2\pi\varepsilon_0(h+z)} \quad |\, z \in (0, h-a)$$

USLOV: $\int_P^R \vec{E}\cdot d\vec{l} = 0$

$$Q'_{ind} = -\frac{2\pi\varepsilon_0}{\ln\frac{2h}{a}} E_0 \cdot h$$

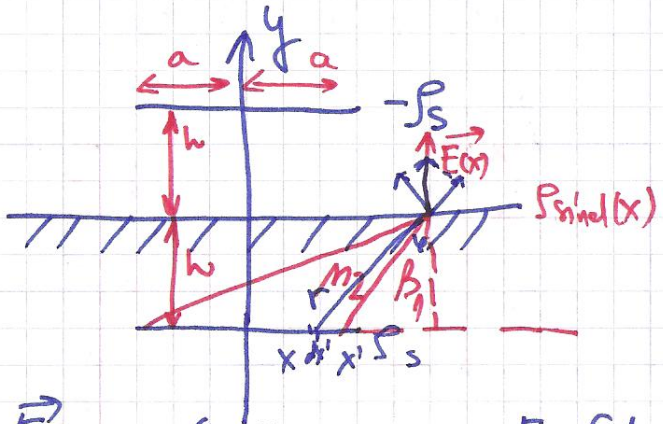
$$E(z) = E_0\left[1 - \frac{h}{\ln\frac{2h}{a}}\left(\frac{1}{h-z} + \frac{1}{h+z}\right)\right], \quad z \in (0, h-a)$$

NA MESTU PROVODNIKA DALEKOVODA:

$$E(h-d) = E_0\left(1 - \frac{2\left(\frac{h}{d}\right)^2}{\left(2\frac{h}{d}-1\right)\ln\frac{2h}{a}}\right)$$

б) $\rho_S = \frac{Q'_{ind}}{2\pi a} = -\varepsilon_0 \frac{h\, E_0}{a \ln\frac{2h}{a}}$

123. $2a, h$



a) $\rho_{sind}(x) = \varepsilon_0 \vec{n} \cdot \vec{E}(x) = -\varepsilon_0 E(x)$

$E = \int dE$

$E = \int 2 dE_1 \cos\varphi$

$$E = \int \frac{2 dQ'}{2\pi\varepsilon_0 r} \frac{x - x'}{r}$$

$dQ' = \frac{dQ}{\ell}$

$\frac{\rho_s dS}{\ell} \Rightarrow \rho_s dx' = dQ$

$$E = \frac{\rho_s}{\pi\varepsilon_0} \int_{-a}^{a} \frac{x - x'}{r^2} dx'$$

$$E = -\frac{\rho_s}{2\pi\varepsilon_0} \int_{h^2+(x+a)^2}^{h^2+(x-a)^2} \frac{dt}{t} \Rightarrow E = \frac{\rho_s}{2\pi\varepsilon_0} \ln\left(\frac{h^2+(x+a)^2}{h^2+(x-a)^2}\right)$$

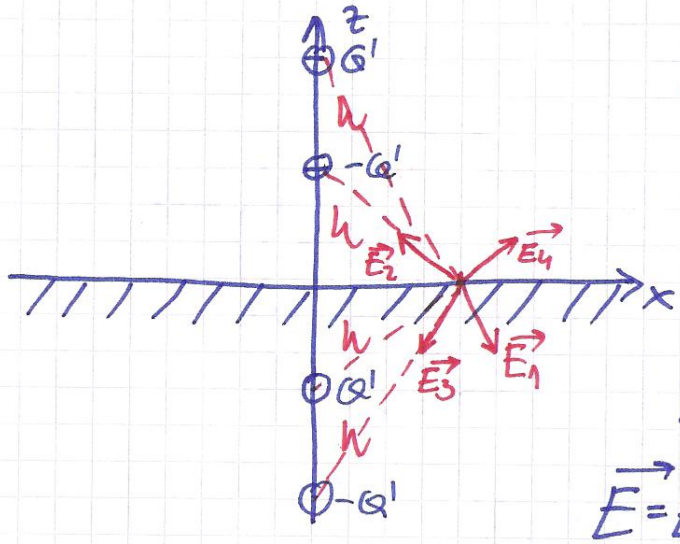
$\Downarrow$

$$\rho_{sind} = -\frac{\rho_s}{2\pi} \ln\left(\frac{h^2+(x+a)^2}{h^2+(x-a)^2}\right)$$

(124.) $a, h \;, h \gg a \;, Q' \;, -Q'$

$|\vec{E_1}| = |\vec{E_3}|$

$|\vec{E_2}| = |\vec{E_4}|$



$E(x)=0$

$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \vec{E_4} = 0$

$$\frac{Q'h}{2\pi\varepsilon_0 r_1} \frac{h}{r_1} = \frac{Q'h}{2\pi\varepsilon_0 r_2} \cdot \frac{2h}{r_2}$$

$$\frac{1}{r_1^2} = \frac{2}{r_2^2}$$

$$r_2^2 = 2r_1^2$$

$$r_2 = x^2 + 4h^2 \;, \; r_1 = x^2 + h^2$$

$$x^2 + 4h^2 = 2x^2 + 2h^2$$

$$x^2 = 2h^2$$

$$x = \pm\sqrt{2}\,h$$

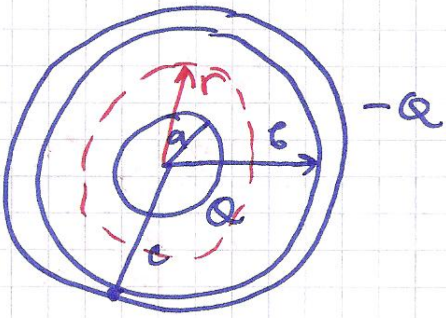
$$\rho_{sind} = \varepsilon_0 \vec{n} \cdot \vec{E} = \frac{Q'}{\pi}\left(\frac{h}{x^2+h^2} - \frac{2h}{x^2+(2h)^2}\right)$$

$\rho_{sin} \to 0 \quad x \to +\infty$

KAPACITIVNOSTI KONDENZATORA

125. $a, b, c, E_{kr}, C = ?$

$$C = \frac{Q}{U}$$



a) $$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}, \quad r \in (a, b)$$

$$U_{AB} = \int_a^b E(r)dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\varepsilon_0} \frac{b-a}{ab}$$

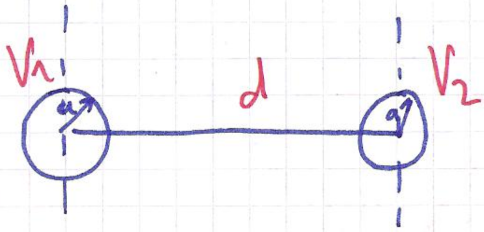
$$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

b) $$E_{MAX} = E(a) = \frac{Q}{4\pi\varepsilon_0 a^2}$$

$$Q_{MAX} = 4\pi\varepsilon_0 a^2 E_{MAX}$$

$$U_{KR} = \frac{Q_{MAX}}{C} = \frac{E_{kr} a(b-a)}{b}$$

(126.) a, d $\qquad C = ?$

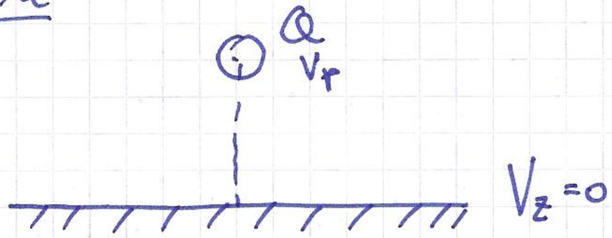


$$V_1 = \frac{Q}{4\pi\varepsilon_0 a} \qquad V_2 = -\frac{Q}{4\pi\varepsilon_0 a}$$

$$U_{12} = V_1 - V_2 = \frac{Q}{2\pi\varepsilon_0 a}$$

$$\boxed{C = \frac{Q}{U_{12}} = 2\pi\varepsilon_0 a}$$

(127.) $\underline{h \gg a}$



$$C = \frac{Q}{V_p}\ , \quad V_p = \frac{Q}{4\pi\varepsilon_0 a} \Rightarrow \boxed{C = 4\pi\varepsilon_0 a}$$

(128.)

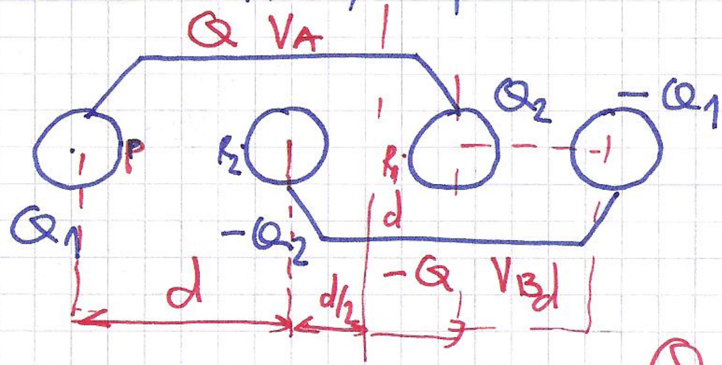
$\vec{E_0}\ , \quad U_0 = E_0 h$

$Q_{ind} = -C_K U_0$

$C_K = 4\pi\varepsilon_0 a$

$Q_{ind} = -4\pi\varepsilon_0 a E_0 h$

129. 4 PROVODNE LOPTE, a, $d \gg a$



$U_{13} = U_{24} = 0$

$Q_1 + Q_2 = Q$

$U_{AB} = V_A - V_B$

$U_{13} = V_P|_{R1}$

$$V_P = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_P} - \frac{1}{r_{R_1}}\right)$$

$$V_P|_{R1} = \frac{1}{4\pi\varepsilon_0}\left(Q_1\left(\frac{1}{a} - \frac{1}{2d-a}\right) - Q_2\cancelto{0}{\left(\frac{1}{d-a} - \frac{1}{d-a}\right)} + Q_2\left(\frac{1}{2d-a} - \frac{1}{a}\right)\right.$$
$$\left. - Q_1\left(\frac{1}{3d-a} - \frac{1}{d+a}\right)\right) \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{a} - \frac{Q_2}{a}\right) = 0$$

$$\Downarrow$$

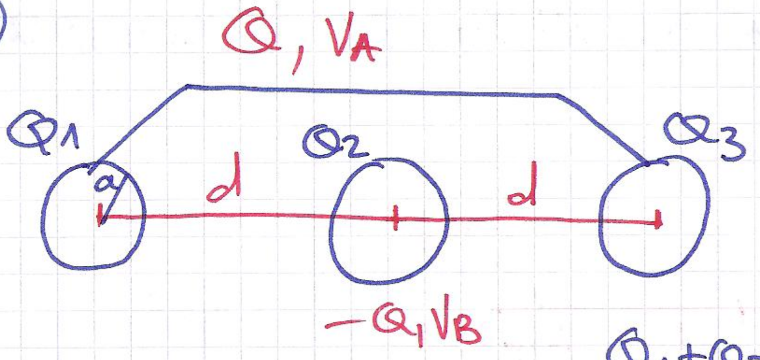
$$\boxed{Q_1 = Q_2}$$

$$V_{12} = V_P|_{R2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{a} + \frac{Q_2}{a}\right) = \frac{Q}{4\pi\varepsilon_0 a}$$

$$\Downarrow$$

$$\boxed{C = 4\pi\varepsilon_0 a}$$

130.



$$Q_1+Q_3=Q=-Q_2$$

$$Q_2=-Q$$

$$Q_1=Q_3=\frac{Q}{2}=-\frac{Q_2}{2}$$

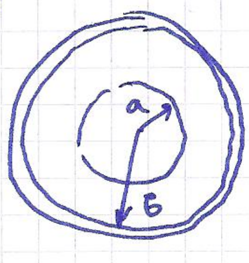
$$V_1=V_A=\frac{Q_1}{4\pi\varepsilon_0 a}=\frac{Q}{8\pi\varepsilon_0 a}$$

$$V_2=V_B=-\frac{Q}{4\pi\varepsilon_0 a}$$

$$U_{12}=V_1-V_2=\frac{Q}{8\pi\varepsilon_0 a}+\frac{Q}{4\pi\varepsilon_0 a}=\frac{3Q}{8\pi\varepsilon_0 a}$$

$$C=\frac{Q}{U_{12}}=\boxed{\frac{8}{3}\pi\varepsilon_0 a}$$

132.



$E_{MAX}, b, a=?$ $\quad C'=?$

$$Q'_{MAX}=2\pi\varepsilon_0 a E_{MAX}$$

JER JE $E_{MAX}=\frac{Q'_{MAX}}{2\pi\varepsilon_0 a}$

$$E_{MAX}=E(a)$$

$$E=\frac{Q'}{2\pi\varepsilon_0 r}$$

$$E=\frac{a}{r}E_{MAX}$$

$$U=\int_a^b E(r)\,dr$$

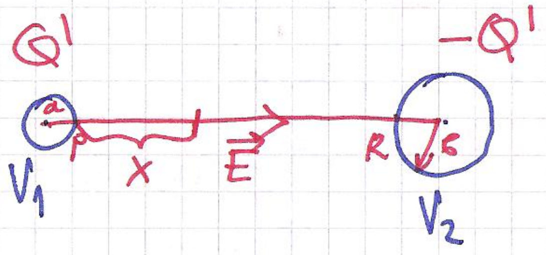
$$U=\int_a^b \frac{aE_{MAX}}{r}dr=aE_{MAX}\ln\frac{b}{a}$$

$\rightarrow$ ODAVDE $a=\ldots$

$$U=\cancel{a}\cdot\frac{Q'_{MAX}}{2\pi\varepsilon_0 \cancel{a}}\ln\frac{b}{a}$$

$$C'=\frac{Q'_{MAX}}{U}=\boxed{\frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}}$$

133. IZVESTI IZRAZ ZA PODUŽNU KAPACITIVNOST NESIMETRIČNOG TANKOG VAZDUŠNOG VODA



$V_1 = V_P$

$V_2 = V_R$

$V_1 = V_P \, , V_2 = V_R$

$$E = \frac{Q'}{2\pi\varepsilon_0 r}$$

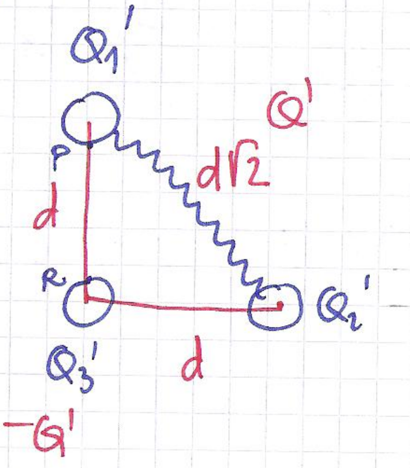
$$U = \frac{Q'}{2\pi\varepsilon_0}\left(\int_a^{d-b}\frac{dx}{x} + \int_a^{d-b}\frac{dx}{d-x}\right)$$

$$U = \frac{Q'}{2\pi\varepsilon_0}\left(\ln|x|\Big|_a^{d-b} - \ln|d-x|\Big|_a^{d-b}\right)$$

$$U = \frac{Q'}{2\pi\varepsilon_0}\left(\ln\left(\frac{d-b}{a}\right) - \ln\left(\frac{\cancel{d}-\cancel{d}+b}{d-a}\right)\right) \approx \boxed{\frac{Q'}{2\pi\varepsilon_0}\ln\frac{d^2}{ab}}$$

$$C' = \frac{Q'}{U} = \frac{2\pi\varepsilon_0}{\ln\frac{d^2}{ab}} = \boxed{\frac{\pi\varepsilon_0}{\ln\frac{d}{\sqrt{ab}}}}$$

(136.)



$$Q' = Q_1' + Q_2'$$

$$Q_3' = -Q'$$

$$Q_1' = Q_2' = \frac{Q'}{2}$$

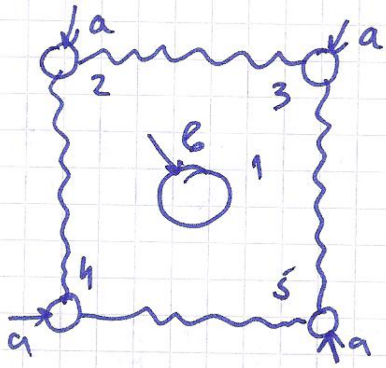
$d \gg a$

$$U_{13} = V_P|_R = \frac{Q_1'}{2\pi\varepsilon_0}\ln\frac{d}{a} + \frac{Q_2'}{2\pi\varepsilon_0}\ln\frac{d}{d\sqrt{2}} - \frac{Q'}{2\pi\varepsilon_0}\ln\frac{a}{d}$$

$$U_{13} = \frac{Q'}{4\pi\varepsilon_0}\ln\frac{d^3}{a^3\sqrt{2}}$$

$$C' = \frac{Q'}{U_{13}} = \frac{4\pi\varepsilon_0}{\ln\frac{d^3}{a^3\sqrt{2}}}$$

(140.)



$C' = ?$

$-\frac{Q'}{4} \rightarrow$ JEDAN a-VOD

$$E = \frac{Q_k'}{2\pi\varepsilon_0 r_k}$$

$$U = V_1 - V_2$$

$$V_2 = -\frac{Q'}{8\pi\varepsilon_0}\ln\frac{r_0}{a} - 2\frac{Q'}{8\pi\varepsilon_0}\ln\frac{r_0}{d-a} - \frac{Q'}{8\pi\varepsilon_0}\ln\frac{r_0}{d\sqrt{2}-a}$$

$$+ \frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{\frac{d\sqrt{2}}{2}-a}$$

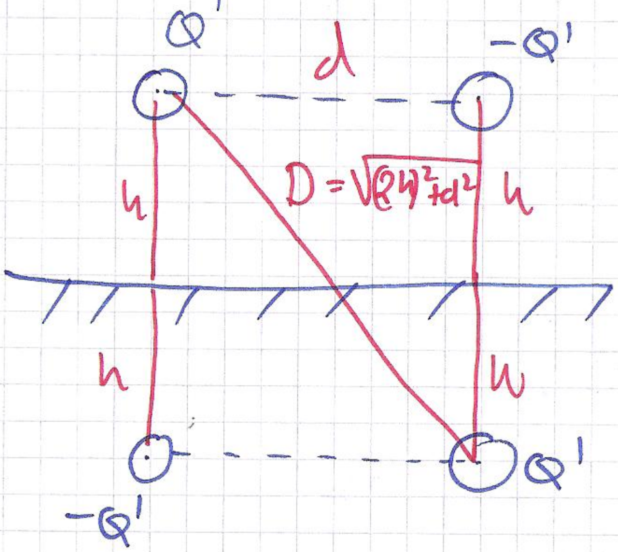
$$V_2 = \frac{Q'}{2\pi\varepsilon_0}\left(\ln\frac{r_0}{\frac{d\sqrt{2}}{2}-a} - \frac{1}{2}\ln\frac{r_0}{d-a} - \frac{1}{4}\ln\frac{r_0}{d\sqrt{2}-a} - \frac{1}{4}\ln\frac{r_0}{a}\right)$$

$$V_1 = \frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{b} - 4\cdot\frac{\frac{Q'}{4}}{2\pi\varepsilon_0}\ln\frac{r_0}{\frac{d\sqrt{2}}{2}-b} = \frac{Q'}{2\pi\varepsilon_0}\ln\frac{d\sqrt{2}-b}{b}$$

$$U = \frac{Q'}{2\pi\varepsilon_0}\left(\ln\frac{d\sqrt{2}}{2b} + \frac{1}{4}\ln\frac{d\sqrt{2}}{2a} + \frac{2}{4}\ln\frac{1}{\sqrt{2}} + \frac{1}{4}\ln\frac{1}{2}\right)$$

$$\boxed{C' = \frac{Q'}{U} = \frac{2\pi\varepsilon_0}{\ln\frac{d}{2b} + \frac{1}{4}\ln\frac{d\sqrt{2}}{2a}}}$$

141.



$$U = V_1 - V_2$$

$$U = \underbrace{\frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{a} - \frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{d} + \frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{D} - \frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{2h}}_{V_1} -$$

$$-\underbrace{\left(\frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{d} - \frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{a} - \frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{D} + \frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_0}{2h}\right)}_{V_2}$$

$$U = \frac{Q'}{2\pi\varepsilon_0}\ln\frac{d\cdot 2h\cdot d\cdot 2h}{a\cdot D\cdot a\cdot D} = \frac{Q'}{\pi\varepsilon_0}\ln\frac{2dh}{aD}$$

$$C' = \frac{Q'}{U} = \frac{\pi\varepsilon_0}{\ln\frac{2dh}{aD}}$$

– POLJE U PRISUSTVU DIELEKTRIKA

147.

$$\oint_C \vec{E}\cdot d\vec{l} = 0 \qquad \vec{D} = \varepsilon\vec{E}$$

$$\oint_S \vec{D}\,d\vec{S} = Q \qquad \oint_S \varepsilon\vec{E}\,d\vec{S} = Q$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \qquad \Downarrow$$

$$\vec{P} = \vec{P}(\vec{E}) \qquad \oint_S \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon}$$

$$\vec{D} = \vec{D}(\vec{E}) \qquad \oint \vec{E}\,d\vec{S} = \frac{Q}{\varepsilon_0} \quad \text{U VAKUMU}$$

148.

$$Q_p = -\oint \vec{P}\,d\vec{S}$$

$$Q_p = -\vec{P}\oint_S d\vec{S} = 0 \Rightarrow Q_p = 0$$

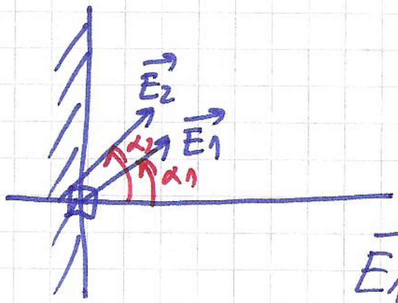
b) DIELEKTRIK LINEARAN

$$\vec{P} = (\varepsilon - \varepsilon_0)\vec{E} = \frac{\varepsilon - \varepsilon_0}{\varepsilon}\vec{D}$$

$$\rho_p = -\frac{(\varepsilon - \varepsilon_0)}{\varepsilon}\rho$$

$$Q_p = -\oint_S \frac{\varepsilon - \varepsilon_0}{\varepsilon}\vec{D}\,d\vec{S} = -\frac{\varepsilon - \varepsilon_0}{\varepsilon}Q$$

149. $\varepsilon_r = 6$, $\alpha_1 = \pi/6$ $E_1 = 2\,kV/m$



$$\vec{E_1} = E_{1x}\vec{i_x} + E_{1z}\vec{i_z} = E_1\cos\alpha_1\vec{i_x} + E_1\sin\alpha_1\vec{i_z}$$

$$\left.\begin{aligned} E_{1t} &= E_{2t} \\ D_{1n} &= D_{2n} \end{aligned}\right\} \text{GRANIČNI USLOVI}$$

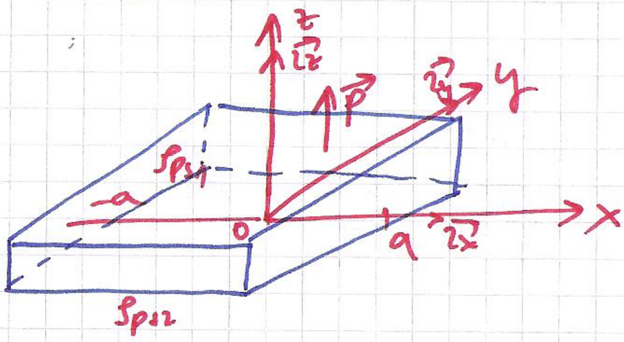
$$E_{2t} = E_{1t} = E_{1z} = 1\frac{kV}{m}$$

$$D_{1n} = D_{1x} = \varepsilon_0 E_{1x} \qquad D_{2x} = \varepsilon_0\varepsilon_r E_{2x}$$

$$\cancel{\varepsilon_0} E_{1x} = \cancel{\varepsilon_0}\varepsilon_r E_{2x} \Rightarrow E_{2x} = \frac{E_{1x}}{\varepsilon_r}$$

$$\vec{E_2} = E_{2x}\vec{i_x} + E_{2z}\vec{i_z}$$

154.



N', $\vec{p} = p\vec{i_z}$, $2a$, h, $M(0,0,z)$ $0<z<h$

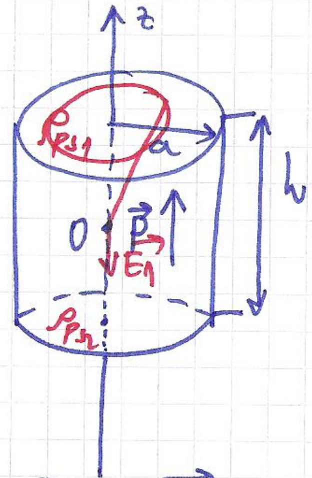
$$\vec{P} = \frac{(\Sigma\vec{p})_{u\Delta v}}{\Delta v} = N'\vec{p} = N'p\vec{i_z} = P\vec{i_z}$$

$$\rho_{ps} = \vec{P}\cdot\vec{n_0} \Rightarrow \rho_{ps1} = P \quad \rho_{ps2} = -P = -\rho_{ps1}$$

$$\vec{E} = E_z\vec{i_z} = \frac{1}{\varepsilon-\varepsilon_0}\vec{P} = -\frac{P}{2\pi\varepsilon_0}\int_{-a}^{a}\left(\frac{z}{x^2+z^2} + \frac{h-z}{x^2+(h-z)^2}\right)dx\ \vec{i_z}$$

$$\vec{E} = \frac{-N'\vec{p}}{\pi\varepsilon_0}\left(\operatorname{arctg}\frac{a}{z} + \operatorname{arctg}\frac{a}{h-z}\right)$$

155.



$a, h, \vec{P}$

$\vec{E}, \vec{D} = ?$ u O

$\rho_{ps1} = P$

$\rho_{ps2} = -P$

SUPERPOZICIJOM POLJA VEZANIH NAELEKTRISANJA

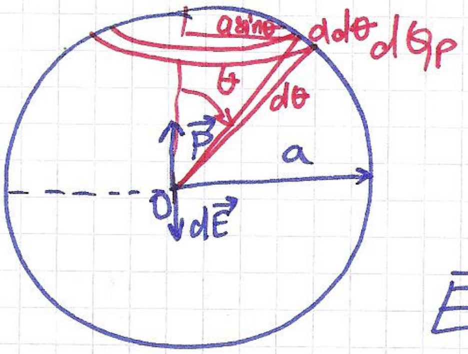
$$\vec{E} = -\frac{\vec{P}}{\varepsilon_0}\left(1 - \frac{h}{\sqrt{4a^2+h^2}}\right)$$

$$\vec{D} = \vec{P} \cdot \frac{h}{\sqrt{4a^2+h^2}}$$

156.

$\vec{E} = ?$
$\vec{D} = ?$ U CENTRU

$\vec{P} = const \Rightarrow \rho_p = 0$



$\rho_{ps} = \vec{n} \cdot \vec{P}$

$\rho_{ps} = P\cos\theta$

$$\vec{E} = \int d\vec{E} = \int \frac{dQ_p \, d\cos\theta}{4\pi\varepsilon_0 a^{\cancel{3}2}}(-\vec{i_z})$$

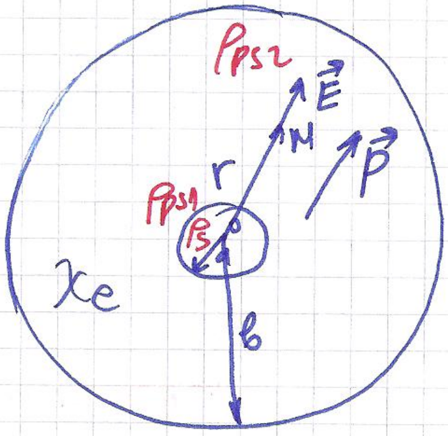
$$\vec{E} = -\int \frac{P\cos\theta \, \cancel{2\pi a^2} \sin\theta \, d\theta \, \cos\theta}{2\cancel{4\pi}\varepsilon_0 \cancel{a^2}} \vec{i_z} = -\int \frac{\rho_{ps} \, dS \cos\theta}{4\pi\varepsilon_0 a^2} \vec{i_z}$$

$$\vec{E} = -\frac{\vec{P}}{2\varepsilon_0} \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta = \left\{ \begin{array}{l} t = \cos\theta \\ dt = -\sin\theta \, d\theta \end{array} \right.$$

$$\vec{E} = \frac{\vec{P}}{2\varepsilon_0} \int_1^{-1} t^2 dt = -\frac{\vec{P}}{2\varepsilon_0} \int_{-1}^{1} t^2 dt = -\frac{\vec{P}}{2\varepsilon_0} \frac{t^3}{3} \Big|_{-1}^{1}$$

$$\vec{E} = -\frac{\vec{P}}{3\varepsilon_0} \qquad \vec{D} = \varepsilon_0 \vec{E} + \vec{P} = -\frac{\vec{P}}{3} + \vec{P} = \frac{2}{3}\vec{P}$$

160.



$$\rho_s = \frac{Q}{4\pi a^2}$$

$$\vec{E} = E \cdot \vec{i_r}$$

$$\vec{P}(r) = P(r)\,\vec{i_r}, \quad r \in (a, b)$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

$$\rho_{ps1} = -P(a^+)$$

$$\rho_{ps2} = P(b^-)$$

$$Q_{p2} = 4\pi b^2 \rho_{ps2}$$

$$Q_{p1} = 4\pi a^2 \rho_{ps1}$$

$$\vec{E} = \begin{cases} 0, & r < a \\ \frac{Q}{4\pi\varepsilon_0 r^2}\vec{i_r}, & r > a \end{cases}$$

$$Q = 4\pi a^2 \rho_s$$

$$E(r) = E_Q + E_{Q_{p1}} = \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{Q_{p1}}{4\pi\varepsilon_0 r^2} = E(a^+)\frac{a^2}{r^2}$$

$$\rho_{ps1} = -\varepsilon_0 \chi_e E(a^+)$$

$$Q_{p1} = -4\pi a^2 \varepsilon_0 \chi_e E(a^+)$$

$$E(a^+)\frac{a^2}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2} - \frac{\chi_e E(a^+) a^2}{r^2}$$

$$E(a^+) = \frac{Q}{4\pi\varepsilon_0 (1+\chi_e) a^2}$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0 (1+\chi_e) r^2}, \quad r \in (a, b)$$

$\varepsilon_r = 1 + \chi_e$

$$E(r) = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2} = \frac{Q}{4\pi\varepsilon r^2}, \quad \varepsilon = \varepsilon_0\varepsilon_r$$

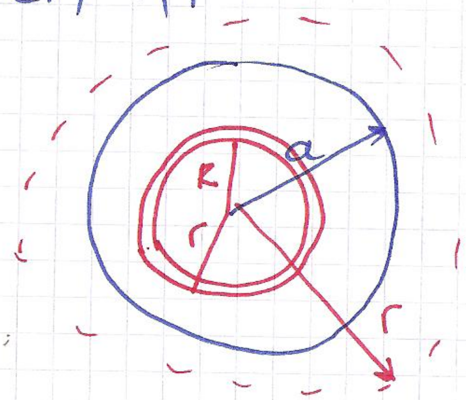
ZA $r > b$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{Q_{p1}}{4\pi\varepsilon_0 r^2} + \frac{Q_{p2}}{4\pi\varepsilon_0 r^2} = \frac{Q + Q_{p1} + Q_{p2}}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$Q_{p1} + Q_{p2} = 0$$

161. LINEARNI HOMOGENI DIELEKTRIK

ε_r, a, ρ



$$\oint \vec{D} \cdot d\vec{S} = Q$$

$$D\cancel{4\pi}r^2 = \frac{\rho\cancel{4\pi}a^3}{3}$$

$$D = \frac{\rho a^3}{3r^2}, \quad r > a$$

$r < a$

$$D\cancel{4\pi}\cancel{r^2} = \frac{\rho\cancel{4\pi}r^{\cancel{2}}}{3}$$

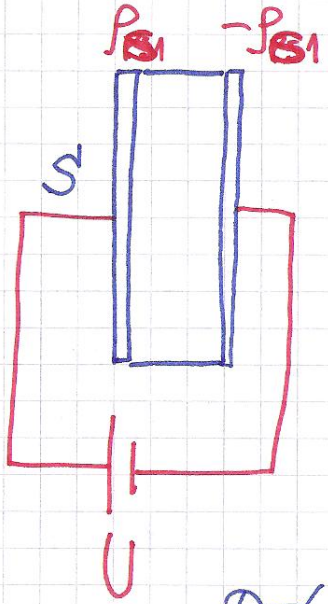
$$D = \frac{\rho r^{\cancel{3}}}{3} = \frac{\rho r}{3}, \quad r < a$$

$$E = \frac{D}{\varepsilon} \longrightarrow \frac{\rho r}{3\varepsilon_r\varepsilon_0}, \quad \underline{r < a}$$

$$E = \frac{D}{\varepsilon_0} = \frac{\rho a^3}{3r^2\varepsilon_0}, \quad \underline{r > a}$$

(SAMO VAZDUH)

162.



ΔQ

$$Q_1 = \rho_{S1} \cdot S$$

$$Q_2 = \rho_{S2} \cdot S$$

$$\Delta Q = Q_2 - Q_1 = (\rho_{S2} - \rho_{S1}) S$$

$$D \cancel{S} = \rho_S \cancel{S}$$

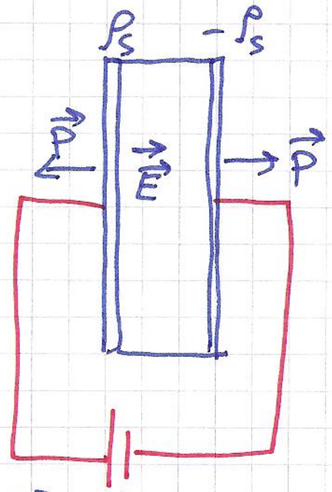
$$D = \rho_S \qquad \Delta \rho_S = \rho_{S2} - \rho_{S1} = D_2 - D_1$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \Delta \rho_S = \cancel{\varepsilon_0 E} - \cancel{E \varepsilon_0} - P = -P$$

$$E = \frac{U}{d} \quad \text{(STALAN NAPON)}$$

$$P = -\frac{\Delta Q}{S}$$

163. PLOČASTI KONDENZATOR, d



$\Delta U, P = ?$

$$D \cancel{S} = \rho_S \cancel{S}$$

$$D = \rho_S$$

$$\Delta U = d \cdot \Delta E$$

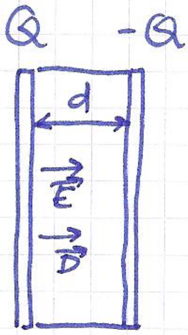
$$\Delta U = d(E_2 - E_1)$$

$$\Delta U = d\left(\frac{D}{\varepsilon_0} - \frac{D-P}{\varepsilon_0}\right) = d\frac{P}{\varepsilon_0}$$

$$\vec{D} = \varepsilon_0 \vec{E_1} + \vec{P}$$

$$\vec{E_1} = \frac{\vec{D} - \vec{P}}{\varepsilon_0} \qquad \vec{E_2} = \frac{\vec{D}}{\varepsilon_0}$$

164.



$d, \varepsilon_r, \varepsilon_0, \Delta U$

$$D\cancel{S} = \rho_{s1}\cancel{S} = -\rho_{s2}\cancel{S}$$

$$D = \rho_{s1} = -\rho_{s2}$$

$$E_0 = \frac{U_0}{d} \qquad D = \varepsilon_0 E_0$$

$$E = \frac{U}{d} \qquad D = \varepsilon_0 \varepsilon_r E$$

$$\Delta U = U - U_0 = Ed - E_0 d = \frac{D}{\varepsilon_0\varepsilon_r}d - \frac{D}{\varepsilon_0}d$$

$$D = -\varepsilon_0\left(\frac{\varepsilon_r}{\varepsilon_r - 1}\right)\frac{\Delta U}{d} = \rho_{s1} = -\rho_{s2}$$

b) $\vec{D}, \vec{P}, \vec{E}$ KOLINEARNI

$$D = \varepsilon_0 E + P$$

$$P = D - \varepsilon_0 E = \varepsilon_0 E_0 - \varepsilon_0 E = -\varepsilon_0 \frac{\Delta U}{d}$$

$$\rho_{ps1} = -P \quad , \rho_{ps2} = P$$

(165.) $a, b, d, \varepsilon_r, E_{kr}, C, Q, E_{kro}$

$$C = \frac{\varepsilon S}{d} = \frac{\varepsilon_r \varepsilon_0 S}{d} = \frac{\varepsilon_r \varepsilon_0 ab}{d}$$

$$E = \frac{Q}{\varepsilon_r \varepsilon_0 S}$$

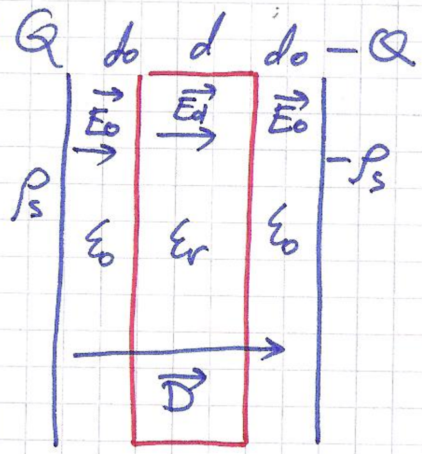
$$\varepsilon_0 E ab\left(\frac{x}{a} + \varepsilon_r\left(1-\frac{x}{a}\right)\right) = Q$$

$$E = \frac{Q}{\varepsilon_0 ab\left(\frac{x}{a} + \varepsilon_r\left(1-\frac{x}{a}\right)\right)} \qquad E_{kr} > E_{kro}$$

$$E < E_{ko}$$

$$\frac{x}{a} = \frac{\varepsilon_r - \frac{Q}{\varepsilon_0 E_{ko} ab}}{\varepsilon_r - 1}$$

(166.)



E_{kr}, d_0, E_{kro}

$U_{MAX} = ?$

$$\rho_s = \frac{Q}{S} \qquad -\rho_s = -\frac{Q}{S}$$

$$\oint \vec{D} \cdot d\vec{S} = Q$$

$$DS = Q \Rightarrow D = \frac{Q}{S} = \rho_s$$

$$E_0 = \frac{D}{\varepsilon_0} = \frac{\rho_s}{\varepsilon_0}$$

$$E_d = \frac{D}{\varepsilon_r \varepsilon_0} = \frac{\rho_s}{\varepsilon_r \varepsilon_0}$$

$$U_{12} = 2E_0 d_0 + E_d \cdot d = 2\frac{\rho_s}{\varepsilon_0} d + \frac{\rho_s}{\varepsilon_r \varepsilon_0} d$$

$$U_{12} = \frac{\rho_s}{\varepsilon_0} d\left(2d_0 + \frac{d}{\varepsilon_r}\right) = \frac{Q}{\varepsilon_0 S} d\left(2d_0 + \frac{d}{\varepsilon_r}\right)$$

$$\boxed{C = \frac{Q}{U} = \frac{\varepsilon_0 S}{d\left(2d_0 + \frac{d}{\varepsilon_r}\right)}}$$

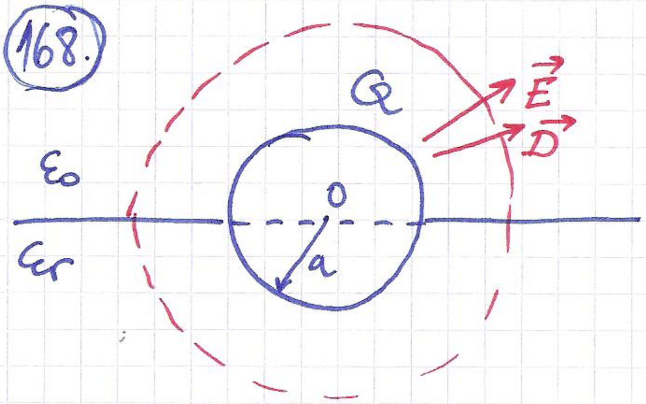
$$\varepsilon_0 E_0 = \varepsilon_0 \varepsilon_r E_d \quad , E_0 = \varepsilon_r E_d$$

$$E_d = \frac{E_{kr0}}{\varepsilon_r} = 7{,}5 \frac{kV}{cm} < 500 \frac{kV}{cm}$$ NIJE DOŠLO DO PROBOJA U DIELEKTRIKU

- PRVO DOLAZI DO PROBOJA U VAZDUHU

$$U_{IZMAX} = 2E_{kr0} d_0 + \frac{E_{kr0}}{\varepsilon_r} d = 6{,}75\,kV$$

168.



$$D_0 = \varepsilon_0 E$$

$$D_1 = \varepsilon_0 \varepsilon_r E$$

$$\oint \vec{D} d\vec{S} = Q$$

$$D_1 S + D_0 S = Q$$

$S = 2\pi r^2$

POLOVINA LOPTE

$$2\pi r^2 (\varepsilon_0 E(r) + \varepsilon_0 \varepsilon_r E(r)) = Q$$

$$\varepsilon_0 E(r)(1+\varepsilon_r) 2\pi r^2 = Q$$

$$E(r) = \frac{Q}{2\pi\varepsilon_0(1+\varepsilon_r) r^2} \quad , r > a$$

$$V = \int_a^\infty E(r) dr = \int_a^\infty \frac{Q}{2\pi\varepsilon_0(1+\varepsilon_r) r^2} dr = \frac{Q}{2\pi\varepsilon_0(1+\varepsilon_r) a}$$

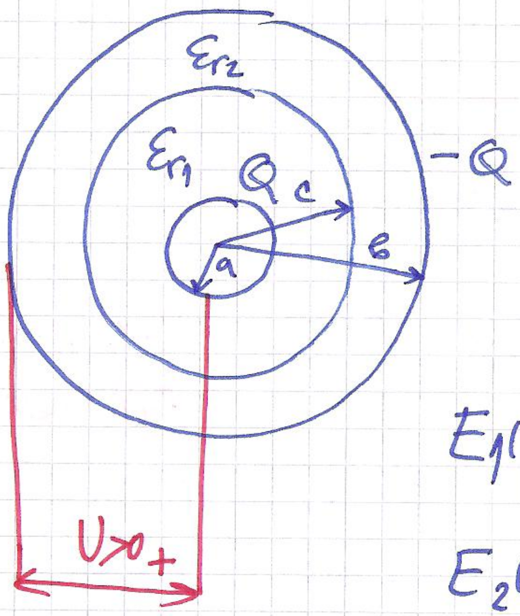
$$C = \frac{Q}{V} = 2\pi\varepsilon_0(1+\varepsilon_r) a$$

$$Q = CV = 2\pi\varepsilon_0(1+\varepsilon_r) a \cdot V$$

$$V_1 = \frac{Q}{4\pi\varepsilon_0 a}$$ KAD JE IZNAD DIELEKTRIKA

$$V_1 = \frac{\cancel{2\pi\varepsilon_0}(1+\varepsilon_r)\cancel{a} \cdot V}{2 \cancel{4\pi\varepsilon_0} \cancel{a}} = \boxed{\frac{1+\varepsilon_r}{2} V}$$

170. SFERNI KONDENZATOR



D(r) ISTO U SVIM DELOVIMA

$$D(r)\cdot S = Q \Rightarrow D(r)=\frac{Q}{4\pi r^2}$$

$$E_1(r)=\frac{D(r)}{\varepsilon_0\varepsilon_{r1}},\ r\in(a,c)$$

$$E_2(r)=\frac{D(r)}{\varepsilon_0\varepsilon_{r2}},\ r\in(c,b)$$

$$U_{12}=\int_a^b E(r)dr=\int_a^c E_1(r)dr+\int_c^b E_2(r)dr$$

$$U_{12}=\int_a^c\frac{Q}{4\pi r^2\varepsilon_0\varepsilon_{r1}}dr+\int_c^b\frac{Q}{4\pi r^2\varepsilon_0\varepsilon_{r2}}dr=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_{r1}}\left(\frac{1}{a}-\frac{1}{c}\right)+\frac{1}{\varepsilon_{r2}}\left(\frac{1}{c}-\frac{1}{b}\right)\right)$$

KAD DIELEKTRIK ε_{r2} ISCURI

$$U_{12}^{(2)}=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_{r1}}\frac{c-a}{ac}+\frac{b-c}{bc}\right)$$

$$\Delta U_{12}=U_{12}^{(2)}-U_{12}=\frac{\varepsilon_{r2}-1}{\varepsilon_{r2}}\frac{Q}{4\pi\varepsilon_0}\frac{(b-c)}{bc}\Rightarrow C=\frac{4\pi\varepsilon_0\varepsilon_{r2}bc}{(\varepsilon_{r2}-1)(b-c)}$$

$$Q=\Delta U_{12}\cdot C=\Delta U_{12}\frac{4\pi\varepsilon_0\varepsilon_{r2}bc}{(\varepsilon_{r2}-1)(b-c)}$$

$$P_2=(\varepsilon_2-\varepsilon_0)E_2=\frac{\varepsilon_{r2}-1}{\varepsilon_{r2}}\frac{Q}{4\pi r^2}=P_2(c)\frac{c^2}{r^2},\ r\in(c,b)$$

$$\rho_{ps2c}=-P_2(c)=-\frac{b\varepsilon_0}{c(b-c)}\Delta U_{12}\qquad \rho_{ps2b}=P_2(c)\frac{c^2}{b^2}$$

171. $a, b, c, U, \varepsilon_{r1}, \varepsilon_{r2}$

$4E_1^{(1)} = E_1^{(2)}$

$E_2^{(1)} = 3E_2^{(2)}$

$$E_1^{(1)} = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_{r1}r^2}, \quad r\in(a,b)$$

$$E_2^{(1)} = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_{r2}r^2}, \quad r\in(b,c)$$

NAPON JE STALAN

$$E_1^{(2)} = \frac{Q_2}{4\pi\varepsilon_0 r^2}, \quad r\in(a,b)$$

$$E_2^{(2)} = \frac{Q_2}{4\pi\varepsilon_0 r^2\varepsilon_{r2}}, \quad r\in(b,c)$$

$$\frac{Q_1}{\cancel{4\pi\varepsilon_0}\varepsilon_{r1}\cancel{r^2}} = \frac{Q_2}{16\cancel{\pi\varepsilon_0 r^2}} \Rightarrow \frac{Q_1}{\varepsilon_{r1}} = \frac{Q_2}{4}$$

$$\frac{Q_1}{\cancel{4\pi\varepsilon_0\varepsilon_{r}r^2}} = \frac{3Q_2}{\cancel{4\pi\varepsilon_0 r^2\varepsilon_{r2}}}$$

$$Q_1 = 3Q_2$$

$$\frac{Q_2}{Q_1} = \frac{1}{3}$$

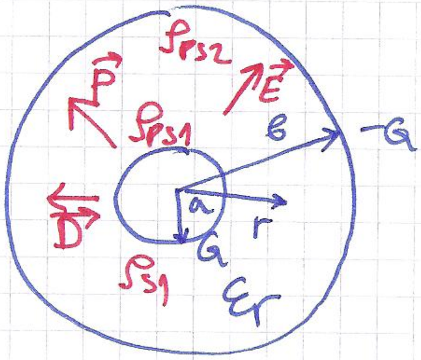
$$\Rightarrow \varepsilon_{r1} = 12$$

$$\Downarrow$$

$$\boxed{\varepsilon_{r2} = \frac{4}{3}}$$

(172) SFERNI KONDENZATOR $\varepsilon_r > 1$

POLOVINA DIELEKTRIKA ISCURI



$$D \cdot 4\pi r^2 = Q \Rightarrow D = \frac{Q}{4\pi r^2}$$

$$E(r) = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{Q}{4\pi \varepsilon_0 \varepsilon_r r^2}$$

$$P(r) = \varepsilon_0(\varepsilon_r - 1)E(r) = \frac{\varepsilon_r - 1}{\varepsilon_r} \frac{Q}{4\pi r^2}$$

$$\rho_{s1} = D(a) = \frac{Q}{4\pi a^2}$$

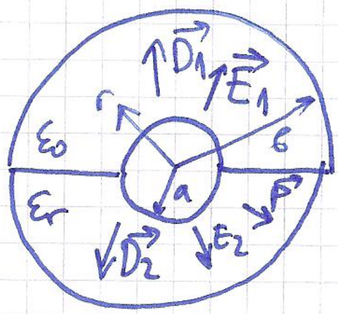
$$\rho_{s2} = -D(b) = -\frac{Q}{4\pi b^2}$$

POVRŠINSKE GUSTINE SLOBODNIH NAELEKTRISANJA ELEKTRODA

$$\rho_{ps1} = -P(a) = -\frac{\varepsilon_r - 1}{\varepsilon_r} \frac{Q}{4\pi a^2}$$

$$\rho_{ps2} = P(b) = \frac{\varepsilon_r - 1}{\varepsilon_r} \frac{Q}{4\pi b^2}$$

DIELEKTRIK ISCURI DO POLOVINE



IZ GRANIČNIH USLOVA

IZRAZ ZA $E(r)$ JE ISTI

U OBA OKRUŽENJA

$$D_1 = \varepsilon_0 E(r)$$

$$D_2 = \varepsilon_0 \varepsilon_r E(r)$$

$$P(r) = \varepsilon_0(\varepsilon_r - 1)E(r)$$

$$\varepsilon_0 E(r) 2\pi r^2 + \varepsilon_0 \varepsilon_r E(r) 2\pi r^2 = Q$$

$$E(r) = \frac{Q}{2\pi\varepsilon_0(\varepsilon_r+1)r^2}$$

$$D_1(r) = \varepsilon_0 E(r) = \frac{Q}{2\pi(\varepsilon_r+1)r^2}$$ VAZDUH

$$D_2(r) = \varepsilon_0 \varepsilon_r E(r) = \frac{\varepsilon_r Q}{2\pi(\varepsilon_r+1)r^2}$$ DIELEKTRIK

$$P(r) = \varepsilon_0(\varepsilon_r-1)E(r) = \frac{(\varepsilon_r-1)Q}{2\pi(\varepsilon_r+1)r^2}$$

$$\rho_{s1}^{(1)} = D_1(a) = \dots$$

$$\rho_{s2}^{(2)} = D_2(a) = \dots$$

$$\frac{\rho_{s1}^{(1)}}{\rho_{s1}} = \frac{2}{\varepsilon_r+1} < 1 \qquad \frac{\rho_{s1}^{(2)}}{\rho_{s1}} = \frac{2\varepsilon_r}{\varepsilon_r+1} > 1$$

$$\rho_{s2}^{(1)} = -D_1(b) = \dots$$

$$\rho_{s2}^{(2)} = -D_2(b) = \dots$$

$$\frac{\rho_{s2}^{(1)}}{\rho_{s2}} = \frac{2}{\varepsilon_r+1} < 1 \qquad \frac{\rho_{s2}^{(2)}}{\rho_{s2}} = \frac{2\varepsilon_r}{\varepsilon_r+1} > 1$$

$$\rho_{ps1}^{(2)} = -P(a) = \dots$$ UZ GORNJU POLOVINU OBE ELEKTRODE NEMA VEZANIH NAELEKTRISANJA

$$\rho_{ps2}^{(2)} = P(b) = \frac{(\varepsilon_r-1)Q}{2\pi(\varepsilon_r+1)b^2}$$

176. KOAKSIJALNI VOD

D - PROBLEM

$$D \cdot 2\pi r \cancel{l} = Q' \cancel{l}$$

$$D = \frac{Q'}{2\pi r} = D(r)$$

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r1}} = \frac{D(r)}{\varepsilon_0 \varepsilon_{r1}} = \frac{Q'}{2\pi \varepsilon_0 \varepsilon_{r1} r} = \frac{C'U}{2\pi \varepsilon_0 \varepsilon_{r2} r}$$

$$E_2 = \frac{Q'}{2\pi \varepsilon_0 \varepsilon_{r2} r} = \frac{C'U}{2\pi \varepsilon_0 \varepsilon_{r2} r}$$

$$E_{1MAX} = E_{KR1} = \frac{C'U}{2\pi \varepsilon_0 \varepsilon_{r1} a}$$

$$E_{2MAX} = E_{KR2} = \frac{C'U}{2\pi \varepsilon_0 \varepsilon_{r2} b}$$

$$U_{KR1} = U_{KR2}$$

$$\frac{\cancel{2\pi \varepsilon_0} \varepsilon_{r1} a}{\cancel{C'}} E_{KR1} = \frac{\cancel{2\pi \varepsilon_0} \varepsilon_{r2} b}{\cancel{C'}} E_{KR2}$$

$$\Downarrow$$

$$\boxed{E_{KR1} \varepsilon_{r1} a = E_{KR2} \varepsilon_{r2} b}$$

TRAŽENA VEZA

*178. KOAKSIJALNI VOD , $a, b, c \ll h$

$\varepsilon_{r1}, \varepsilon_{r2}$

$$U = \int_a^c \vec{E}(r) d\vec{r}$$

$D_1 = D_2 = D$

$E_1 \neq E_2$

$$\oint \vec{D} \cdot d\vec{S} = Q$$

$$DS = Q \Rightarrow D = \frac{Q}{S} = \frac{Q}{2\pi r h}$$

$$E_1(r) = \frac{D}{\varepsilon_1} = \frac{D}{2\pi \varepsilon_0 \varepsilon_{r1} h r}$$

$$E_2(r) = \frac{D}{\varepsilon_2} = \frac{D}{2\pi \varepsilon_0 \varepsilon_{r2} h r}$$

$$U = \int_a^b E_1(r) dr + \int_b^c E_2(r) dr$$

$$U = \frac{Q}{2\pi h \varepsilon_0} \left(\frac{1}{\varepsilon_{r1}} \int_a^b \frac{dr}{r} + \frac{1}{\varepsilon_{r2}} \int_b^c \frac{dr}{r} \right) = \frac{Q}{2\pi h \varepsilon_0} \left(\frac{1}{\varepsilon_r} \ln \frac{b}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{c}{b} \right.$$

$$C = \frac{Q}{U} = \frac{2\pi h \varepsilon_0}{\frac{1}{\varepsilon_{r1}} \ln \frac{b}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{c}{b}} \Rightarrow C' = \frac{C}{h} = \frac{2\pi \varepsilon_0}{\frac{1}{\varepsilon_{r1}} \ln \frac{b}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{c}{b}}$$

$$Q'^{(1)}_{kr} = 2\pi \varepsilon_{r1} \varepsilon_0 a E_{kr1}$$

$$U^{(1)}_{kr} = \frac{Q'}{C'} = \varepsilon_{r1} E_{kr1} a \left(\frac{1}{\varepsilon_{r1}} \ln \frac{b}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{c}{b} \right)$$

$$U^{(2)}_{kr} = \varepsilon_{r2} E_{kr2} b \left(\frac{1}{\varepsilon_{r1}} \ln \frac{b}{a} + \frac{1}{\varepsilon_{r2}} \ln \frac{c}{b} \right)$$

$$U_{kr} = \min \left\{ U^{(1)}_{kr}, U^{(2)}_{kr} \right\}$$

181. $a, b,$ E - PROBLEM

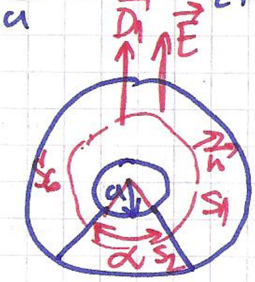
I) SAMO VAZDUH

$D_0 \cdot S = Q' h$

$D_0 2\pi r h = Q' h$

$D_0 = \frac{Q'}{2\pi r} \Rightarrow E_0(r) = \frac{Q'}{2\pi\varepsilon_0 r} \Rightarrow U = \int_a^b E(r)dr = \frac{Q'}{2\pi\varepsilon_0} \ln\frac{b}{a}$

$C_0' = \frac{Q'}{U} = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}$



II) UBAČEN DIELEKTRIK

$E_1 = E_2 \quad , D_1 \neq D_2$

$D_1 S_1 + D_2 S_2 = Q' h$

$D_1(r) = \varepsilon_0 E(r)$

$D_2(r) = \varepsilon_0 \varepsilon_r E(r)$

$E(r) = \frac{Q'}{2\pi\varepsilon_0\left(1 + (\varepsilon_r - 1)\frac{\alpha}{2\pi}\right) r}, \quad r \in (a, b)$

$U = \int_a^b E(r)dr = \frac{Q'}{2\pi\varepsilon_0\left(1 + (\varepsilon_r - 1)\frac{\alpha}{2\pi}\right)\cancel{r}} \ln\frac{b}{a}$

$C' = \frac{Q'}{U} = \frac{2\pi\varepsilon_0\left(1 + (\varepsilon_r - 1)\frac{\alpha}{2\pi}\right)}{\ln\frac{b}{a}}$

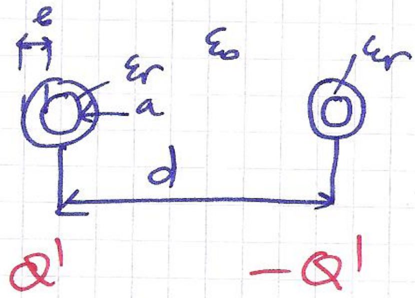
$\Delta C' = C' - C_0' = \frac{(\varepsilon_r - 1)\varepsilon_0 \alpha}{\ln\frac{b}{a}}$

$S_1 = S_b - S_2$

$S_1 = 2\pi r h - r\alpha h = 2\pi r h\left(1 - \frac{\alpha}{2\pi}\right)$

$S_2 = \cancel{2\pi} r h \frac{\alpha}{\cancel{2\pi}} = r h \alpha$

182.



$d \gg a, b$

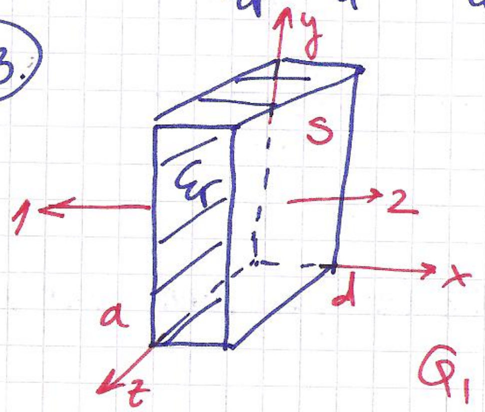
$$D \cdot 2\pi r h = Q' h \Rightarrow D = \frac{Q}{2\pi r}, \; r > a + b$$

$$E = \begin{cases} \dfrac{Q'}{2\pi\varepsilon_r\varepsilon_0 r}, & a < r < a+b \\ \dfrac{Q'}{2\pi\varepsilon_0 r}, & r > a+b \end{cases}$$

$$U_{12} = \int_1^2 E(r)\,dr = \frac{Q'}{\pi\varepsilon_0}\left(\frac{1}{\varepsilon_r}\ln\frac{a+b}{a} + \ln\frac{d}{a+b}\right), \; d \gg a, b$$

$$C' = \frac{Q'}{U} = \frac{\pi\varepsilon_0}{\frac{1}{\varepsilon_r}\ln\frac{a+b}{a} + \ln\frac{d}{a+b}}$$

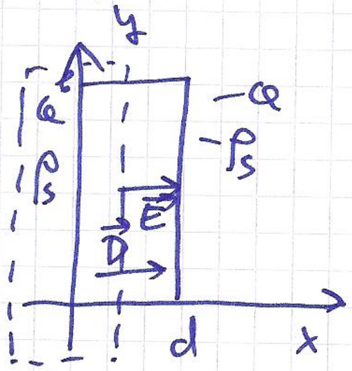
***183.**



$Q, -Q$

$\vec{D}$ i $\vec{E}$ NORMALNI NA ELEKTRODE

$\varepsilon_r = \varepsilon_r(x)$



POLJE VEKTORA D JE HOMOGENO

D – PROBLEM
(IZ GRANIČNIH USLOVA)

$$\vec{E} = \frac{\vec{D}}{\varepsilon_r(x)\varepsilon_0} \qquad D_x = \rho_s = \frac{Q}{S}$$

$$E_x(x) = \frac{D_x}{\varepsilon_r(x)\varepsilon_0} = \frac{Q}{\varepsilon_r(x)\varepsilon_0 S}$$

$$U_{12} = \int_1^2 E_x(x)dx = \frac{Q}{\varepsilon_0 S}\int_0^d \frac{dx}{\varepsilon_r(x)}$$

$$C = \frac{Q}{U} = \frac{\varepsilon_0 S}{\int_0^d \frac{dx}{\varepsilon_r(x)}}$$

II NAČIN – KAO REDNA VEZA MALIH KONDENZATORA

б) KADA ZAVISI OD y-OSE $\varepsilon_r = \varepsilon_r(y)$

$$\vec{E_x} = E_x \vec{i_x} = \vec{E}$$

$$\vec{D}(y) = \varepsilon_0 \varepsilon_r(y)\vec{E} = \varepsilon_0 \varepsilon_r(y) E_x \vec{i_x} = D_x(y)\vec{i_x}$$

$$\int_0^b \varepsilon_0 \varepsilon_r(y) E_x a\, dy = Q \Rightarrow E_x = \frac{Q}{a\varepsilon_0 \int_0^b \varepsilon_r(y)dy}, \quad 0<x<d$$

$$U_{12} = E_x d = \frac{Qd}{a\varepsilon_0 \int_0^b \varepsilon_r(y)dy} \Rightarrow C = \frac{Q}{U_{12}} = \dots$$

v) KADA ZAVISI OD z-OSE

$$C = C_0 \frac{1}{a}\int_a^b \varepsilon_r(z)dz$$, POSTUPAK KAO POD б.

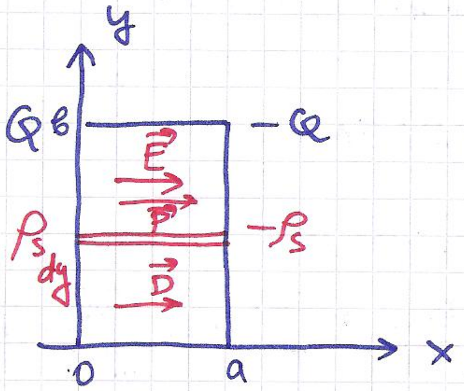
186.

$\varepsilon_r(y)=1+\frac{y}{b}$ $\quad \underline{a\ y, b} \quad 0<y<b$

a, b, Q, d

a) $C=?$

b) $\rho_{ps}=?$ $Q_p=?$



E-PROBLEM

$\vec{D}=\varepsilon_r(y)\varepsilon_0\vec{E}$

$dS=a\,dy$

$$\oint_S \vec{D}\,d\vec{S}=Q$$

$$\oint_S D\,dS=Q$$

$$\int_{y=0}^{b}\varepsilon_0\varepsilon_r(y)E\,a\,dy=Q$$

$$\varepsilon_0 E a\int_0^b\left(1+\frac{y}{b}\right)=Q$$

$$E=\frac{Q}{\varepsilon_0 ab\cdot\frac{3}{2}} \Rightarrow U=E\cdot d=\frac{Qd}{\varepsilon_0 ab\frac{3}{2}}\ ;\ C=\frac{Q}{U}$$

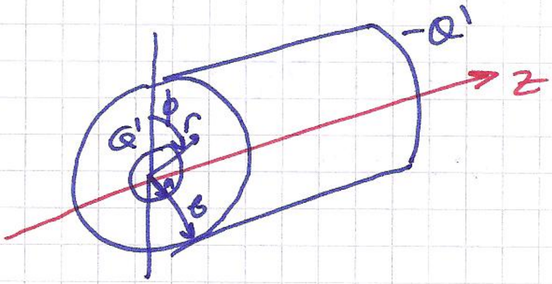
$$\boxed{C=\frac{3}{2}\frac{\varepsilon_0 ab}{d}}$$

b) $\vec{P}=\varepsilon_0(\varepsilon_r(y)-1)\vec{E}=\frac{2}{3}\frac{y}{b}\frac{Q}{ab}\vec{i}_x$

$\rho_{ps1}(y)=-\rho_{ps2}(y)=-P(y)$

$Q_{p1}=-Q_{p2}=-a\int_0^b P(y)\,dy=-\frac{Q}{3}$

*187.



a) $\varepsilon_r(r)$

b) $\varepsilon_r(\phi)$

a) $\vec{D} = D_r \vec{2r} \quad \vec{E} = E_r \vec{2r}$

D-PROBLEM

$$D \cdot 2\pi r \cancel{h} = Q' \cancel{h}$$

$$D = \frac{Q'}{2\pi r} \Rightarrow \vec{D}(r) = \frac{Q'}{2\pi r}\vec{2r}$$

$$\vec{E}(r) = \frac{Q'}{2\pi \varepsilon_r(r)\varepsilon_0 r}\vec{2r} \qquad a<r<b$$

$$U_{12} = \int_a^b E(r)dr = \frac{Q'}{2\pi\varepsilon_0}\int_a^b \frac{dr}{\varepsilon_r(r)\cdot r}$$

$$\Downarrow$$

$$C' = \frac{Q'}{U_{12}} = \dots$$

b) E-PROBLEM

$$\vec{D} = \varepsilon_r(\phi)\varepsilon_0\vec{E}$$

$$\int_0^{2\pi} D_r(r,\phi)\cdot \underbrace{h r d\phi}_{dS} = Q'h$$

$$\int_0^{2\pi} \varepsilon_r\phi\,\varepsilon_0 E \cancel{h} r d\phi = Q'\cancel{h}$$

$$E(r) = \frac{Q'}{\varepsilon_0 r\int_0^{2\pi}\varepsilon_r\phi\, d\phi} \Rightarrow U_{12} = \int_a^b E(r)dr = \dots$$

$$C' = \frac{Q'}{U_{12}} = \dots$$

(188.) KOAKSIJALNI VOD $\varepsilon_r(r)=\varepsilon_r(b)\frac{b}{r}$

D- PROBLEM

IZ GRANIČNIH USLOVA

$$D\cdot 2\pi r \cancel{h} = Q' \cancel{h}$$

$$D=\frac{Q'}{2\pi r} \Rightarrow E=\frac{Q'}{2\pi r\varepsilon_0\varepsilon_r(r)}=\frac{Q'}{2\pi\varepsilon_0\varepsilon_r(b)\frac{b}{\cancel{r}}\cdot\cancel{r}}$$

$$U=\int_a^b E(r)dr=\int_a^b\frac{Q'}{2\pi\varepsilon_0\varepsilon_r(b)b}dr=\frac{Q'}{2\pi\varepsilon_0\varepsilon_r(b)}\, r\Big|_a^b=\frac{Q'(b-a)}{2\pi\varepsilon_0\varepsilon_r(b)b}$$

$$\boxed{C'=\frac{Q'}{U}=\frac{2\pi\varepsilon_0\varepsilon_r(b)b}{b-a}}$$

(192.)

$$\varepsilon_r(r)=\frac{1}{2}\varepsilon_r(a)\left(1+\frac{a}{r}\right) \quad r>a$$

a) $C=?$

b) $\rho_s=?$, V

D-PROBLEM

$$D\cdot 4\pi r^2 = Q \quad D(r)=\frac{Q}{4\pi r^2}$$

$$E(r)=\frac{D(r)}{\varepsilon_r(r)\varepsilon_0}=\frac{Q}{4\pi r^2\varepsilon_0\varepsilon_r(r)}=\frac{Q}{2\pi r^2\varepsilon_0\varepsilon_r(a)(1+\frac{a}{r})}$$

$$V=\int_a^\infty E(r)dr=\frac{Q}{2\pi\varepsilon_0\varepsilon_r(a)}\int_a^\infty\frac{dr}{r(a+r)}=\left\{\frac{1}{a}\left(\frac{1}{r}-\frac{1}{r+a}\right)\right.$$

$$V=\frac{Q}{2\pi\varepsilon_0\varepsilon_r(a)a}\left(\ln\frac{Ref}{a}-\ln\frac{Ref+a}{2a}\right) \qquad Reff>>a$$

$$\ln\frac{\frac{\cancel{Ref}}{\cancel{a}}}{\frac{\cancel{Ref}}{2\cancel{a}}}=\ln 2$$

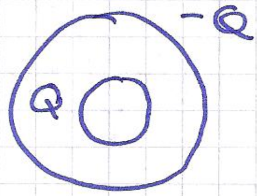
$$V=\frac{Q}{2\pi\varepsilon_0\varepsilon_r(a)a}\ln 2$$

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 \varepsilon_r(a) a}{\ln 2}$$

$$б) \quad \rho_S = \frac{Q}{4\pi a^2} = \frac{\varepsilon_0 \varepsilon_r(a) V}{2a \ln 2}$$

*193. SFERNI KONDENZATOR, a, b $\quad \varepsilon_r = \varepsilon_r(r)$

D-PROBLEM



$$D(r) = \frac{Q}{4\pi r^2} \qquad a < r < b$$

$$\vec{E} = E(r)\vec{i_r}$$

$$\vec{D} = D(r)\vec{i_r}$$

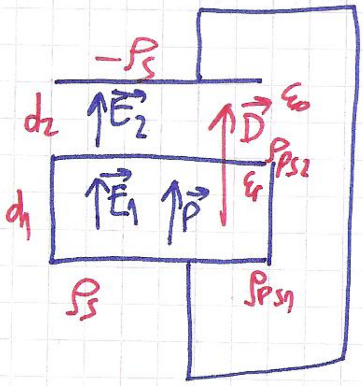
$$E(r) = \frac{Q}{4\pi \varepsilon_r(r) \varepsilon_0 r^2}$$

$$U_{12} = \int_a^b E(r)dr = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{\varepsilon_r(r) r^2}$$

$$C = \frac{Q}{U_{12}} = \frac{4\pi\varepsilon_0}{\int_a^b \frac{dr}{\varepsilon_r(r) r^2}}$$

196.

D-PROBLEM



$$D \cdot S = Q$$

$$D \cdot \cancel{S} = \rho_s \int dS = \rho_s \cancel{S}$$

$$D = \rho_s$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\left.\begin{array}{l} \rho_s = \varepsilon_0 E_1 + P_0 \\ \rho_s = \varepsilon_0 E_2 \end{array}\right\} \Rightarrow \int_0^{d_1} E_1 \, dz + \int_{d_1}^{d_1+d_2} E_2 \, dz = 0$$

NA ISTOM POTENCIJALU

$$E_1 = \frac{\rho_s - P_0}{\varepsilon_0}$$

$$E_2 = \frac{\rho_s}{\varepsilon_0}$$

$$\frac{\rho_s - P_0}{\varepsilon_0} d_1 + \frac{\rho_s}{\varepsilon_0} d_2 = 0$$

$$\Downarrow$$

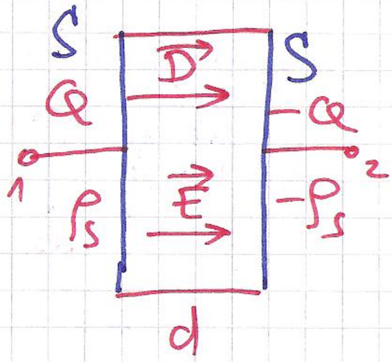
$$\rho_s = \frac{P_0 d_1}{d_1 + d_2} \Rightarrow -\rho_s = -\frac{P_0 d_1}{d_1 + d_2}$$

б) $\vec{E}_1 = -\frac{d_2 P_0}{\varepsilon_0 (d_1 + d_1)} \vec{i}_z$

$\vec{E}_2 = \frac{d_1 P_0}{\varepsilon_0 (d_1 + d_2)} \vec{i}_z$

v) $\rho_{ps1} = -P_0$, $\rho_{ps2} = P_0$

(200.) PLOČASTI KONDENZATOR , S, d, ε_r $W_e = ?$



$$D = \rho_s = \frac{Q}{S}$$

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{Q}{\varepsilon_0 \varepsilon_r S} \qquad w_e = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}DE = \frac{1}{2}\varepsilon_0\varepsilon_r E^2 = \frac{1}{2}\frac{Q^2}{\varepsilon_0\varepsilon_r S^2}$$

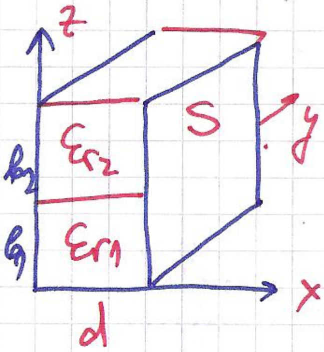
$$= \frac{1}{2}\frac{\rho_s^2}{\varepsilon_0\varepsilon_r}$$

$$W_e = \int_V w_e \, dV = w_e \int_V dV = w_e \cdot V = w_e S d$$

$$W_e = \frac{1}{2}\frac{Q^2 d}{\varepsilon_0 \varepsilon_r S} \quad C$$

$$W_e = \frac{1}{2} Q \cdot U = \frac{1}{2} C \cdot U^2 = \frac{1}{2}\frac{Q^2}{C}$$

(202.)



$$E = \frac{U}{d}$$

E-PROBLEM

$$D_1 = \varepsilon_0 \varepsilon_{r1} E = \frac{U}{d}\varepsilon_0\varepsilon_{r1} \quad , \ z \in (0, b_1)$$

$$D_2 = \varepsilon_0 \varepsilon_{r2} E = \frac{U}{d}\varepsilon_0\varepsilon_{r2} \quad , \ z \in (b_1, b_1 + b_2)$$

$$w_{e1} = \frac{1}{2}\vec{D}_1\cdot\vec{E} = \frac{1}{2} D_1 E = \frac{1}{2}\varepsilon_0\varepsilon_{r1}\frac{U^2}{d^2} \qquad w_{e2} = \frac{1}{2}\varepsilon_0\varepsilon_{r2}\frac{U^2}{d^2}$$

$$W_e = W_{e1} + W_{e2} = \frac{1}{2}\varepsilon_0 \frac{U^2}{d}(\varepsilon_{r1} S_1 + \varepsilon_{r2} S_2)$$

$$S_1 = \frac{b_1}{b_1 + b_2} S \ ; \ S_2 = \frac{b_2}{b_1 + b_2} S$$